Opportunities for Cognitive Activation

Intended, Enacted, and Experienced Practices in Mathematics Education

Jimmy Karlsson

Faculty of Health, Science and Technology

Mathematics

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Abstract

At the core of mathematics education lies the development of students' mathematical thinking. In classroom research, opportunities for such thinking can be conceptualised through cognitive activation, which entails practices that support students' engagement with challenging tasks, subject discourse, and reasoning. The aim of this thesis is to explore cognitive activation as a multidimensional opportunity structure in mathematics classrooms, focusing on intended, enacted, and experienced opportunities, and on how students' experienced opportunities relate to self-efficacy, test anxiety, and mathematical achievement.

The thesis consists of four empirical papers conducted in Swedish secondary mathematics education. It examines classroom-level and individually perceived cognitive activation in relation to self-efficacy, test anxiety, and achievement. Furthermore, it explores teachers' intended facilitation in lesson plans involving a challenging task, and how observed instructional features co-occur to form lesson segment types across lessons and classrooms.

The findings show that cognitive activation can be understood as an opportunity structure constituted by interrelated dimensions, including task and interaction demands, teachers' facilitation, and subject discourse. Experienced cognitive activation was positively associated with self-efficacy, and self-efficacy may mediate the relation between cognitive activation and achievement. Teachers' facilitation of challenging tasks, analysed as regulation of learning, varied across planned instructional events, and enacted opportunities formed distinct lesson segment types in terms of cognitive activation and instructional clarity, working format, and lesson phase.

Overall, the thesis contributes to how cognitive activation can be theorised and studied as a multidimensional opportunity structure, and offers insights into how it can inform teachers' didactical decision-making.

Foreword from the management of the graduate school

Jimmy Karlsson's thesis Opportunities for Cognitive Activation: Intended, Enacted, and Experienced Practices in Mathematics Education is a product of the KÄKK graduate school – a collaboration between Karlstad University, Halmstad University and University West aimed at teacher educators. The graduate school is funded via a project grant from the Swedish Research Council (ref. 2019-04419).

KÄKK's ambition is to strengthen the scientific underpinnings of teacher education by honing teacher educators' and teachers' basic competence in and understanding of how subject knowledge can be transformed into teaching content.

The aim of the graduate school is to promote the theoretical development of primary and secondary teacher education in Sweden and internationally via a practice-oriented approach. This is done by focusing on how different aspects of subject transformation can occur in different subjects and stages of education.

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List of papers

Paper 1. Karlsson, J., Liljekvist, Y. Cognitive Activation and Links to Mathematics Learning. [Under review]

Paper 2. Karlsson, J. (2025). Exploring the Relationships Between Individual Cognitive Activation, Self-efficacy and Test Anxiety in Upper Secondary Mathematics. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.), *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education: Research Reports* (Vol. 2). PME.

Paper 3. Karlsson, J. & Liljekvist, Y. Regulation of Learning in Lesson Planning around a Challenging Task. [Manuscript]

Paper 4. Karlsson, J. & van Bommel, J. Unpacking Mathematics Lessons: A Latent Class Analysis of Instructional Features Within and Across Lessons and Classrooms. [Submitted]

Declaration of author contributions

For Papers 1–3, I was responsible for the research design, instrument development, data generation, and analyses. In addition, for Paper 3, I was responsible for developing the coding framework, while both authors were involved in coding, revising coding framework and interpreting the analysis. For Paper 4, I was responsible for conceptualisation, formal analysis and writing the original draft, while van Bommel was responsible for data curation and formal analysis related to the PLATO framework. For co-authored papers, all authors reviewed and revised the manuscripts.

1 Introduction

At the core of mathematics education lies development of mathematical thinking. Mathematical thinking provides means for students to engage with the world empowered by the tools of reasoning from the mathematics discipline. As such, mathematical thinking is powerful knowledge when working with ideas and conjectures, making connections and giving reasons for why something is true. In classrooms, this becomes visible when students solve problems, explain solution strategies, justify claims, and evaluate alternative approaches.

However, developing this form of thinking is demanding. It requires moving beyond right and wrong answers, towards reasoning and justification on mathematical grounds. Further, it is shaped in interaction, through what is expected, what counts as an acceptable explanation, and engagement in practices with potential to develop reasoning.

This makes teaching central. Mathematical instruction influences whether students encounter tasks that invite them to engage in problem solving, whether they are expected to explain their reasoning, and whether the classroom discourse supports sense-making and reasoning. Hence, teaching and interaction around content create opportunities for engaging with mathematics, which act as structured ways for students to engage in, and develop, mathematical ways of thinking.

In mathematics education, these opportunities have been conceptualised through the construct of cognitive activation. Cognitive activation has been characterised by classroom practices involving challenging activities and participation in discourse that requires explanation, justification and comparison of solution strategies, and mathematical reasoning (Mu et al., 2022; Praetorius et al., 2018; Spreitzer et al., 2022). Cognitive activation can therefore be understood as an opportunity structure for mathematical thinking: what students are invited and required to do with mathematical ideas. Its focus lies in thinking processes, moving beyond behavioural activity towards deep cognitive engagement with central mathematical ideas.

Despite being identified as an essential dimension of teaching quality, cognitive activation poses several challenges in conceptualisation, measurement, and explanations for relations to student outcomes (Praetorius et al., 2020). As Praetorius et al. (2014, p. 14) argue, “In the long term, it is of paramount importance to advance our theoretical understanding of cognitive activation".

First, cognitive activation spans several dimensions, such as: challenging tasks, challenging questions, exploration of students' thinking, and support for metacognition (Mu et al., 2022; Praetorius et al., 2018) giving a range of conceptual foundations and operationalisations (Herbert et al., 2024; Mu et al., 2022). This has given rise to questions about which dimensions are central (Lipowsky et al., 2009; Wemmer-Rogh et al., 2024) and to calls for greater clarity regarding its conceptual core and theoretical groundings (Praetorius et al., 2020; Wemmer-Rogh et al., 2024).

Secondly, cognitive activation is commonly framed as an opportunity provided through teaching and related to student outcomes through students' use of these opportunities (Praetorius et al., 2020). The assumed mechanism is through students' depth of processing; however, evidence for the mediating pathways remains mixed (Alp Christ et al., 2024; Praetorius et al., 2020). Depth of processing has been identified as under-specified in research (Praetorius et al., 2020) and other pathways via motivational dimensions have been suggested for examination (Alp Christ et al., 2024; Praetorius et al., 2020) where self-related perceptions such as self-efficacy can be explored as one possible pathway.

Third, questions have been raised regarding cognitive activation and its place in different types of lessons and lesson sequences. For example, Praetorius et al. (2014) contrasted introducing new content with a consolidation phase, and suggested that cognitive activation might not be suitable for all kinds of phases. Hence, it would be worthwhile to also explore how cognitive activation dimensions could shift across lessons.

Fourth, cognitive activation pertains to one domain of mathematics opportunities in classrooms related to cognitive engagement. However, classroom practices focusing on content selection, representation and instructional clarity have also been identified as important for teaching quality (Charalambous & Praetorius, 2018; Klette et al., 2017; Nilsen & Gustafsson, 2016). As such, how the subject discourse is established depends on teachers' facilitation of cognitive activation (Mu et al., 2022) but also on how content is selected and presented. Just providing challenging tasks is not sufficient, and cognitive activation might also benefit from clarity in instruction (Pauli et al., 2008).

Teaching is inherently complex and research typically reduces dimensions into multiple constructs, but these are rarely cleanly separable or independent (Praetorius et al., 2024; Vieluf & Klieme, 2023). Since the use of multiple constructs is both necessary and challenging, examining them in conjunction can provide further understanding of how they relate to the constitution of productive classroom practices. For example, teachers' facilitation of cognitive activation also relates to how they intend to regulate students' learning by selecting tasks, presenting content, organising the learning environment and the type of expectations for students' participation. As cognitive activation together with content-specific instructional clarity has been found beneficial for students' learning gains (Pauli et al., 2008), how teachers facilitate cognitively activating classroom practices can also be examined together with other content-specific features.

Taken together, these issues point to a need for research that (i) addresses how dimensions of cognitive activation are constituted in classroom practices, (ii) examines mechanisms linking cognitive activation to student outcomes, and (iii) explores how cognitively activating practices could vary across lessons and co-occur with other instructional features.

Therefore, this thesis sets out to empirically explore the multidimensional construct of cognitive activation, framed as opportunity structures formed in classrooms that can support students to develop mathematical thinking.

1.1 Aim

The aim of this thesis is to contribute to understanding of cognitive activation as a multidimensional opportunity structure in mathematics classrooms, by examining how such opportunities are constituted in teaching and experienced by students, and how students' experienced opportunities relate to mathematics achievement and self-related perceptions.

To approach the aim, two overarching questions are posed:

- How are opportunities for cognitively activating classroom practices intended, enacted, and experienced in mathematics classrooms?
- How are experienced opportunities for cognitively activating classroom practices related to self-related perceptions and mathematics achievement?

1.2 Thesis structure and overview

The kappa should be read as the integrative frame of the thesis. It first introduces the overarching aim, research questions, and background. The methodology chapter then explains the design, empirical context, and different approaches of the papers. The results chapter summarises the findings of each paper. The kappa concludes with a discussion chapter, which addresses the research questions and discusses the findings in relation to contributions, limitations, implications, and suggested directions for further research.

Across four papers, the thesis examines cognitive activation as a multidimensional opportunity structure. The papers provide the full empirical studies that form the foundation of the thesis. Furthermore, the papers are complementary as they approach cognitive activation through intended, enacted, and experienced perspectives. Thus, the four papers have different empirical bases and analytical approaches, which together contribute to the thesis aim and research questions.

Paper 1 addresses the experienced perspective and its links to students' outcomes by examining how students' perceived classroom-level cognitive activation relates to self-efficacy, test anxiety and achievement. Paper 1 also examines both direct and indirect relationships via mediation and moderation.

Paper 2 further examines the experienced perspective but shifts the referent from the teacher/classroom to the individual student to examine individually perceived cognitive activation. The paper examines dimensions of individual cognitive activation and relations to self-efficacy and test anxiety.

Paper 3 primarily focuses on the intended perspective via teachers' lesson planning around a challenging task. The paper further explores potential relations between teachers' intended regulation and students' perceptions of classroom practices.

Paper 4 addresses the enacted perspective by using classroom observation to identify types of mathematics lesson segments based on co-occurring instructional features related to cognitive activation and instructional clarity together with working format. Additionally, the paper also explores how lesson segment types vary across lesson phases and classrooms. An overview of the approaches and main contributions of each paper is given in Table 1.

