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Mathematical activities in school-age educare

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ABSTRACT

Since 2016, the Swedish school-age educare has had a clarified mission regarding knowledge and teaching, and mathematics has been an explicit part of the education. Drawing on six universal mathematical activities, this study aims to contribute knowledge about the experiences and lessons learned within a project in which school-age educare teachers, in collaboration with researchers, plan, implement, and evaluate mathematical activities. The research process was guided by an action learning model and involved four teachers working at one school and represented four departments. The results indicate that the challenges arising during the processes were threefold: (i) gaining a sufficiently clear understanding of the teachers' knowledge and experience with mathematical activities to provide optimal support during the planning phase, (ii) striking a balance between the teachers' own ideas and the level of guidance provided by the researchers in designing a teaching session, and (iii) planning, conducting, and evaluating instruction within an educational context that is defined only by central content, without specific learning objectives for students. Nonetheless, the study demonstrates the potential of school-age educare to offer students subject-based teaching through practical-aesthetic forms of expression and a play-based pedagogical approach, as well as the opportunity to learn and develop in an educational context where their performance is not formally assessed.

1. Introduction

Over the last 20 years, interest in students' time outside of the compulsory school day and how it can be used to support and stimulate their learning and development has increased across the world. To refer to this period of learning time after school, various terms have been used, such as school-age educare (Australia), after-school programs (the US), extra-curriculum programs (Japan), and all-day schools (Germany and Switzerland), all of which are covered by the umbrella term *extended education* (Schuepbach, 2018). In some countries, extended education programs are organized as an extension of the school day, which implies that students are offered assistance with homework and/or various school subjects. Other programs, such as Swedish school-age educare centers,¹ provide space for play and creative forms of expression (Swedish Institute for Educational Research, 2021). SAEs should stimulate students' holistic learning and development, offer them meaningful leisure time, and complement the compulsory school's teaching with students' needs, interests, and initiatives forming the basis of the activities.

In Sweden, extended education through SAEs is an important part of many students' education and childhood (Hjalmarsson & Odenbring, 2020). Up to 85 % of all younger students are enrolled and have access to SAEs before and after the school day and during school holidays. Thus, SAEs have significant potential to support the learning and development of the vast majority of younger students in Sweden. SAEs are governed by the Education Act (SFS, 2010, pp. 800, 2010, p. 800) and the curricula for compulsory school, preschool class, and school-age educare centers (Swedish National Agency for Education, 2022). In 2016, the curriculum was revised, and the commission of SAEs was clarified (Swedish National Agency for Education, 2016). Since then, children enrolled in SAEs have been considered students, and the content offered is no longer regarded as activities but as teaching, with SAEs becoming a more prominent part of the school system. However, the school-age educare center lacks knowledge objectives for students to achieve. Teachers may have an idea that students should learn something specific during an activity, but the learning is not to be assessed or evaluated.

Teachers should safeguard the traditional focus on care and ensure

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E-mail addresses: maria.hjalmarsson@kau.se (M. Hjalmarsson), maria.fahlgren@kau.se (M. Fahlgren), karin.vage@kau.se (K. Våge).¹ Henceforward abbreviated SAEs.

that students' interests, initiatives, and needs are considered. Furthermore, they should conduct teaching that corresponds to SAECs' core content, as outlined in the curriculum: (a) language and communication; (b) creative and aesthetic forms of expression; (c) nature and society; and (d) games, physical activities, and outdoor excursions (Swedish National Agency for Education, 2022). These four categories of knowledge include several aspects that do not necessarily have to be regarded as part of SAEC teaching; instead, they function as guidelines for what the teaching should cover. Some of these aspects are related more or less explicitly to mathematics: (a) mathematics as a tool to describe ordinary phenomena and solve ordinary problems; (b) design and construction using different materials, tools, and techniques; and (c) different materials, tools, and technologies for creating and expressing oneself. By offering different ways of approaching mathematics, SAECs can foster the emergence of relations with mathematics other than those established in the mathematics classroom during compulsory schooling. However, two years after the implementation of the revised curriculum, the Swedish School Inspectorate found very few examples of teaching that stimulated students' mathematical thinking (Swedish School Inspectorate, 2018). It should be mentioned here that SAEC teachers are not trained subject teachers and that their training includes only shorter subject courses. Their teacher education is instead mainly directed toward practical aesthetic subjects and leisure pedagogy. Despite their SAEC work before and after school hours, as well as during school holidays, the teachers also teach during the compulsory school day.