Table 1. Summary of perspective, empirical focus, and main contribution to the research questions for each paper.

Paper 1	
Perspective	Experienced, classroom-level
Empirical focus	Aggregated student perceptions, achievement, self-efficacy, test anxiety
Main contribution	Examines relations between classroom-level cognitive activation and outcomes, contributing to RQ2
Paper 2	
Perspective	Experienced, individual-level
Empirical focus	Individual perceived cognitive activation, self-efficacy, test anxiety
Main contribution	Distinguishes dimensions of experienced cognitive activation and links to self-efficacy and test anxiety, contributing to RQ1 and RQ2
Paper 3	
Perspective	Intended
Empirical focus	Teachers’ lesson plans around a challenging task
Main contribution	Examines intended regulation and facilitation of cognitive activation, contributing to RQ1
Paper 4	
Perspective	Enacted
Empirical focus	Observed lesson segments rated on instructional features
Main contribution	Identifies segment types and patterns across lesson phases and classrooms, contributing to RQ1

1.2.1 Delimitations

In this thesis, an important delimitation pertains to the alignment between the three perspectives on cognitive activation as an opportunity structure. While the papers address the perspectives, no direct alignment is possible within the current scope of the thesis. Hence, the papers are jointly contributing through a conceptual triangulation, and are not directly aligned across the same classrooms, lessons or instructional units.

Another delimitation is the term self-related perceptions which is used as an umbrella for students' self-reported beliefs and perceptions of affective experiences related to mathematics. Empirically, this domain is delimited to self-efficacy and mathematics test anxiety. Hence, the thesis makes no broader claim related to students' affective or motivational domains than what is empirically examined in the papers.

2 Background

Mathematics can be described as constituted by ways of understanding and ways of thinking (Harel, 2008). Ways of understanding consist of all institutionalised products such as definitions, theorems, and proofs. Ways of thinking are the characteristics of the mental acts through which ways of understanding are generated, justified, and used. The most central process of mathematical thinking pertains to reasoning, underpinned by practices of justifying, generalising, and connecting ideas on mathematical grounds (Harel, 2008; Lithner, 2008). Hence, mathematical thinking, and in particular reasoning, can be regarded as powerful knowledge (Young & Muller, 2016) that is specialised, context-independent and enables new ways of thinking beyond everyday experience.

However, disciplinary mathematics (i.e., the institutionalised knowledge) requires transformation for teaching and student learning (Gericke et al., 2018; Harel, 2008). For example, deductive formal reasoning as in proof schemes might not be readily available for younger students (Harel, 2008) and as such the powerful knowledge of mathematics needs to undergo transformation processes for teaching and learning in schools (Gericke et al., 2018). First, there is a transformation process in the planning phase, when teachers select content and how it will be represented in a lesson. Second, another transformation process occurs in the teaching and learning situation, when teachers and students interact around the content (Gericke et al., 2018). Thus, teaching is central in transforming the powerful ways of thinking in mathematics to be accessible to students in mathematics classrooms. Accordingly, teaching is highly influential in forming opportunities to learn mathematics through the classroom activities by which students are invited to engage with practices such as explanation, justification, and sense-making.

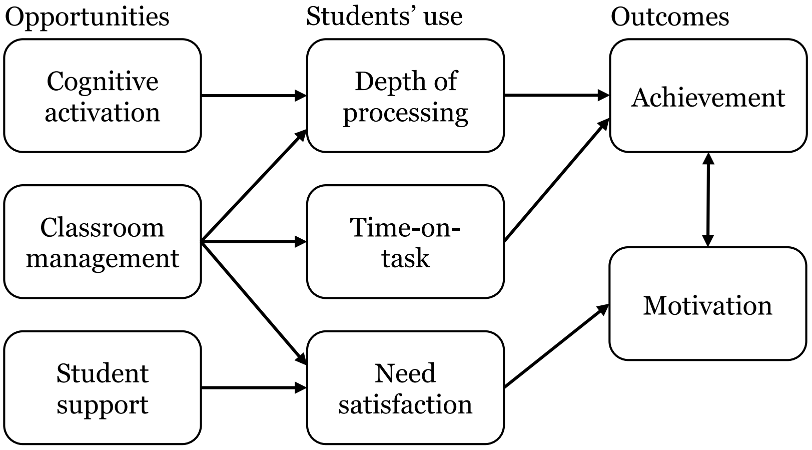
In this thesis, opportunities for engaging in mathematical thinking are approached through the construct of cognitive activation. Cognitive activation is framed as an opportunity structure for students to engage in higher-level thinking by classroom practices that invite students to connect mathematical ideas, explain and compare solution strategies, justify claims, and engage in problem-solving (Lipowsky et al., 2009;

Mu et al., 2022; Praetorius et al., 2018). Furthermore, by the framing of opportunity, the quality of teaching can be understood as an offer, which can vary depending on the students' use (Charalambous, Grob, et al., 2025). This framing also makes it possible to distinguish between opportunities that are intended, enacted, and experienced, and between opportunities offered through interactions between students, the content, and the teacher, and students' use of these opportunities.

2.1 Three Basic Dimensions of teaching quality

The construct of cognitive activation was first introduced as one of Three Basic Dimensions of teaching quality (Klieme et al., 2001, Praetorius et al., 2020). The framework was developed as a parsimonious model for describing essential features of teaching quality, through three deep structures, see Figure 1. The framework was first established through an exploratory factor analysis by Klieme et al. (2001) for the German extension of the Third International Mathematics and Science Study (TIMSS). In this work, Klieme et al. (2001) were able to reduce 21 scales into three separate dimensions: classroom management, student support, and cognitive activation (Praetorius et al., 2020). The dimension of classroom management focuses on student behaviour, supporting positive and reducing negative behaviour. It includes aspects such as effective use of time, communicating rules, and establishing routines. The dimension of student support focuses on classroom climate, including positive relationships in classrooms and teachers helping student learning. Since the introduction of Three Basic Dimensions in German speaking contexts, it has been used in other contexts and included in large-scale studies such as PISA (Klieme & Nilsen, 2022; Schreyer & Charalambous, 2024).

Figure 1. The Three Basic Dimensions of teaching quality and proposed relations to students' use and outcomes, adapted from Klieme et al. (2009).



Theoretically, the Three Basic Dimension framework has been grounded in both traditional didactics and educational psychology (Praetorius et al., 2018, 2020; Vieluf & Klieme, 2023). Cognitive activation was linked to constructivist and socio-constructivist theories and student support to Self-Determination Theory (Praetorius et al., 2018, 2020).

The Three Basic Dimensions are structurally related to student outcomes via intermediary processes, described as students' use or uptake (see middle column of Figure 1). Cognitive activation is, within Three Basic Dimensions, related to achievement via students' depth of processing. Student support, as based in Self-Determination Theory, is assumed to support students' experience of autonomy, competence, and relatedness which are influential for motivation (Ryan & Deci, 2002). Classroom management is structurally related to all three levels of student mediation. An orderly classroom is seen as a prerequisite for processing the content, time-on-task, and motivation via students' need satisfaction (Alp Christ et al., 2024; Praetorius et al., 2020).

The structural parts of the Three Basic Dimensions framework are described as offer and use to connect teaching practices to student outcomes, extending beyond process-product research (Praetorius et al., 2020). Teaching and classroom practices are thus considered as giving opportunities in which the students can engage, representing the offer side. Within this distinction, cognitive activation refers to classroom practices aimed at stimulating students' cognitive engagement, whereas cognitive engagement refers to the cognitive activities students engage in during learning. Although the distinction has been stated since the inception of the framework, students' uptake of the opportunities is often under-specified and less researched (Praetorius et al., 2020).

Although the Three Basic Dimensions framework positions cognitive activation at the core of learning as a central dimension of teaching quality, the construct itself is multifaceted and often inconsistently defined (Praetorius et al., 2020). It is therefore necessary to attempt to frame its conceptual core and dimensions as well as different operationalisations.

2.2 Conceptual core and dimensions of cognitive activation

In its initial conception, cognitive activation was grounded in constructivist and socio-constructivist theories of learning focusing on conceptual understanding (Klieme et al., 2009; Lipowsky et al., 2009). In this framing, cognitive activation refers both to conditions for knowledge construction and indications of students' deep cognitive engagement with the content (Praetorius et al., 2018).

In Klieme et al. (2009), the core consisted of challenging tasks, activating prior knowledge, and content-related discourse. These types of practices are assumed to engage students in "(co-)-constructive and reflective higher-level thinking and thus to develop an elaborated, content-related knowledge base" (Klieme et al., 2009, pp. 140–141).

Although it has been argued that cognitive activation has a recognisable conceptual core (Wemmer-Rogh et al., 2024), recent reviews have identified variation in conceptualisation of subdimensions and operationalisations (Herbert et al., 2024; Mu et al., 2022; Praetorius et al., 2018). Praetorius et al. (2018) identified seven subdimensions of cognitive activation, including challenging tasks and questions, activating and exploring prior knowledge, eliciting student thinking, discursive and co-constructive learning, genetic-Socratic teaching, support for metacognition, and receptive learning as a negative indicator. In their citation analysis, Schreyer and Charalambous (2024) found that, among the dimensions identified by Praetorius et al. (2018), challenging tasks and questions were the most frequently cited, whereas receptive learning, genetic-Socratic teaching, and supporting metacognition were the least cited.

Other reviews have proposed more condensed organisations. Spreitzer et al. (2022) organised cognitive activation into four subdimensions: challenging tasks, challenging questions/class discussions, activation of prior knowledge, and supporting metacognition. Similarly, Mu et al. (2022) proposed to organise cognitive activation by distinguishing teachers' cognitive facilitation, the choice of challenging tasks, and students' cognitive engagement in higher-order thinking. For teacher facilitation, they include aspects such as activating prior knowledge, exploring ways of thinking, and asking challenging questions. This distinction is useful since it separates the task potential, teacher facilitation and students' engagement with cognitively demanding mathematical work.

The presented reviews, taken together, suggest that cognitive activation has a shared conceptual core, but not a single uniform conceptualisation or operationalisation. Higher-level thinking, challenging tasks, subject discourse, and activation of prior knowledge recur across previous studies.

2.3 Cognitive activation, instructional clarity, and lesson sequencing

Cognitive activation is not only a matter of whether challenging tasks or questions are present. It also depends on how mathematical content is made available, how expectations for mathematical work are clarified, and how opportunities for reasoning are sequenced across lessons.