This study is part of an overall project whose purpose is to explore and develop teaching in the SAEC setting. The study aims to contribute knowledge about the experiences and lessons learned within a project in which SAEC teachers, in collaboration with researchers, plan, implement, and evaluate mathematical activities.

The following research questions are focused on:

- (i) What challenges emerged during the project's processes?
- (ii) How can these challenges be understood in relation to the unique educational context of school-age educare centers?

2. Mathematics: the universal activities

To analyze the mathematics work in SAEC, we used Bishop's (1991) cultural perspective on mathematics education. In total, he identified six cross-cultural "activities and processes which lead to the development of mathematics" (Bishop, 1991, p. 22). Whereas some of the activities are related to our physical environment (*counting*, *measuring*, *location*, and *designing*), others are concerned with our social environment (*playing* and *explaining*). Bishop (1988) called these six activities "universal" because they reflect mathematical ideas generated by all cultural groups. The six mathematical activities are briefly described below.

Counting is probably the most obvious activity and perhaps also the most fundamental mathematical idea, as it forms the basis of number systems. The activity of counting concerns answering the question, "How many?"

Locating is related to the spatial environment and the positioning of "oneself or other objects within the spatial environment" (Bishop, 1991, p. 38). Locating can be achieved by using maps or models of the physical environment or by describing locations in words.

Measuring, a significant activity for the development of mathematical knowledge, concerns comparing, ordering, and quantifying and is a prerequisite for developing measurement units.

Like *locating*, *designing* concerns the spatial environment, but with a focus on creating shapes or objects that are part of the spatial environment. Designing also involves reshaping the natural environment based on imagined shapes and forms. According to Bishop (1991), the idea of geometrical shapes is a central part of *designing*.

Whereas the aforementioned activities can easily be recognized as mathematical, the relationship between play and mathematics is not that obvious at first glance. However, when one considers the rule-

governed nature of mathematics, this relationship becomes more evident. Engaging in play is a social activity in which participants agree on more or less formalized rules. For example, Bishop (1991) regarded *playing* games as "a formalization of playing" (p. 44). Moreover, Bishop indicated that the central mathematical idea of abstraction was a feature of playing. In play and games, certain aspects of reality are abstracted while others are neglected, which enables distancing oneself from reality and practicing abstract thinking.

According to Bishop (1991), mathematics in itself is "a very particular and powerful means of explanation" (Bishop, 1991, p. 58), in which the ultimate form of *explanation* is (formal) proof. Consequently, the activity of explaining is central to learning mathematics. This activity involves answering "the complex question of 'Why?'" (p. 48), which goes beyond one's experience of action in the environment. Explaining entails making logical connections between different phenomena. Bishop (1991) states that similarities and classifications are central to this activity.

3. Mathematical activities in early childhood education

Swedish SAECs offer a more informal learning environment than compulsory schools, which does not mean that teaching is unplanned or incomplete (Swedish Institution for Educational Research, 2021). Rather, situational and experience-based teaching should promote every student's imagination and ability to learn with others through creativity and play. The role of play and games in mathematical learning has been the focus of a large body of international literature. However, regarding play and mathematics, much of the research has been conducted in the field of early childhood mathematics education (e.g., Helenius et al., 2016; Seo & Ginsburg, 2004). For example, in a study of 90 five-year-old preschool children, Seo and Ginsburg (2004) found that young children engaged in a wide range of mathematical activities during their free play. In addition to counting, the children engaged in grouping and sorting objects by attributes (*classification*), comparing the size of objects either directly or side by side (*magnitude*), exploring geometric shapes or patterns and relationships, and describing or locating spatial relations. Although, as Ginsburg (2006) argued, this kind of play is based on children's everyday mathematics in terms of informal skills and other everyday experiences, the activities observed in Seo and Ginsburg (2004) were closely connected to school mathematics content.

Helenius et al. (2016) pointed out that, in research on students' play, such play is often regarded as a vehicle for students' learning, including the learning of mathematical content. However, considering that play is closely connected to mathematical processes rather than to content, Helenius et al. (2016) suggested that a mathematician's view of play is more appropriate. For a mathematician, processes such as creativity and problem solving are important. This view matches what De Holton et al. (2001) termed "mathematical play," which they regarded as a mathematical problem-solving process. From this perspective, the focus is on play as "doing mathematics" rather than being a vehicle for learning mathematical content (Helenius et al., 2016).

Drawing on the literature on the relationship between play and mathematics, Helenius et al. (2016) suggested three groups of interrelated features that make young children's play mathematical: participatory, creative, and rule-governed. These features stem from Bishop's (1991) description of playing as a cross-cultural mathematical activity. He stated that "playing is a form of social activity" (p. 43), in which the participants agree on "social procedures and rules of performance" (p. 23). Play cannot occur unless all participants follow the rules negotiated in the play situation. Creativity is closely connected to the view of play by a mathematician (Helenius et al., 2016), which aligns with De Holton, Ahmed, Williams, and Hill (2001) term "mathematical play," which is, according to them, "that part of the process used to solve mathematical problems, which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion" (p. 403).

In this view, play can support children's creativity and conjecturing, which resembles a mathematician's way of working. According to Ginsburg (2006), when playing, young children pose and reflect on deep questions while trying to solve real problems. In doing this, they both use mathematics (mathematical strategies and ideas) and "play with mathematics" (Ginsburg, 2006, p. 158).

For preschool children, free play is key, as reflected in the literature. Regarding the play of older children, which is the focus of this article, such play is more structured and organized (Sarama & Clements, 2009). It can involve imagining a shop or a café, with participants taking on different roles and negotiating rules. Games with prearranged rules are even more structured (Sarama & Clements, 2009). Bishop (1991) argued that the idea of a game, which he regarded as a formalization of play, has been present in all cultures for a long time. He also acknowledged that a large body of literature suggests various classifications of games, thus reflecting the wide range of existing games.

Since the late 1960s, researchers have acknowledged the potential of using games for learning school mathematics (Mousoulides & Sriraman, 2014). In addition to using games for educational purposes, such as developing conceptual knowledge or practicing skills, many teachers use games to motivate and engage students (Russo et al., 2021). In a survey that explored 248 primary school teachers' use of games in mathematics classrooms, Russo et al. (2021) reported that they frequently used them. According to this study, the teachers preferred simple games that required few or no materials, such as cards and dice, although one unexpected result was that only two of the surveyed teachers mentioned digital games as their favorite option for working with students. This was surprising, given the rapid development of educational digital games over the past two decades (e.g., Pan et al., 2022).

There are many different types of games and reasons for using them. Concerning mathematics education, some games are intended to develop mathematical knowledge in specific domains, such as arithmetic, geometry, and probability. The predominant mathematical topic has been arithmetic (e.g., Pan et al., 2022). There are also games that are not primarily developed for mathematical learning, but are still used in school mathematics. Researchers have also highlighted the possibility of using games to improve generic knowledge, such as collaboration and problem-solving skills (Mousoulides & Sriraman, 2014).

4. Mathematical activities in extended education

Research on mathematics in Swedish SAECs is scarce (Wallin, 2022). However, international studies conducted in extended or extracurricular education have actualized the mathematical activities upon which Bishop (1991) reflects. The *counting* activity occurred in Barreto et al.'s (2017) study, in which a pair of students played two games embedded in the virtual online game *Club Penguin*. In one of the games, the students were encouraged to count "bolts" to earn virtual coins. In a study by DeLiema (2017), students received individual help with their homework dealing with the four arithmetic operations (i.e., addition, subtraction, multiplication, and division). Stott and Graven (2013) focused on arithmetic in their mathematics clubs, which they described as "informal" learning environments guided by national curriculum documents. For example, they introduced manipulatives and card games to reinforce students' basic number sense. Other studies included fractions and proportional reasoning (Mueller et al., 2012; Razfar, 2013; Turner et al., 2013; Vomvoridi-Ivanović, 2012). In studies focused on developing students' critical mathematical literacy skills, students were encouraged to work on projects with a real-world context. The students engaged in counting and calculating fractions and percentages, which were then organized into tables (Slayton et al., 2018; Turner et al., 2009).