Although the instructional features in the Three Basic Dimensions are often empirically distinguishable (Praetorius et al., 2018), additional features have been suggested. Instructional clarity, content selection, and content presentation are examples which have been proposed as complementary dimensions of instructional quality (Charalambous & Praetorius, 2020; Nilsen et al., 2016). Related to this, in an exploratory factor analysis, Jentsch et al. (2021) identified mathematics educational structuring, including features such as teacher explanations, structuring, and mathematical correctness, as a complementary dimension to the Three Basic Dimensions. This is important because cognitively activating practices, such as including challenging tasks and questions, are unlikely to be sufficient on their own. As Pauli et al. (2008) argued, they are likely to be most productive when coupled with structural clarity in teaching and rich subject discourse. Similarly, classroom management is often considered a prerequisite for cognitive activation (Charalambous & Praetorius, 2020). For PISA 2012, Caro et al. (2016) found a positive interaction between disciplinary climate and cognitive activation, indicating that the association between cognitive activation and achievement was stronger when disciplinary climate was higher.

A related issue concerns whether cognitive activation should be expected in the same way across all lessons and lesson phases. Praetorius et al. (2014), for example, questioned whether cognitive demands should be similar when introducing new content and when students work in consolidation phases. Herbert et al. (2024) similarly emphasised the temporal frame of measurement, arguing that cognitive activation cannot be expected to appear in the same way across lesson phases.

This suggests that cognitive activation could be examined not only as a general characteristic of teaching, but also in relation to other instructional features and to how mathematical work is organised across lesson phases.

2.4 Cognitive activation and links to student outcomes

Given its focus on students' opportunities for engagement in higher-level thinking, cognitive activation has mainly been studied in relation to student achievement. However, empirical findings are not uniform, and more recent research has also examined relations to motivational and affective outcomes. This makes it important to consider not only whether cognitive activation is related to outcomes, but also through which student processes such relations may occur.

Early studies provided support for a positive relationship between cognitive activation and mathematical achievement. Klieme et al. (2001) reported initial evidence for this association, which was later corroborated by findings from the Pythagoras Study (Lipowsky et al., 2009) and the COACTIV project (Baumert et al., 2010; Kunter et al., 2013). However, later work has shown more divergent results. In the review by Spreitzer et al. (2022), 11 of 18 studies reported positive associations with achievement for the full construct or specific aspects. For two frequently occurring aspects of challenging tasks and challenging questions/classroom discussion, 10 of 12 studies including these aspects reported positive associations. Applying a more restrictive inclusion criteria to longitudinal multilevel studies, Praetorius et al. (2018) identified cognitive activation to be a significant predictor of achievement in three of six studies. In the international comparison of PISA, Caro et al. (2016) found a positive association between cognitive activation and achievement in 27 education systems, negative in six, and non-significant in 29. Caro et al. (2016) also identified a curvilinear relationship in a majority of education systems, suggesting that too high levels of cognitive activation reduce the association with achievement. Thus, the empirical literature supports the relevance of cognitive activation while also suggesting that its relation to achievement depends on design, operationalisation, and level of analysis.

2.4.1 Links to affective and motivational domains

Although cognitive activation was initially posed as structurally related to achievement, later empirical work has examined associations with affective and motivational constructs. Cognitive activation has been found related to interest (Lazarides et al., 2023), self-efficacy (Li et al., 2021), self-concept (Ramazan et al., 2023), enjoyment (Lazarides & Buchholz, 2019), and perseverance (Zhang et al., 2021). It has also been found to be negatively related to boredom (Lazarides & Buchholz, 2019). For anxiety, however, findings vary. Some studies report no significant relation between cognitive activation and anxiety (Kunter & Voss, 2013; Lazarides & Buchholz, 2019), whereas others report a significant negative relationship (Liu et al., 2022; Zuo et al., 2024).

Together, this suggests a growing empirical base for considering cognitive activation in relation to affective and motivation-related outcomes. This is theoretically relevant because the dimensions of teaching quality show some conceptual overlap, and mediating pathways linking teaching quality to student outcomes remain under-specified (Alp Christ et al., 2024; Praetorius et al., 2020). Thus, examining cognitive activation in relation to self-related perceptions can extend research beyond direct relations with achievement by considering possible cognitive and motivational pathways.

2.4.2 Self-efficacy as a potential pathway

One suggested pathway linking cognitive activation, motivation, and achievement is self-efficacy. In Social Cognitive Theory, self-efficacy refers to an individual's perception of capability to perform in a specific domain (Bandura, 1997; Bong & Skaalvik, 2003). Self-efficacy is shaped mainly through mastery experiences, where previous success in relevant tasks influences students' beliefs about what they are capable of doing. Such experiences are particularly influential when students succeed in challenging activities that require effort and persistence (Usher & Pajares, 2008).

This may connect self-efficacy to cognitive activation. Cognitively activating classroom practices expect students to engage with demanding mathematical work, such as solving challenging tasks, explaining, reasoning, comparing strategies, and justifying claims. When these demands are experienced as manageable, they may provide mastery experiences and support students' beliefs in their mathematical capability. When demands are experienced as unmanageable, they may instead contribute to uncertainty or anxiety. Emerging empirical support for the relation between cognitive activation and self-efficacy has been reported by Liu et al. (2022) and Zuo et al. (2024). Self-efficacy may therefore be a theoretically relevant pathway for understanding how cognitively activating opportunities relate to achievement and affective outcomes.

2.4.3 Offer-use, uptake and regulation of learning

A major premise in the Three Basic Dimensions framework is the offer-use perspective, where teaching quality is understood as opportunities offered through instruction and learning depends on students' use of these opportunities (Praetorius et al., 2020). For cognitive activation, the assumed pathway to achievement has often been described as through students' depth of processing (Klieme et al., 2009). However, depth of processing and students' uptake of cognitively activating opportunities remain under-specified compared with the offer side of cognitive activation (Charalambous, Grob, et al., 2025; Praetorius et al., 2020).

Recent empirical work has begun to address this issue, and the findings show that opportunity and use need to be distinguished. Alp Christ et al. (2024) examined mediating pathways in the Three Basic Dimensions model and found mediation between cognitive activation and depth of processing at the individual level, but not at the classroom level. Charalambous, Grob, et al. (2025) similarly distinguished between cognitive activation as the offer side and cognitive activity as the student side, finding that cognitive activity had a stronger relation to performance. These findings suggest that cognitive activation cannot be understood as the presence of demanding tasks or questions; it also depends on whether and how the students engage with the opportunities made available.

The offer-use perspective also raises questions about how cognitively activating opportunities are intended and facilitated as these may influence students' engagement. Challenging tasks, for example, do not automatically lead to productive cognitive engagement. Even though a task has high potential, the cognitive demands can either be maintained or reduced depending on implementation (Henningsen & Stein, 1997). Thus, teachers need to decide how mathematical work should be introduced, supported, shared, or transferred to students. This makes regulation of learning relevant for examining intended cognitive activation.

Vermunt and Verloop's (1999) conceptualisation of regulation of learning provides a framework for examining how responsibility for learning functions is distributed between teacher and students. They conceptualise that in strong regulation, the teacher carries out many learning functions, for example by explaining, modelling, structuring, or rehearsing subject matter. In shared regulation, responsibility is distributed between teacher and students, for example through guiding questions, joint work, and discussion of strategies. In loose regulation, students carry greater responsibility for regulating their own work, for example through exploration, independent problem solving, or collaboration.

The framework also distinguishes between congruence and friction in the relation between teacher regulation and students' self-regulation. Congruence refers to a productive match between the degree of teacher regulation and students' capacity to regulate their own learning. Friction may occur when this relation is misaligned, for example when students are expected to work independently with demanding mathematical content without sufficient support. At the same time, friction can also be productive when teacher regulation challenges students slightly beyond their current ways of working. For this thesis, regulation of learning is therefore useful for examining how teachers intend to facilitate students' engagement with cognitively demanding tasks.

2.5 Summary and thesis positioning

Taken together, the reviewed literature positions cognitive activation as a central but complex construct in mathematics education. It concerns opportunities for students to engage in higher-level thinking, but these opportunities are shaped by tasks, discourse, teacher facilitation, instructional clarity, lesson sequencing, and students' uptake. The thesis therefore treats cognitive activation as a multidimensional opportunity structure.

3 Methodology

The empirical work in this thesis revolves around cognitive activation, explored from three complementary perspectives: intended, enacted, and experienced mathematics teaching. The methodological design follows from the multidimensional nature of cognitive activation, including opportunities provided through teaching, and also how such opportunities are experienced and taken up by students. Since no single data source can provide the empirical basis for how such opportunities are intended, enacted, and experienced, the thesis combines several empirical sources. The thesis therefore combines teacher planning data, classroom observation data, student questionnaire data, and achievement data, to examine cognitive activation from different perspectives.

This chapter first introduces the empirical context by describing the settings, participants, and data generation processes. It then presents the methodological approach of each paper, explaining the operationalisations and analyses undertaken. Finally, the chapter concludes with a synthesis of how the four papers come together to form the methodological foundation of the thesis. Together, these sections clarify the empirical basis, design, and methodological approach of the thesis, through which cognitive activation is examined across the four papers.

3.1 Empirical context

Two projects generated the data which form the empirical basis of the thesis. The first project involved students and teachers in upper secondary school and data were generated within this thesis project and form the empirical basis for Papers 1–3. For Paper 4, the data stem from the Linking Instruction and Student Achievement (LISA) project (Klette et al., 2017). Thus, the two empirical contexts provide complementary empirical bases for examining cognitive activation across intended, enacted, and experienced perspectives. In the following sections, the settings, participants and data generation process are presented.

3.1.1 Upper secondary

The upper secondary setting includes students and teachers from both social science and science tracks in Sweden. At the time of this project, mathematics in upper secondary was organised into separate courses, where students receive a final grade in each course. Students and teachers participating in this project were involved in the second mathematics course, which typically comprises 81–90 hours of teaching and is commonly studied during the first or second year of upper secondary school. Students in these years are typically 16–18 years old, depending on programme and course progression.

Partnerships were established by first contacting school principals and, after their approval, mathematics teachers. Teachers received written information followed by a school visit where the project was introduced and responsibilities for participation were discussed. Once partnerships had been established, each participating class and their teacher was visited.

During these class visits, students received information about the aim and process of the project, including that participation was voluntary and why identifiers were needed to link datasets. They were also informed how generated data would be handled and that results would be reported so that no individual student, teacher, or school would be identifiable in scientific dissemination. The study design and data generation procedures were reviewed in accordance with the ethical review process at Karlstad University (Dnr HNT 2022/619). Before data generation, teachers and students provided written consent.

All included schools were located in mid-Sweden and were selected to provide variation in school and background characteristics, such as SES, achievement level, proportion of migrant students and school types. The regional focus was due to the possibility of school visits, as the project setup required several visits to each participating class.