Previous studies that actualized the mathematical activities of *locating*, *measuring*, and *designing* were all related to the topic of geometry. The math games used in Barreto et al.'s (2017) study encouraged

students to identify geometric shapes to build ramps out of geometrical snow blocks. Moreover, the game offered hints in terms of pop-up drawings that included geometric symbols, such as right angles and angle congruence. In studies focusing on recipes (Razfar, 2013; Vomvoridi-Ivanović, 2012), the students used kitchen measurement units, such as cups and teaspoons, to determine the volume of various ingredients. Lundbäck and Egerhag (2020) described an activity in which students were encouraged to estimate and measure the weight of various objects to become familiar with the units of grams and kilograms.

To increase numeracy proficiency among young learners in South Africa, Stott and Graven (2013) and Graven (2015) organized informal after-school mathematics clubs. In contrast to the ordinary classroom, Stott and Graven (2013) argued that the activities were less teacher-controlled and that the learners had more choice regarding which activities to engage in. Typically, activities included *playing* mathematical games using cards and manipulatives (Stott & Graven, 2013). The games were simple, requiring cards or dice, which meant that the students could practice the games at home (Graven, 2015). In their investigations of the linguistic resources used by bilingual students when handling probability problems, LópezLeiva et al. (2013) used game-like tasks. For example, they observed the communication between three students playing a spinner game. In the study by Marshall (2004), the students played the mathematical puzzle game *Logix*. As in the study by LópezLeiva et al. (2013), the students solved mathematical problems, although in this case in geometry instead of probability.

The mathematical *explaining* activity became evident in studies on students' proportional reasoning. The findings highlighted the importance of linguistic (LópezLeiva et al., 2013; Razfar, 2013) and cultural resources (Vomvoridi-Ivanović, 2012) for these students in making sense of the problems. These findings are aligned with those of Turner et al. (2009, 2013). For example, to discuss and explain their thinking when working with their peers, the students used "multiple discourses and forms of communication, including oral explanations, physical enactments, gestures, diagrams, symbols, inscriptions, everyday idioms, and the use of two languages" (Turner et al., 2013, p. 367). All these studies suggest a tension between formal learning environments and students' out-of-school experiences, which can be said to correspond to the borderland of leisure and school that the Swedish SAEC constitutes.

5. Methodology and methods

In relation to the Swedish SAEC perspective, Wallin (2022) argued that when mathematics is allowed to emerge from students' and teachers' genuine engagements and interests, concrete tools for teachers to recognize and understand "mathematics" become available. The research process was inspired by an action learning model with close collaboration between researchers and teachers (Hjalmarsson & Söderström, 2014), in which mathematical activities were planned, carried out, and then reflected upon and evaluated during dialogue meetings before being developed, tested, and re-evaluated. This approach to practical action learning, which involves both researchers and teachers as active participants in the research process, offers tools for creating knowledge. Thus, the project contributes to creating practical improvements and the development of teachers' understanding of their own teaching practices (McNiff, 2013; Timperley, 2013; Zuber-Skerritt, 2002).

6. Context and participants

This study was conducted at a single school with four SAEC departments. Initially, we invited the headmaster to discuss the project and what participation would entail. The focus would be on the teachers planning and evaluating teaching in the area of mathematical activities together with the researchers, as well as teaching on their own without the involvement of researchers. The headmaster showed great willingness to let teachers from the school's four SAEC departments be

involved, not least because the local authority had made demands on the municipality SAECs to develop teaching, including that of mathematics, on a scientific basis. Participation in the project would thus be part of such work. The headmaster gave us permission to contact the teachers themselves to inform them about the project and its ethical principles. Four teachers, one from each of the SAEC departments, agreed to participate in audio-recorded dialogue meetings in which teaching in the area of mathematical activities would be planned and evaluated in collaboration between teachers and researchers. They also agreed to carry out mathematical activities in the groups of students between the dialogue meetings. The process planning was colored by the headmaster's instruction that the teachers could deviate from their regular SAEC work on a total of seven occasions of 2 h each during the academic year.

The teachers' educational backgrounds varied. Some were qualified SAEC teachers, while others had other teaching qualifications. All four had an interest in mathematical activities, and three had, during their education, studied courses specifically aimed at creative mathematics. The teachers represented SAEC departments in which students between the ages of six and eleven were enrolled. At the time of the start of the study, these departments had no formal collaboration regarding teaching.

The research project was scrutinized by the university's ethical board, and the research process was led by Swedish guidelines for good research practice (Vetenskapsrådet, 2017). The ethics committee did not require that the study be reviewed further by the Swedish Ethical Review Authority. The teachers were given both written and verbal information about the aim of the project, the use of empirical data, and their right to withdraw their consent to participate. All teachers provided informed consent in writing before the first dialogue meeting began.

7. The process of planning, implementing, and evaluating teaching

At the first meeting, the researchers gave short lectures about leisure pedagogy perspectives on teaching and Bishop's (1991) theory on the mathematical activities (*count, measure, locate, design, play, and explain*). A short film about how these activities could be expressed in preschool was shown because a corresponding film recorded in an SAEC was not available.

Furthermore, the tool used to plan teaching at the SAEC was discussed. The tool relates to the SAECs' core content as outlined in the curriculum, and consists mainly of tickable pre-printed checkboxes (Appendix 1). As some of the teachers were unaware of the tool or had never used it themselves, they were tasked with discussing what they saw of these activities in their educational settings, ready for presentation and discussion at the next meeting.

At the second meeting, the teachers told the researchers about their observations, creating a starting point for reflective dialogues. The researchers introduced a pedagogical planning tool that had been developed within the framework of a research circle conducted in another study within the overall research project in collaboration between researchers and SAEC teachers in another municipality (Carlman, submitted). This tool was characterized, among other aspects, by a greater scope for the teachers' own formulations in relation to the teaching's overall theme, as well as ideas for, and identification of, learning components. The researchers encouraged the teachers to plan an activity rooted in their regular teaching, to base the planning on Bishop's (1991) framework, and to involve the students in the planning process. Based on this, the teachers started on their own, with the support of the tool, to plan a mathematical activity that they would carry out on their own together with the students (Appendix 2).

The intended mathematical objectives of the first activity were:

- To use and reinforce methods for calculating with natural numbers using the three arithmetic operations—addition, subtraction, and division—within the number range 1–20.
- To apply proportional relationships, including doubling and halving.
- To interpret word problems and select appropriate methods to solve them.
- To develop an understanding of various strategies for arithmetic operations, thereby expanding one's own repertoire.
- To use common words to describe numbers (e.g., more, fewer, half, double, decrease, increase).

At the third meeting, the teachers presented their planning to the researchers, who gave their feedback. One of the decisions based on this was that one of the teachers would act as an observer of the activity to be carried out of the other teachers. The teachers chose to implement the activity outdoors during the autumn holidays.

At the fourth meeting, at the close of the beginning of the following semester, the teachers told the researchers about what had happened during the implementation, after which the teachers and the researchers conducted reflective and evaluative dialogues on the subject. The implementation of the activity lasted for approximately 30 min, during which the students worked in mixed-aged groups of 3–5 students per group. This was the first of two teaching sessions which the teachers planned within the framework of the project.

During the following semester, the project continued, starting with what the researchers had picked up in the teachers' reflections about their teaching in relation to mathematics during the previous semester: collaboration, "talking mathematics," and the importance of the outdoor environment. The researchers rehearsed Bishop's math activities for the teachers and presented a revised planning tool that they had created. This planning tool became the basis for planning yet another mathematical activity, taking into account the teachers' focus on collaboration, "talking mathematics," and the importance of the outdoor environment (Appendix 3). Since the researchers, during the reflection discussion following the implementation of the first activity, noted that the students' involvement in the planning stage had been insufficient, the teachers were encouraged to pay particular attention to the importance of student influence in the planning of the second activity.