Papers 1–3 use partially overlapping subsets of these data, depending on the research question and relevant data sources, as described further in the paper-specific summaries and contrasted in the concluding section of this chapter. Three different data sets were generated in the upper secondary setting: student questionnaires, student achievement data, and teacher questionnaires. Because the papers draw on different combinations of these data sources, they also differ in sample size, focal variables and unit of analysis, which is explained in more detail in the summary of this chapter.

Student questionnaire data

The purpose of the student questionnaire was to examine three main domains: students' self-related perceptions, experience of classroom practices, and background variables. It was administered during the final month of students' second mathematics course during scheduled class visits. Students answered the questionnaire via an online survey using a computer or smartphone.

Student achievement data

Mathematics achievement data were obtained from students' results on the national mathematics tests in the first and second mathematics course. The national test was taken during the final part of each course. Each test contained three parts, with a maximum testing time of 240 minutes, usually including a lunch break before the final part. The national tests aim at providing grounds to evaluate students' mathematical competencies relating to concepts, procedures, problem solving, reasoning and communication applied to different core topic areas, such as arithmetic, algebra, geometry, functions, probability, and statistics.

Teacher questionnaire data

Teacher questionnaire data were generated through an online questionnaire comprising two main parts. The first part focused on contextual factors of teaching, including teacher qualifications and experience, teaching hours, textbook use, and participation in professional development. The other main part, which is in focus in Paper 3, related to teachers' perception, interpretation, and decision-making in relation to planning a lesson around a cognitively challenging task. The teachers analysed a challenging task and then produced a written lesson plan in which the task was central. These written lesson plans constitute the planning documents used in Paper 3, where the focus lies on teachers' intended regulation of learning.

3.1.2 Lower secondary

Paper 4 draws on video-recorded grade 7 mathematics lessons from the Swedish extension of the LISA project (Klette et al., 2017; Tengberg et al., 2022). The sample includes 35 classrooms across 15 schools and comprises a total of 127 mathematics lessons. In grade 7 in Sweden, students are typically 13–14 years old. Teachers, students, and guardians received information about the purpose of the study. Participating teachers, students (and their guardians) signed informed consent before data generation.

Classroom video data

The classroom observation dataset includes 127 video-recorded lessons. Lessons were recorded with a two-camera setup, one facing front and one facing back of the classroom. One microphone was placed in the ceiling and the teacher wore one microphone. This setup enabled the capture of both teacher and student talk and interactions in the classroom. For students who did not consent to participation, they were placed in the classroom so that they would not be captured on video. Lessons were split into 15-minute episodes, resulting in a total of 403 segments. Each lesson segment was coded in the LISA project using a standardised observation manual, the Protocol For Language Arts Teaching Observation (PLATO), used to capture instructional features in several subjects, including mathematics, within the LISA project (Grossman, 2015; Klette et al., 2017; Tengberg et al., 2022). The coded lesson segments were the main unit of analysis in Paper 4.

Summary data generation

In summary, the four papers draw on four main data sources. The relationship between data sources and papers is given in Table 2. Papers 1–3 share the upper secondary setting, but use different combinations of student questionnaire data, achievement data, and teacher questionnaire data. Paper 1 combines student questionnaire data with achievement scores, Paper 2 focuses solely on the student questionnaire dataset, and Paper 3 primarily uses teaching planning documents together with linked student questionnaire data. Paper 4 is based on a separate lower secondary classroom observation dataset.

Table 2. Data sources and focal variables in relation to the papers included in the thesis.

Data source	Focal variables	Paper			
		P1	P2	P3	P4
Student questionnaire	Student reports of classroom practices, self-efficacy, test anxiety and background variables	X	X	X	
National test	Student achievement scores	X			
Teacher questionnaire	Planning documents around a challenging task			X	
Classroom observations	Observer-rated indicators for instructional features				X

3.2 Methods and analytic approaches in Papers 1–4

The four papers examine cognitive activation through complementary perspectives and therefore differ in data sources, units of analysis, and analytic approaches. This reflects the overall design of the work in the thesis, in which cognitive activation is approached through intended, enacted, and experienced mathematics teaching.

To make the overlaps and differences explicit, and to explain how they form the methodological foundation of the thesis, each approach is described paper by paper before the chapter concludes with an overall methodological synthesis. The paper-specific descriptions therefore show how the thesis aim is approached through different empirical and analytical approaches.

3.2.1 Paper 1: Cognitive Activation and Links to Mathematics Learning

Paper 1 examines how classroom-perceived cognitive activation relates to students' mathematics achievement, self-efficacy, and test anxiety, and whether self-efficacy and test anxiety function as mediators in the link to achievement. The paper also explores whether associations differed by gender and parental education.

Design, data sources, sample, and measures

The sample included 531 students from 28 classrooms in upper secondary school.

Mathematics achievement measures were based on results from national tests separated by one year. Both prior and later achievement measures showed high reliability (ω = 0.93 for test 1, and ω = 0.93 for test 2).

Classroom-level cognitive activation was measured using nine items from the PISA 2012 questionnaire with a teacher/classroom referent. The items had the stem: "How often do the following occur during mathematics lessons?". Example item: "The teacher gives us problems which can be solved in different ways". Items capture practices such as reasoning, explaining solutions, and engagement with challenging tasks. Students' responses were aggregated to the classroom level and treated as a shared classroom construct. Aggregation was supported by ICC(1)= .20 and ICC(2)= .83, and high within-class agreement (mean $r_{WG(j)}$=.897).

Course-specific self-efficacy was assessed with a newly developed eight-item instrument aligned with Social Cognitive Theory's (Bandura, 1997) domain-specific framing. Students rated how confident they were in solving tasks central for the mathematics course (range 1–4). Example item: "Determine the axis of symmetry for a quadratic function". Reliability was acceptable (ω = .86).

Mathematics test anxiety was measured with three items adapted from the Global Teaching InSights study (OECD, 2020). Example item: "I am often worried that a mathematics test will be difficult for me". Items were rated on a four-point scale ranging from "applies poorly" to "applies very well". Reliability was acceptable (ω = .81).

Gender and parental education were included as covariates and examined as potential moderators. Gender was reported as students' legal gender. Parental education was based on the longest educational level of either parent or caregiver in the household, classified according to ISCED levels and dichotomised as ≥3 years higher education vs otherwise. See Paper 1 and the corresponding appendix for the full item specifications.

Analytic approach

Due to students being nested in classrooms, analyses used multilevel regression models. The use of classic multilevel models over full multilevel structural equation modelling was based on the number of level-2 units (McNeish, 2017).

Models were estimated with REML, with inference based on the Kenward-Roger approximation to reduce type I errors by more accurate standard errors with smaller level-2 samples. Mediation was examined using the Krull and MacKinnon (2001) approach by sequential model specification. Indirect effects were computed using distribution-of-the-product confidence intervals via RMediation (Tofighi & MacKinnon, 2011).

For the exploratory moderation analysis, interactions between cognitive activation and gender/parental education were followed by contrast analysis via emmeans (Lenth & Piaskowski, 2025) with cluster-robust standard errors and Holm correction for multiple comparisons, to reduce false discovery rates.

Methodologically, Paper 1 provides the thesis with a classroom-level perspective on cognitive activation and its relation to achievement and self-related perceptions.

3.2.2 Paper 2: Exploring the Relationships between Individual Cognitive Activation, Self-efficacy and Test Anxiety in Upper Secondary Mathematics

Paper 2 examines individual perceived cognitive activation and its relationships with students' self-efficacy and mathematics test anxiety. The paper first evaluates the measurement structure of a newly developed individual-referent cognitive activation instrument and then tests direct and indirect relations with self-efficacy and test anxiety.

Design, data sources, sample, and measures

The paper used student questionnaire data from 1009 upper secondary students from 49 classes.

For individual perceived cognitive activation, eight items were developed by shifting the referent from the teacher/classroom to the individual students. Accordingly, all items were phrased with the individual student as referent, for example: “I work with tasks where I have to think thoroughly” and “I need to explain how I think”. Students rated how often they experienced this during mathematics lessons.

Both mathematics self-efficacy and test anxiety were measured using the same instruments as in Paper 1. See Paper 2 for full item specifications.

Analytic approach

Analyses were conducted in Mplus version 8.5 (Muthén & Muthén, 1998–2017). The analysis proceeded with two steps: first a CFA of cognitive activation, followed by SEM to explore direct and indirect relations between cognitive activation, self-efficacy, and test anxiety. Global fit was evaluated using χ^2, RMSEA, CFI, and SRMR. Because items were ordered categorical, models were estimated using WLSMV. Missingness was low (≤ 1.2% per item). Due to the nested structure of students in classrooms, TYPE = COMPLEX in Mplus was used to adjust standard errors and chi-square related indices, affecting inferences and fit indices.

Methodologically, Paper 2 shifts the referent to the individual student and thereby provides an approach to students’ reports of uptake of cognitively activating opportunities.

3.2.3 Paper 3: Regulation of learning in lesson planning around a challenging task

Paper 3 examines how mathematics teachers intend to regulate students’ learning when planning a lesson around a challenging task, how this regulation shifts across planned instructional events, and how teachers’ overall regulation pattern relates to students’ reported classroom practices.

Design, data sources, sample, and measures

Complete task analyses and lesson plans were provided by 26 teachers. The final corpus included 174 coded instructional events across all lesson plans (mean 7.29, min 2, max 19). Questionnaire responses from

656 students regarding classroom practices were linked to the participating teachers.

Students' reported classroom practices were examined through nine questionnaire items focusing on teacher explanation and modelling (five items), together with opportunities and expectations for student participation and discussion (four items). These items stemmed from the Global Teaching InSights project (OECD, 2020). This design made it possible to examine intended regulation of learning as the primary analysis, while also exploring how overall planning patterns related to students' reported classroom practices. See Paper 3 for full item specifications.

Analytic approach

The main analysis followed a deductive qualitative content analysis (Mayring, 2014) of lesson-plan segments. The coding unit was defined as distinct planned instructional events, forming segments in the lesson plan. As the challenging task was held constant across lesson plans, the analysis could focus on intended regulation of learning as an indicator of teacher facilitation and subject discourse. The analytical framework was based on Vermunt and Verloop's (1999) regulation of learning functions. This theoretical basis separates teaching actions into strong, shared, and loose regulation of learning functions. After initial rounds of coding, two aspects were discussed and refined: boundaries of the coding unit, and how to interpret teachers' intended actions by expected student engagement (e.g., distinguishing rehearsal/monitoring and elaborative guiding questions). Text of a general character, for example referring to a previous or a forthcoming lesson, was excluded from the lesson segment corpus. Interrater agreement after the second round of coding was 84%. Disagreements were resolved via adjudication for consensus coding (Stemler, 2004) for the final dataset. The adjudication process followed: i) the theoretical framing of regulation of learning from Vermunt and Verloop (1999), ii) the coding rules, and iii) anchor examples, which were applied successively item by item.