The intended mathematical objectives of the second activity were:

- To use common positional words to describe length (e.g., long, short, longer, shorter, the same length).
- To compare and estimate lengths.
- To measure length using standard, non-standard, and historical units of measurement.

The teachers planned and carried out a mathematical activity in the absence of the researchers. This was the second of two teaching sessions designed by the teachers as part of the of the project framework.

In short, the methodology meant that the teachers largely carried out the planning work on their own, and that the researchers' role was to prepare them (through lectures and film) for the planning, as well as to provide feedback on the written plans before the activities were implemented.

The project ended with a reflective and evaluative dialogue focusing on experiences of using the revised planning tool and experiences after the implementation of the teaching element. The process is illustrated in Appendix 4, and the instructional design steps presented in Appendix 5.

The empirical data consist of three teachers' and two researchers' audio-recorded oral reflections on planning, evaluation, and completed teaching in the form of mathematical activities.

8. Analysis

The process of analysis was guided by Braun and Clarke (2006), holding a movement between raw data, codes, and themes. First, each

researcher listened repeatedly to the audio recordings, after which the researchers jointly formulated initial codes. Thereafter, the codes were collated into potential themes to be reviewed in the next step, which involved checking whether the themes worked in relation to the coded extracts as well as to the dataset as a whole. In the last step before starting the actual writing process, the identified themes were named as the researchers' governance of the teachers' ideas, formulating aim and goal connected to the activities and the ambition to let the unique SAEC educational context leave its mark on the activities. In this way, the analysis was carried out through an inductive approach, in which the strongly identified themes were linked to the data themselves.

9. Results

This section discusses three themes with particular focus on the challenges that emerged during the processes of planning, implementing, and evaluating mathematical activities in the SAEC: the researchers' governance of the teachers' ideas, formulating aim and goal connected to the activities and the ambition to let the unique SAEC educational context leave its mark on the activities.

10. Researchers' governance of the teachers' ideas

During the research process, the researchers strove for an approach whereby the teachers' own ideas would be given considerable space, while the researchers would be able to provide qualified feedback on the teachers' planning of activities and their subsequent reflections.

The researchers' stated ambition was that the teachers would plan an activity that was or could have been part of their regular teaching. One reason for this was that the teachers' participation in the project would not entail any additional burden on an already stressed professional group because the planned activity would be linked to regular everyday practice. With this approach, the researchers also wanted to let the students' needs and the teachers' competence in teaching creative mathematics in the SAEC be the guiding principle. In addition, the researchers were clear that activity planning should take into account the students' interests and initiatives, as this is a central point of departure for SAEC education. Another expressed ambition was that the teachers would connect the planning and implementation of the task to Bishop's (1991) mathematical activities.

After hearing the researchers' lectures about SAEC teaching and Bishop's theoretical framework, the teachers began planning a mathematical activity to carry out with the students outdoors during the upcoming school holiday. The teachers used the planning tool developed in a collaboration between teachers and researchers in a substudy within the larger research project (Carlman, submitted). The researchers gave feedback on the planning, both in terms of form and content, which the teachers then developed.

The activity was planned to be based on a number of "Halloween cards" that the teachers would hide and the students would have to find. It emerged that the teachers started the planning with the central content for SAEC teaching in mind, but they also selected central content from the mathematics curriculum for the compulsory school. Questions were prepared to be asked orally to the students throughout the activity, based on the number of cards. Examples of questions were: *How many cards have you found?*, *If you remove seven cards, how many do you have left?*, and *If you divide all the cards evenly into two piles, how many cards are in each pile?* The mathematical content was addition, subtraction, and division in the number range 1–20, and the current concepts expected to be used were *number*, *more*, *fewer*, *half*, *double*, *decrease*, and *increase*. The researchers challenged the teachers' ideas by encouraging them to extend the activity to include both aspects of subtraction, *comparing* and *subtracting*, and to let the students formulate their own questions around the cards. The researchers also noted that the teachers had not designated which of them would be responsible for the actual implementation of the activity and which would observe the implementation. They then

carried out the activity during an upcoming school holiday without researchers present.

At the subsequent dialogue meeting, the teachers told the researchers that, for various reasons, they had been obliged to carry out the activity indoors instead of outdoors, and that the number of participating students had become fewer than planned. Nine students between the ages of six and nine years had participated. Experiences of working with mathematics at the SAEC and with the two planning tools—one used by some of the teachers in the past and the other presented by the researchers to the teachers, who tried it out—were discussed. The teachers saw strengths and weaknesses in both planning tools from different aspects and concluded that a compromise between the two variants could be a good alternative. The researchers created a third planning tool for the teachers to try in the next process.

11. Formulating aim and goal connected to the activities

The teachers reported that during the Halloween card activity, when dividing by two, the students had used two different strategies. Most of the students knew that half of 20 was 10 and solved the problem immediately, while others distributed the cards into two piles, one at a time. The activity also seemed to provide an opportunity to challenge the students' solution strategies at different levels; this became apparent when one of the students explained division by 4 as "first half of 20 and then half again." Another student said, " $4 \times 5 = 20$, so there will be 5 in each pile."

This also became evident when the students were asked to calculate $8 + 5$. Some solved the task by counting on their fingers (one by one), while another student solved it mentally by breaking 5 into 2 and 3, thereby using the strategy of "bridging through ten" with $8 + 2 + 3$.

Similarly, when solving the subtraction $20 - 7$, students discovered different ways to approach the task. Some used the method of removing one card at a time, while another student solved it mentally by applying the number bond of 10 for 7 (i.e., 3).

The teachers said that the students had appreciated the activity, saying that "it didn't feel like math."

The teachers' accounts of the implementation of the activity showed that it had not been expanded with the proposals launched by the researchers at the previous dialogue meeting. Therefore, at the next dialogue meeting, the researchers chose to repeat these activities for the teachers, urging them to more clearly include and stage some of Bishop's six mathematical activities when planning the next teaching session. The teachers decided that the activity should concern measurement, because they knew that the students had worked a lot with this during the compulsory school mathematics lessons.

In the implementation, which took place outdoors, 10 students between the ages of seven and eight years participated. The teachers told us that they chose to divide the students into pairs who were given the task of "measuring themselves with sticks." It turned out that the task created greater commitment in the students than the teachers had expected. Only a few students lay down on the ground and measured themselves with several short sticks, a strategy that the teachers believed would be more common. The teachers went around between the pairs and watched and documented with a camera how the students chose to solve the task. Concepts such as *short*, *long*, *shorter*, *longer*, *the same length*, *more*, and *fewer* were used by the students. The pairs worked some distance from each other in the forest and could only partially see how the other pairs chose to solve the task.

During the implementation, the teachers discovered a possible further development of the activity by starting a joint discussion in the student group about the consequences of the different lengths of their measuring sticks. During the dialogue meeting, the teachers further reflected that another development of the activity might have been to measure with sticks of equal length or to restrict the measurement to a certain fixed number of sticks. Since no accounting or discussion of the pairs' measurements was carried out, the activity could have been

poorer in mathematics than it actually was. However, in a subsequent discussion, when the teachers brought the student group together, one student brought up units of measurement, such as cubits and feet. The teachers had not really planned to touch on the area at all, but chose to pick up on the student's input and led the discussion further towards contemporary and accepted measurement tools and units of measurement. In other words, the teachers captured the students' interest and initiative in the moment and further developed the activity in real time. By doing so, the activity gained more mathematical content, thanks to the students' input, than the teachers had planned. The pros and cons of the various measuring tools talked about during the activity were discussed with the students. Alternative measuring tools in the forest, such as needles, cones, and stones, came up for discussion. One student summed it all up as follows: "Spruce cones are better than pine cones because then we don't need so many."

At the subsequent dialogue meeting, when the teachers discussed, reflected on, and evaluated the implementation with the researchers, the teachers said that the conversation with the students had been good, but also noted the requirement for tact to meet and capture the students' ideas in the moment in a way that would promote learning.