Each instructional event was coded for regulation of learning, and each lesson plan received an overall categorisation. Sequencing within lesson plans as shifts in intended regulation were also analysed together with transitions between consecutive planned instructional events.

The final step in the analysis included linking teachers' intended regulation to students' experienced classroom practices. Item-level differences were explored by Kruskal-Wallis tests, followed by pairwise Wilcoxon rank-sum tests with Holm correction, to reduce false discovery rates.

Methodologically, Paper 3 provides access to intended opportunities for cognitive activation by examining how teachers plan to regulate learning around the same challenging mathematical task.

3.2.4 Paper 4: Unpacking Mathematics Lessons: A Latent Class Analysis of Instructional Features Within and Across Lessons and Classrooms

Paper 4 examines types of mathematics lesson segments based on observer-rated instructional features related to cognitive activation, instructional clarity, and working format. The paper also explores how these segment types vary across lesson phases and classrooms.

Design, data sources, sample, and measures

The sample comprised 35 classrooms across 15 schools and 127 lessons, which were segmented into 15-minute units, yielding 403 rated segments as the primary unit of analysis.

The analysis used eight indicators of instructional features from the PLATO observation framework. Intellectual Challenge, Classroom Discourse, and Connections to Prior Knowledge were treated as indicators related to cognitive activation. Strategy Use and Instruction, Modelling, Representation of Content, Feedback, and Purpose were used as indicators related to instructional clarity.

Each segment was rated on the indicators using a four-point scale reflecting the extent/strength of evidence for each instructional feature, ranging from "no/almost no evidence" to "consistent strong evidence", with element-specific criteria (see Grossman, 2015). In addition to indicators of instructional features, each segment was also coded for dominant working format, including whole-class instruction, group work, pair work, and individual seatwork. The dominant working format was determined by the activity format that occupied the most time within each lesson segment.

Analytic approach

Mathematics lesson segment types were identified using latent class analysis (LCA) based on categorical indicators of instructional features and working format. Model evaluation and selection combined information criteria (BIC, SABIC, CAIC, and AWE), likelihood tests (VLMR, BLRT), classification diagnostics (smallest class size, entropy), with an emphasis on interpretability of resulting profiles of mathematics segment types. After retaining a solution, differences between segment types were interpreted using class-specific probabilities of high ratings across indicators and working format. The nesting of segments in classrooms was handled using cluster-robust estimation in Mplus.

Segment position was derived from the segment order within each lesson (start/middle/end) and used to test whether segment types systematically varied across lessons. To explore association between segment types and lesson position, a multinomial logistic regression using a three-step approach (Vermunt, 2010) was used.

Classroom exposure was estimated by aggregating estimated segment-level types to the classroom level. Paper 4 therefore combines segment-level analysis of observed teaching with classroom-level exposure patterns, providing a link between within-lesson structuring and between-classroom variation.

Methodologically, Paper 4 provides an enacted perspective by identifying recurring patterns of instructional features in observed lesson segments.

3.3 Methodological summary

The four papers provide complementary methodological approaches to examine cognitive activation in mathematics teaching. By exploring how cognitively activating classroom practices are planned, observed and experienced, the thesis can examine cognitive activation through intended, enacted, and experienced perspectives. Consequently, the papers differ in their empirical basis, unit of analysis, time scale, and operationalisation of cognitive activation.

Empirical basis

The papers differ in their empirical basis. These differences follow from the different research questions, data requirements, and units of analysis in each paper. Papers 1–3 draw on partially overlapping subsets of the upper secondary dataset, whereas Paper 4 is based on a separate lower secondary classroom observation dataset. In particular for Papers 1–3, the differences in using different subsets of student questionnaire data stem from the need of linking different data sources. Paper 1 includes only students and classrooms for which the required combination of questionnaire data and achievement data was available. Paper 2 could include a larger part of the student questionnaire data because all focal variables stemmed from that source. Paper 3 is based primarily on teachers' completed lesson plans with linked student questionnaire data. The different empirical bases of the papers are therefore consequences of methodological design.

Time scales and unit of analysis

The papers also differ in time scale and unit of analysis. Papers 1 and 2 focus on students' experiences across the second mathematics course, spanning approximately 80–90 hours of teaching. Papers 3 and 4 use a narrower temporal scale by focusing on lesson segments. In Paper 3, these are planned distinct instructional events, whereas in Paper 4 they are fixed 15-minute observed lesson segments. This combination of course-level and segment-level approaches makes it possible to examine cognitive activation both through student reports of recurring classroom practices and more fine-grained analyses of lesson sequencing and structuring.

Opportunity and use

The papers also operationalise cognitive activation in partly different ways. Paper 2 comes closest to the uptake side of cognitive activation by focusing on individual students' engagement in cognitively activating practices. This approach aligns with what the Three Basic Dimensions framework refers to as students' use of opportunities and what student engagement research commonly describes as cognitive engagement (Fredricks et al., 2016). Papers 1, 3, and 4 instead focus primarily on the opportunity side of teaching and learning by examining classroom-level student perception, intended teacher regulation of learning, and observed instructional features. Thus, the thesis combines opportunity and use perspectives, while also showing that these are not interchangeable representations of cognitive activation.

Taken together, these differences in empirical basis, operationalisation, time scale, and unit of analysis provide grounds for examining cognitive activation as a multidimensional opportunity structure and how it is shaped by tasks, discourse, teacher facilitation, and instructional sequencing and structuring. The design enables conceptual triangulation across intended, enacted, and experienced perspectives, but it does not support direct alignment between the perspectives. The papers are therefore complementary and should not be considered as interchangeable.

4 Results

This chapter presents the main findings from the four papers.

Sections 4.1–4.4 summarise the results of each paper in relation to its aim/research questions, focal variables and analytical approach, which together form the empirical results for the thesis.

4.1 Paper 1: Cognitive Activation and Links to Mathematics Learning

Paper 1 examines cognitive activation as a classroom-level construct, based on aggregated student ratings, and its relations to self-efficacy, test anxiety, and later mathematical achievement. Prior achievement was included as a predictor, while gender and parental education were included as covariates and examined as potential moderators.

Cognitive activation was positively associated with later achievement ($\beta = 0.198$) and self-efficacy ($\beta = 0.299$), but was not significantly related to test anxiety, while controlling for prior achievement, gender, and parental education. Prior achievement predicted later achievement, self-efficacy, and test anxiety (full modelling results are reported in Table 2 of Paper 1).

Self-efficacy mediated the relation between cognitive activation and achievement (CogAct → SelfEff → Ach, $\beta = 0.0647$), while test anxiety did not mediate this relation (see Table 3 of Paper 1). Thus, the findings support self-efficacy, but not test anxiety, as an indirect pathway linking cognitive activation to achievement.

Exploratory contrast analysis suggested that the gender gap in self-efficacy, with male students reporting higher self-efficacy, was larger in classrooms with low cognitive activation and was not identified at higher levels of cognitive activation. No significant contrasts were identified for students' parental education.

4.2 Paper 2: Exploring the Relationships Between Individual Cognitive Activation, Self-efficacy and Test Anxiety in Upper Secondary Mathematics

Paper 2 examines individual perceived cognitive activation and its relations with self-efficacy and mathematics test anxiety. The analysis first examined the latent structure of a newly developed individual-referent cognitive activation instrument and then used SEM to test direct and indirect relations with self-efficacy and test anxiety.

A unidimensional latent variable of individual cognitive activation did not fit the data well, $\chi^2(20) = 631.297$, $p < .001$, RMSEA = .175, CFI = .818 and SRMR = .094. Instead, a two-factor model showed improved fit, $\chi^2(19) = 166.541$, $p < .001$, RMSEA = .088, CFI = .956 and SRMR = .046, identifying two dimensions: task demands and interaction demands. The final SEM including task demands, interaction demands, self-efficacy, and test anxiety showed acceptable fit, $\chi^2(39) = 165.166$, $p < .001$, RMSEA = .057, CFI = .970 and SRMR = .039.

Both dimensions showed positive associations with self-efficacy, although task demands ($\beta = 0.235$) showed a stronger association than interaction demands ($\beta = 0.176$). Task demands also had a positive direct association with test anxiety ($\beta = 0.109$), but a negative indirect association via self-efficacy (TaskDemand → SelfEff → TestAnx, $\beta = -0.087$), rendering the total effect non-significant.

Interaction demands were positively related to self-efficacy ($\beta = 0.176$), and were related to test anxiety only indirectly via self-efficacy ($\beta = -0.065$). Full reporting of direct and indirect relationships is given in Table 3 of Paper 2. Thus, Paper 2 suggests that different dimensions of individually perceived cognitive activation are differentially associated with self-efficacy and test anxiety.

4.3 Paper 3: Regulation of Learning in Lesson Planning Around a Challenging Task

Paper 3 examines how teachers intended to regulate students' learning when planning a lesson around a challenging task, how intended regulation shifts across planned instructional events, and how intended regulation relates to students' perceived classroom practices. In this paper, the challenging-task domain of cognitive activation was held constant, while the focus was on teachers' intended facilitation and subject discourse, analysed through regulation of learning functions.

Teachers' planned segments included all three regulation modes. At the overall lesson-plan level, strong regulation emerged as the most consistent pattern, whereas plans with overall categorisation as shared or loose were more internally mixed.

At the segment level, strong regulation was characterised by teacher explanation, modelling, stepwise procedures, review, and rehearsal. Shared regulation was characterised by elaborative questioning, whole-class discussion, comparison of strategies, and joint work on student solutions. Loose-regulation segments placed greater responsibility on students' own activity, including individual or collective exploration, practice, and consolidation.

The regulation of learning was not static across lesson plans. Across transitions between planned instructional events, 54% remained in the same regulation mode, while 21% involved an increase and 24% involved a decrease in regulation. Strong-to-strong was the most common transition (33%), and shifts between strong and loose were relatively uncommon.

In the exploratory analysis linking intended regulation to students' perceived classroom practices, students in loose-regulation classrooms reported lower frequencies of teacher explanation and modelling than students in classrooms whose teachers' lesson plans were categorised overall as shared or strong regulation. For items related to student participation, the findings were more mixed, although some discussion-related items were reported more often for teachers with loose-regulation lesson plans than shared regulation.

4.4 Paper 4: Unpacking Mathematics Lessons: A Latent Class Analysis of Instructional Features Within and Across Lessons and Classrooms

Paper 4 identifies types of mathematics lesson segments based on instructional features related to cognitive activation and instructional clarity together with dominant working format, and explores how segment types vary across lesson phases and classrooms.

The latent class analysis resulted in a three-class solution, giving three types of mathematics lesson segments.

Type 1, whole-class strategy instruction, accounted for 36.3% of segments. This type showed the strongest profile across instructional features, with high probabilities of high ratings for Strategy Use and Instruction, Representation of Content, Modelling, and Classroom Discourse. This type was also most strongly associated with whole-class instruction.

Type 2, individual seatwork, accounted for 36.2% of segments. This type retained some features of instructional clarity, especially Representation of Content and Strategy Use and Instruction, but showed almost no probability of a high rating of Classroom Discourse.

Type 3, low probability of high ratings, accounted for 27.5% of segments. This type was characterised by generally low probabilities of high ratings across instructional indicators and was not clearly associated with one dominant working format, although it was somewhat more likely to occur during individual seatwork.

Segment types were systematically related to lesson position. As lessons progressed, the probability of Type 1 decreased, whereas the probability of Type 2 increased. Exposure to segment types also varied substantially across classrooms. In 20 of the 35 classrooms, one segment type accounted for more than half of predicted segment exposure, meaning that a majority of observed segments in those classrooms were assigned to the same type.

5 Discussion

Across four papers, this thesis examines cognitively activating classroom practices as a multidimensional construct from intended, enacted, and experienced perspectives. The multi-method design sheds light on how cognitive activation opportunity structures are constituted and related to other instructional features, lesson structuring, and student outcomes.

Taken together, the findings address the first research question by showing how opportunities for cognitive activation can be constituted through several interrelated dimensions of mathematics teaching: task demands, interaction demands, subject discourse, regulation of learning, teacher facilitation, and lesson sequencing. Furthermore, dimensions co-occurred to different extents with other instructional features related to instructional clarity and appeared differently depending on whether cognitive activation was examined as intended, enacted, or experienced.

The findings address the second research question by showing that experienced cognitive activation was positively associated with self-efficacy and mathematics achievement. In addition, self-efficacy emerged as a plausible mediator between cognitive activation and mathematics achievement. The findings related to test anxiety were more mixed, and depended on the dimension of cognitive activation and the modelling approach.

The chapter first provides an integrative summary of the findings in relation to previous research and identified themes across the papers. It then summarises the thesis contributions, discusses limitations, outlines implications, and concludes with directions for future research.

5.1 Integrative summary

The integrative summary is structured around six main themes identified across the papers.

5.1.1 Cognitive activation and links to self-efficacy and achievement

Paper 1 and Paper 2 identified cognitive activation as positively associated with self-efficacy. This holds both when cognitive activation is conceptualised as a shared classroom construct (Paper 1) and when it is measured at the individual level (Paper 2). Within Social Cognitive Theory, self-efficacy is considered predictive of behaviour as it shapes effort, persistence, and resilience (Bandura, 1997). Self-efficacy is mainly driven by mastery experiences stemming from success on increasingly difficult tasks that required persistence (Bandura, 1997; Bong & Skaalvik, 2003). Paper 1 further examined links to achievement from cognitive activation via self-efficacy. While controlling for prior knowledge, an indirect effect of cognitive activation on mathematics achievement via self-efficacy was supported, in line with findings from Liu et al. (2022) and Zuo et al. (2024). This responds to calls to examine mediating processes by testing self-efficacy as a pathway towards motivation and achievement from cognitive activation (Alp Christ et al., 2024; Praetorius et al., 2018, 2020). Thus, cognitively activating opportunities may matter not only through cognitive engagement, but also through students' perceived capability to engage with demanding mathematical practices.

5.1.2 Cognitive activation and mathematics anxiety

The relation between cognitive activation and different anxiety measures has previously shown diverging results. For example, in Kunter and Voss (2013), there was no identified association between cognitive activation and achievement anxiety, whereas in Zuo et al. (2024) there was a significant negative association. The results from two papers in this thesis add to previous findings regarding cognitive activation and different anxiety measures. In Paper 1, there was no identified significant association between cognitive activation and test anxiety. In Paper 2, the task demands dimension was found to be positively associated with test anxiety. However, when taking into account self-efficacy as a mediator, the total association was non-significant. Taken together, these results suggest that anxiety relations depend on which dimension of cognitive activation is measured, and whether models distinguish between direct and indirect pathways via self-efficacy. Furthermore, it is of importance to also consider the type of anxiety measure used, and its operationalisation. In Papers 1 and 2, it was labelled mathematics test anxiety and included both emotional and state-related dimensions, which are oftentimes mixed in research (Dowker et al., 2016).

5.1.3 Opportunity and use

A central tenet in the Three Basic Dimensions model is that cognitive activation is commonly framed as an opportunity offered through teaching. In this thesis, Paper 1, 3, and 4 primarily examine such opportunities, although from different perspectives: experienced classroom-level, intended teacher regulation, and enacted segment types. Paper 2 provides a different perspective, closer to students' uptake, by shifting referent to the individual student. Paper 2 identified two dimensions of individually perceived cognitive activation: task demands and interaction demands. The results in Paper 2 show inter-individual differences in experienced demands, moving beyond the shared classroom level examined in Paper 1 and indicating differences on tasks and interaction, as well as different relationships with self-efficacy and test anxiety. This aligns with recent calls for separating students' cognitive engagement and learning opportunities (Mu et al., 2022) and also exploring students' uptake of cognitively activating classroom practices (Charalambous, Grob, et al., 2025; Praetorius et al., 2024).

5.1.4 Tasks, subject discourse, and teachers' facilitation

Across the papers, the results point towards the potential value of separating task-related demands from discourse-related demands. Furthermore, the findings suggest that cognitively activating opportunities cannot be inferred from only task demands. The same task potential may be transformed differently depending on how teachers regulate learning, elicit student thinking, and also by the extent of modelling instruction. In Paper 2, from the students' perspective, the interaction demands were separated from the task demands. In Paper 4, cognitive activation was operationalised through three indicators of instructional features (Intellectual Challenge, Classroom Discourse, and Connections to Prior Knowledge), and the type of lesson segment profiles identified suggested larger differentiation on subject discourse than on intellectual challenge from activities. Paper 3 further complements this as even though the challenging task was held constant, teachers' intended facilitation through their planned regulation of learning showed a wide diversity across the lesson plans. Teachers' intended instructional moves from strong and shared regulations are also in line with the type of segments from Paper 4 in terms of subject discourse and modelling co-occurring.

These results are in line with the distinction by Mu et al. (2022) between activities set and teachers' cognitive facilitation within cognitive activation. The findings from the papers suggest that cognitive activation can be an overarching construct where tasks and subject-discourse are related but distinguishable subdimensions.

5.1.5 Cognitive activation and co-occurrence with other instructional features

Cognitively activating opportunities did not appear in isolation. The findings suggest that these opportunities were intended and enacted through combinations of task demands, discourse, and teacher facilitation including explanations, modelling and questioning. In Paper 4, the segment type characterised by the strongest cognitive activation also showed the highest probability of instructional clarity features. In Paper 3, strong regulation was characterised by modelling and explanations, while shared regulation more often involved elaborative questioning and joint work on student solutions. Together, this suggests that cognitive activation is intended and enacted through practices combining demands, subject discourse, and aspects of instructional clarity. For example, exploring students' ways of thinking could give different cognitive activation opportunities depending on the extent of rehearsing, elaborative questioning or students' own exploration. In instructional quality frameworks, notions such as instructional clarity, content selection and presentation have been proposed as complementary dimensions to the Three Basic Dimensions (Klette et al., 2017; Mu et al., 2022; Nilsen & Gustafsson, 2016). Cognitive activation has been framed as independent of instructional approaches such as discovery or guided instruction (Pauli et al., 2008). However, Pauli et al. (2008) suggested that productive discourse and demanding tasks are best coupled with instructional clarity, as just providing students with challenging tasks and questions is unlikely to be sufficient for productive cognitive engagement. Previously it has also been shown that even though a task has mathematical potential, this potential is contingent on the implementation to maintain the cognitive demands (Henningsen & Stein, 1997). Thus, the co-occurrence of cognitive activation and instructional clarity features in Paper 4 is consistent with calls for including instructional clarity and the results suggest that they may work as mutually supportive while remaining as distinct features of teaching dimensions.

5.1.6 Cognitive activation and sequencing

Paper 3 and Paper 4 also highlight that cognitively activating opportunities vary across lesson segments. Teachers' lesson plans shifted in regulation across instructional events, providing different cognitive facilitation by teachers. In Paper 4, it was found that types of lesson segments were systematically related to position within lessons. These findings align with previous research showing substantial variability of cognitive activation across lessons (Praetorius et al., 2014) and questions have been raised regarding measurement issues across lessons and phases (Herbert et al., 2024). Furthermore, Pauli et al. (2008) considered potential bias towards discussion segments for cognitive activation measurement, as they tended to be rated higher. This could also be observed in Paper 4, where there could be similar task demands, but whole class segments had much higher probability of being rated high in discourse compared to individual seatwork dominated segments. Together, Paper 3 and Paper 4 suggest that potential cognitive activation depends on phases and organisation, for example dimensions relating to discourse are more prevalent in whole-class segments. This may also have implications for measurement, as averages across lessons can miss systematic within-lesson structure.

5.2 Contributions

Based on the results and integrative discussion, the thesis contributes to knowledge about opportunities for cognitive activation in mathematics education in four ways, which are briefly summarised below.

5.2.1 Cognitive activation as a multidimensional opportunity structure

The papers in this thesis contribute to understanding cognitive activation as a multidimensional opportunity structure. Instead of treating cognitive activation as one instructional feature, the findings consider distinguishable but related dimensions, including task demands, interaction demands, teachers' cognitive facilitation, and subject discourse. Thus, the thesis can provide insights into important conditions that may support students' cognitive engagement.

5.2.2 Mechanisms linking cognitive activation and student outcomes

By combining cognitive activation with self-efficacy grounded in Social Cognitive Theory, the thesis contributes to research on potential mechanisms linking instructional quality to student outcomes. Papers 1 and 2 suggest that self-efficacy may function as an explanatory pathway between cognitive activation, anxiety, and mathematical achievement. Thus, cognitive activation may relate to mathematical development both directly and indirectly through students' self-related perceptions.

5.2.3 Cognitive activation, instructional support and sequencing

The thesis also contributes by showing how opportunities for cognitive activation can form from decisions relating to instructional support and lesson organisation. The findings from Papers 3 and 4 suggest that potential cognitive activation is shaped by how teachers structure, model, question, and organise mathematical work, and by how these classroom practices are sequenced across lesson phases.

5.2.4 Methodological approaches to examine cognitive activation

Methodologically, the thesis contributes by examining cognitive activation across intended, enacted, and experienced perspectives. Paper 2 contributes by utilising a new instrument for capturing students' uptake of cognitively activating practices, by focusing on individually perceived cognitive activity. By keeping a challenging task constant, Paper 3 provides a novel approach to examine teachers' intended facilitation by focusing on regulation of learning. Paper 4, using latent class analysis, provides an approach to examine how different aspects of cognitive activation relate to other instructional features and form lesson segment types. Furthermore, Papers 3 and 4 also show how aspects of cognitive activation form different patterns within lessons by utilising segment-level analysis.

5.3 Limitations

The papers included in this thesis explored intended, enacted, and experienced aspects of cognitive activation and relations with other instructional features, self-related perceptions, and mathematics achievement. However, no paper examined all three perspectives simultaneously, which limits claims about alignment between intended, enacted, and experienced cognitive activation. This also constrains interpretations regarding mechanisms across perspectives, as the data do not allow for direct comparisons across the same instructional units.

In addition, the empirical contexts differed across papers, but were all situated within Swedish secondary mathematics education. Whereas Papers 1–3 draw on Swedish upper secondary education, Paper 4 focuses on grade 7 classrooms. Most research on cognitive activation has been conducted in lower secondary contexts, and fewer studies have examined upper secondary students (Praetorius et al., 2020). While this thesis contributes to the empirical grounding of cognitive activation in upper secondary education, it is restricted to Swedish secondary education, which warrants caution before extending the findings to other contexts. First, the papers draw on context-specific measures (e.g., national test scores, course-specific self-efficacy), which should be compared with other measures in future studies. Of special interest are the principles underlying the design of the measures, which need replication in other contexts. The mathematics tests were closely aligned with curricula, and the self-efficacy measure focused on concepts and procedures central to the mathematics course. Second, both student samples were compared with Swedish population characteristics and showed only minor deviations (e.g., parents' educational levels). Third, recruitment was based on voluntary partnerships for teachers, which may have introduced selection effects.

5.3.1 Cognitive activation as a multidimensional construct

Cognitive activation is operationalised differently across the papers. In this thesis, cognitive activation functions as an opportunity structure with different dimensions and operationalisations. In Paper 1, the operationalisation is based on items originating from the PISA questionnaire; in Paper 2, it is measured using a newly developed individual-referent instrument; in Paper 3, it is approached through content analysis of lesson plans, focusing on teachers' intended facilitation; and in Paper 4, it is operationalised using PLATO indicators of Intellectual Challenge, Classroom Discourse and Connections to Prior Knowledge. Consequently, the thesis does not treat the different operationalisations as interchangeable, and convergent validity across papers is not examined. Rather than claiming measurement equivalence, the thesis should be understood as a conceptual triangulation of dimensions of cognitive activation, intended to shed light on different aspects of the construct.

In relation to the above, the coverage of cognitive activation as a multidimensional construct is limited. Across the papers, the main dimensions examined concern tasks and subject discourse (including teachers' facilitation), whereas other proposed indicators are not included. For example, support for metacognition is not operationalised across the studies. This restricts the scope of conclusions and ties findings to the specific conceptual indicators included. Thus, calls to identify the most central dimensions (e.g., Lipowsky et al., 2009) and clarify the conceptual core of cognitive activation (Praetorius et al., 2024) can only be partly addressed in this thesis.

Finally, as Praetorius et al. (2024) and Charalambous, Sergiou, et al. (2025) underscore, it is important to distinguish the opportunity and use sides of cognitive activation. No paper directly measured students' cognitive processing. Paper 2 approached the use side through students' self-reports of practices during mathematics lessons, but this remains an indirect proxy for cognitive processing and uptake.

5.3.2 Perspectives and methods

A further limitation concerns potential systematic method effects. Both Papers 1 and 2 rely on student self-reports for cognitive activation, self-efficacy, and test anxiety from the same data source, which may be affected by common-method bias. In Paper 1, achievement measures are separated by one year, but there is no time separation or repeated measurement of cognitive activation or self-efficacy, as they stem from the same data source. This limits mediation interpretations. In Paper 2, the mediation model is based on cross-sectional self-reported data, raising concerns about reverse causality and common-method bias. Although it is difficult to rule out reciprocal relations (e.g., self-efficacy shaping perceptions of cognitive activation), the structural ordering of constructs was based on theory. Moreover, as Podsakoff et al. (2012) argue, method bias may inflate observed associations, which may lead to an overestimation of indirect effects in mediation models.

Paper 3 relies on written lesson plans, which represent intentions rather than actual enacted instruction. Furthermore, teachers varied in their level of detail and number of presented segments (min 2 to max

19). Although no systematic pattern with regard to regulation modes and length of plans was identified, further work is needed to rule out consequences of variation in level of detail and length of lesson plans. Paper 4 used the standardised observation manual of PLATO. However, all frameworks include constraints such as reduction to operationalised variables with consequences for measurement and interpretation (Charalambous & Praetorius, 2018). For example, an indicator such as Classroom Discourse may be more visible in whole-class formats than in individual work, which may favour cognitive activation in terms of lesson format.

As Hiebert and Grouws (2007) argue, teaching is a system of interacting features, which presents a challenge for research on teaching and learning. Although the Three Basic Dimensions are often examined together, a hierarchical or systematic ordering is not consistently specified (Praetorius et al., 2020). For example, some (e.g., Klieme et al., 2009) have proposed that classroom management may function as a prerequisite for cognitive activation. In this thesis, cognitive activation is the focal construct, and only Paper 4 systematically examined cognitive activation in relation to other instructional features. Thus, the scope of the thesis mainly concerns cognitive activation, and inferences regarding instructional systems and interacting instructional features are limited.

Furthermore, all included papers use observational, non-experimental designs, which can limit causal interpretations. Although Paper 1 utilised a longitudinal design with three time points and repeated measurement of achievement, cognitive activation was not measured repeatedly. Thus, while temporal order between cognitive activation and later achievement is supported, the temporal ordering between cognitive activation and self-efficacy is not, as these were measured at the same time. This limits mediation claims in both Paper 1 and Paper 2, and implies that mediation should be interpreted with caution.

5.3.3 Analytical and unit of analysis constraints

In Paper 1, the number of level-2 units (i.e., classrooms) is limited, which constrains statistical power. Charalambous, Sergiou, et al. (2025) conducted Monte Carlo simulations with similar characteristics (though more level 1 variables) and reported that 31 classes with 15 students would yield power of .80. Paper 1 had an average cluster size of 19 students but only 28 classes, which may reduce statistical power. In addition to limitations of 28 classes, linear associations between cognitive activation and achievement were modelled, and curvilinear relations as proposed by earlier research (Caro et al., 2016) were not examined.

In Paper 2, analyses used TYPE = COMPLEX, which adjusts standard errors for clustering but does not explicitly model between-classroom variance. Therefore, Paper 2 supports conclusions regarding individual perceived cognitive activation and self-related perceptions, but limits conclusions regarding classroom-level opportunity structures and related factors.

In Paper 4, latent class analysis was used to explore lesson segment types. The indicators were based on the standardised PLATO protocol and treated as separate instructional features. Because the indicators are intended to be distinct, the local independence assumption is important and requires scrutiny. Although cluster-robust standard errors were used to account for classroom clustering, segment dependency within lessons is of importance, as segment types were found to be systematically related to sequencing in lessons.

Finally, segmentation decisions may shape results. In Paper 3, segments were defined as distinct instructional events, but segment length and level of detail varied considerably across lesson plans. Teachers also differed in the total number of segments (min 2, max 19), which may influence transition analyses. Although no systematic relation between segment length and regulation type was identified, it is difficult to rule out such relations based on the available data. In Paper 4, segments were defined as fixed 15-minute episodes. Even though the term "segment" is used in both Papers 3 and 4, these units should not be treated as interchangeable, as they originate from different data sources and represent different perspectives. Generalisations across Papers 3 and 4 should therefore be made cautiously.

5.4 Implications

The findings in this thesis have potential implications for how cognitive activation is theorised, studied, and supported in mathematics classrooms. Three brief suggested implications based on the findings and discussion are summarised below.

First, the findings imply that theoretical models including cognitive activation should consider the multidimensional nature of the construct. The included papers have shown that cognitive activation can be understood as a configuration of, and interaction between, task demands, teacher facilitation, and students' participation in mathematical discourse and their cognitive engagement. Thus, future studies should make explicit how, and which, dimensions of cognitive activation are configured to support interpretations of findings and development of theoretical models including cognitive activation.

Second, the thesis has methodological implications for future studies of cognitive activation. Future research should distinguish between provided opportunities for cognitive activation and students' use of these opportunities. As the thesis has shown, student reports, teacher plans, and classroom observations each capture different aspects of the teaching and learning process and thus contribute in different ways to understanding potential cognitive activation.

Third, the thesis has implications for mathematics teaching and teachers' didactical decision-making. For teaching, there are important considerations regarding not only whether tasks are cognitively demanding, but also how students are supported to engage with those demands. Thus, there are important didactical decisions regarding how this engagement can be supported through explanations, questions, modelling, comparison of strategies, and opportunities for mathematical reasoning. Task potential is a starting condition, but is not sufficient for students' mathematical development, as cognitive activation depends on the interaction between content, students, and teacher facilitation, and ultimately, on students' own cognitive engagement.

The thesis also suggests that cognitively activating teaching does not require the teacher to step back. Instead, cognitive demand may need to be combined with clear representations, modelling, and appropriate structuring and scaffolding to support students' engagement. Practice and consolidation work are not inherently low quality, and whole-class discussion is not inherently high quality. What seems to matter is the function of different classroom practices, how they are structured and how they support students' engagement. Thus, another consideration for didactical decision-making pertains to how classroom practices in different lesson and learning phases provide different opportunities for reasoning, discussion, and consolidation.

5.5 Directions for future research

Based on the findings and limitations, several potential directions for future research are apparent.

One direction for future research is to examine the relationships between intended, enacted, and experienced cognitive activation within a sequence of teaching and learning. This would allow a direct analysis of how teachers' intentions are transformed into enacted classroom opportunities and how these opportunities are experienced and used by students. The present thesis examined these perspectives separately, which enabled triangulation of different aspects of cognitive activation, but did not make it possible to follow the full chain of transformations. Thus, such an approach would provide conditions for examining transformation of cognitive activation from the stage of planning to students' mathematical learning. Furthermore, research is needed to distinguish between different opportunities and students' uptake relating to both cognitive and affective outcomes.

Another direction for future research pertains to potential mechanisms through which cognitive activation is related to students' mathematical development. This includes further examinations of cognitive and motivational pathways together, in order to disentangle relationships and provide further understanding of how cognitive activation can be related to different student outcomes. In the present thesis, cognitive activation and self-efficacy were measured at one time point. Future research could therefore utilise longitudinal designs to examine causal relationships more directly, and also explore reciprocal relationships between cognitive activation, motivational domains, and mathematical achievement. Targeted experimental designs could also contribute to the knowledge base of cognitively activating classroom practices by changing specific dimensions of cognitive activation and examining changes in affective and cognitive outcomes. As the present thesis identified different configurations of dimensions of cognitive activation, this could act as a starting point to distinguish which dimensions are more central, and how they interact, responding to calls by Lipowsky et al. (2009) and Praetorius, Brinkmann, et al. (2024) regarding the conceptual core of cognitive activation.

Examining mechanisms also involves exploring potential moderating factors. A key question for educational research is to understand how teaching and learning relationships shape equity. This includes both differential effects of teaching practices and the interplay between students' dispositions and the classroom environment. Examining the extent to which classroom practices related to cognitive activation may mitigate or exacerbate inequalities is therefore important for supporting the development of theory and instructional models. Future research should thus investigate for whom and under what conditions cognitive activation supports learning and self-related perceptions, by focusing on potential moderators and mediators.

Finally, future research should attend to the sequencing and structuring of mathematics teaching and learning while examining cognitive activation. This could provide insights into how cognitively activating practices can be distributed across learning phases, clarifying how productive cognitive demands can be manifested and maintained when new content is introduced, during collective explorations, and during consolidation phases. Such research could provide knowledge pertaining to how different aspects of cognitive activation are needed across different phases of learning.

6 Svensk sammanfattning

Matematiskt tänkande är centralt för undervisning och lärande i matematik. Det omfattar exempelvis att kunna resonera, motivera och jämföra lösningsstrategier och se och använda samband mellan matematiska idéer. I matematikklassrum formas elevernas möjligheter till matematiskt tänkande genom de aktiviteter och innehåll de möter, de frågor som ställs, hur matematik diskuteras och hur lärare organiserar och stödjer arbetet med matematiska idéer.

I denna avhandling undersöks sådana möjligheter genom begreppet kognitiv aktivering. Kognitiv aktivering avser undervisningspraktiker som kan stödja elevers engagemang i matematiskt tänkande, exempelvis genom utmanande uppgifter, resonemang, förklaringar och diskussion av olika lösningsstrategier. Avhandlingen behandlar kognitiv aktivering som en flerdimensionell struktur för möjligheter i matematikklassrum. Detta innebär att kognitiv aktivering inte betraktas som en specifik undervisningsmetod, utan istället som något som formas genom samspelet mellan ämnesinnehåll, aktiviteter, lärares stöd och elevers kognitiva engagemang.

Avhandlingen syftar till att bidra till förståelsen av kognitiv aktivering i matematikundervisning genom att undersöka hur sådana möjligheter planeras, genomförs och erfars, samt hur elevers erfarenheter relaterar till deras matematikprestation, självförmåga och matematikängslan. Självförmåga behandlas i avhandlingen som elevers tilltro till sin förmåga i matematik.

Avhandlingen bygger på fyra empiriska delstudier. De tre första delstudierna berör gymnasiets andra kurs, medan den sista delstudien handlar om undervisning i årskurs 7.

I den första delstudien undersöktes kognitiv aktivering på klassrumsnivå, baserat på elevers gemensamma erfarenheter av undervisningen. I denna delstudie ingick 531 elever från 28 olika klasser på studieförberedande program på gymnasiet. Analysen byggde på flernivåregressionsmodellering där samband mellan kognitiv aktivering, matematikprestation, självförmåga och matematisk testängslan undersöktes. Resultaten visade att högre grad av kognitiv aktivering på klassrumsnivå var positivt relaterat till elevers självförmåga och matematikprestation, när det kontrollerades för tidigare prestation i matematik, kön och elevernas föräldrars utbildningsnivå. Vidare visade resultaten inget samband mellan kognitiv aktivering och testängslan i matematik. Resultaten visade också att självförmåga kan fungera som en länk mellan kognitiv aktivering och matematikprestation, på så vis att självförmåga kan mediera mellan kognitiv aktivering och matematikprestation. I delstudien hade pojkar i genomsnitt signifikant högre tilltro till sin förmåga i matematik än flickor, men i klasser med högre kognitiv aktivering fanns ingen signifikant skillnad mellan pojkar och flickor. Inga signifikanta skillnader identifierades gällande sambandet mellan kognitiv aktivering och självförmåga eller prestation i matematik beroende på elevernas föräldrars utbildningsbakgrund.

I den andra delstudien fördjupades det erfarna perspektivet genom att rikta fokus mot den enskilda elevens erfarenheter, istället för kognitiv aktivering på klassrumsnivå. För denna delstudie användes strukturell ekvationsmodellering för att undersöka kognitiv aktivering på elevnivå och dess samband med självförmåga och matematisk testängslan. I analysen ingick 1009 elever från studieförberedande program på gymnasiet. Delstudien identifierade två dimensioner av individuellt upplevd kognitiv aktivering: kognitiva krav kopplade till uppgifter och kognitiva krav kopplade till klassrumsinteraktion. Båda dimensionerna visade ett positivt samband med elevernas tilltro till sin förmåga. Samtidigt visade resultaten att uppgiftsrelaterade krav kan vara relaterade till testängslan i matematik, men att den indirekta relationen via självförmåga var negativ. Sammantaget visar detta att kognitivt aktiverande undervisning kan ha ett positivt samband med självförmåga men olika samband med matematikängslan beroende på vilka dimensioner som undersöks.

Den tredje delstudien undersökte det planerade perspektivet genom lärares lektionsplaneringar kring en utmanande matematikuppgift. För denna analys ingick 26 lärares kompletta uppgiftsanalys och lektionsplanering tillsammans med deras 656 elevers erfarenheter av klassrumspraktiker. Analysen utgick från hur lärare avsåg att organisera lektionen genom stark, delad eller lös reglering av lärande. Totalt kodades 174 planerade lektionssegment. Resultaten visade att lärare planerade för olika former av stöd och fördelning av ansvar för det matematiska arbetet. Stark reglering kännetecknades av förklaringar, modellering och stegvisa genomgångar. Delad reglering kännetecknades av mer utforskande frågor, gemensamma diskussioner och jämförelser av lösningsstrategier. Lös reglering gav större ansvar till eleverna i det matematiska arbetet genom enskilt arbete, grupparbete och utforskande av uppgiften på egen hand. Delstudien visade att en utmanande uppgift i sig inte avgör undervisningens kognitiva potential utan det är lärarens planerade stöd och reglering av aktiviteter som är avgörande för hur uppgiften kan bli en möjlighet till matematiskt tänkande.

Den fjärde delstudien undersökte det genomförda perspektivet genom videoinspelade matematiklektioner i årskurs 7. Här ingick 127 lektioner från 35 olika klassrum. Dessa 127 lektioner delades upp i segment om 15 minuter, vilket gav 403 olika lektionssegment som skattades och analyserades utifrån åtta olika undervisningsfaktorer och arbetsform. Utifrån en latent klassanalys identifierades tre olika typer av lektionssegment utifrån undervisningsfaktorer relaterade till kognitiv aktivering, tydlighet i undervisning och arbetsform. Den första segmenttypen (36.3%) kännetecknades av helklassundervisning med högre sannolikhet för strategiundervisning, modellering och gemensam diskussion. Den andra typen av lektionssegment (36.2%) kännetecknades främst av individuellt arbete med vissa drag av undervisningstydlighet i form av exempel, förklaringar och strategiundervisning. Den tredje och sista typen (27.5%) kännetecknades av låg sannolikhet för hög skattning av de ingående undervisningsfaktorerna. Resultaten visade också att de tre segmenttyperna var systematiskt relaterade till olika lektionsfaser och klassrum. Den första segmenttypen hade högst sannolikhet att förekomma i början av lektioner, och därefter minskade sannolikheten, medan den andra segmenttypen ökade i sannolikhet längre in i lektionen. På klassrumsnivå visade det sig att 20 av de 35 undersökta klassrummen hade en dominerande (>50%) förekomst av en viss segmenttyp.

Sammantaget visar avhandlingen att kognitiv aktivering kan förstås som en mångdimensionell struktur för möjligheter i matematikklassrum. Resultaten pekar på att uppgifter, interaktion, lärares reglering, ämnesdiskurs, tydlighet i undervisning och lektionssekvensering samspelar i hur möjligheter till matematiskt tänkande formas. Avhandlingen visar också att självförmåga kan utgöra en del för att förstå relationen mellan kognitiv aktivering och elevers matematiska utveckling. Relationen till matematisk testängslan visade sig mer komplex och beroende av vilka dimensioner av kognitiv aktivering som undersöks.

Sammantaget bidrar avhandlingen till forskning om undervisningskvalitet genom att visa hur kognitiv aktivering kan undersökas och förstås genom planerade, genomförda och erfarna perspektiv på matematikundervisning. Avhandlingen visar att kognitiv aktivering har ett positivt samband med elevers självförmåga och matematikprestation. Avhandlingen visar också att kognitiv aktivering ofta kombineras med tydliga presentationer, modellering, frågor, struktur och möjligheter för elever att delta i en matematisk diskurs. För matematikundervisning innebär detta att didaktiska val inte bara handlar om att välja utmanande uppgifter, utan också om hur arbetet med dessa uppgifter introduceras, stöds, diskuteras och regleras under lektioner.

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Opportunities for Cognitive Activation

Mathematical thinking is central in mathematics education. Opportunities for such thinking can be conceptualized through cognitive activation: practices that support students' engagement with challenging tasks, subject discourse, and reasoning. This thesis explores intended, enacted, and experienced opportunities of cognitive activation, and their relation to self-efficacy, test anxiety, and achievement.

Across four empirical papers in Swedish secondary education, the thesis examines classroom-level and individually perceived cognitive activation, teachers' intended facilitation of a challenging task, and observed segment types across lessons and classrooms.

Findings show that cognitive activation is constituted by interrelated dimensions, including task and interaction demands, teachers' facilitation, and subject discourse. Experienced cognitive activation was positively associated with self-efficacy, which may mediate between cognitive activation and achievement. Planned facilitation varied across instructional events, and observed lesson segments differed in cognitive activation and instructional clarity.

Overall, the thesis contributes to theorising and studying cognitive activation as a multidimensional opportunity structure and offers insights for didactical decision-making.

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