To a very limited extent, the two activities (the one with the Halloween cards and the one involving "measuring oneself with sticks") were planned using Bishop's (1991) theory of the six mathematical activities. It can be understood that the researchers had too much faith in the teachers' knowledge of (in the researchers' words) "creative mathematics," and therefore did not sufficiently guide the planning. This, in turn, meant that the researchers' ambition for the teachers' knowledge of SAEC teaching to set the tone in planning the mathematical activities had a negative impact on the quality of those activities. In other words, one of the challenges that emerged during the process was about an obvious difficulty for the teachers to formulate the aim and goal of the activities and to then connect it with a mathematical content. The objective of the first activity was to observe how the students use different calculation methods, as well as to practice sorting and collaboration. The second activity was formulated in terms of the students measuring, collaborating and conversing, without specifying what the students would learn. Overall, this spotlights a problem that can arise in research projects when the researchers' insufficient knowledge of the teachers' prior knowledge in a teaching area can lead to uncertainty about how much control of the teachers' planning the researchers need to exercise while maintaining the ambition that the initiative must be rooted as far as possible in the teachers' mathematical knowledge and their knowledge of the needs of the student group.

12. Letting the unique educational context leave its mark on the activities

The researchers insisted that pedagogical planning tools should be used to make the activities as thoughtful as possible. The use of planning tools was a challenge in itself, as the teachers were not used to using them. They had the easiest time using the planning tool that was already available at their own school and that actualized abilities in relation to the four central contents for SAEC teaching as well as the teachers' division of responsibilities in the implementation of the activity. This tool simply required the teachers to tick off the abilities to which the various activities related, and did not require any other wording about the activity's purpose or goals. This challenge became even clearer when the researchers introduced another planning tool developed by teachers and researchers within the framework of a research circle in another sub-study within the framework of the larger research project. This tool required the teachers to express themselves in free text to a greater extent about aspects that included the overall theme and idea, the identification of learning components, the teaching goals, and how to take the students' backgrounds and experiences into account in the activity. The challenge for the teachers to use the last-mentioned planning tool in particular seemed to be rooted in the fact that they were not used

to formulating a pedagogical plan in writing, but also in that the headings in this tool contained words and themes that the teachers did not feel were natural in relation to the wider SAEC teaching framework.

Another challenge that emerged stems from the fact that the SAEC is only covered by aspirational goals and not knowledge goals for students to achieve. The part of the curriculum that specifies the purpose and content of the SAEC presents the four central contents to which the teaching must relate. However, the content connected to mathematics is vaguely formulated and open to interpretation, which means that the SAEC teaching with a connection to mathematics can take on very different expressions and be directed at different mathematical aspects. Consequently, the activities that the teachers planned did not include a precise statement of what the students were meant to learn during implementation. As the SAEC is supposed to complement compulsory school teaching, it requires teachers to know the learning objectives in the curricula for the grades covered by the SAEC. The teachers connected with the content of the compulsory school mathematics education, albeit to a rather low degree, when planning the Halloween activity. When planning the "measuring oneself with sticks" activity, they explicitly related to work with measurement that the students had been involved in at school. The teachers identified a challenge in creating the conditions for more systematic planning together with the teachers in the compulsory school, which in itself is important in order to use the SAEC's potential to contribute to the students' learning and development.

SAEC teaching must, among other things, offer students meaningful leisure. The meaning of *meaningfulness* is not defined in the curriculum, which raises questions about whether it is the teachers' or students' perspectives that should be given interpretive priority. For the teachers, it was important that the students had fun during the activities. This goal seemed superior to the importance of the purely mathematical content. The teachers identified a challenge in implementing mathematical activities that the students would be attracted to and want to participate in. When the students expressed after the implementation that "it didn't feel like mathematics," the teachers interpreted this as a sign that the activity had been successful. The teachers themselves repeatedly emphasized at the planning stage that the activities "would not be school."

13. Discussion

The study shows examples of SAEC's potential to contribute to students' learning and development through play pedagogical and practical aesthetic methods with a different emphasis and starting point than compulsory school teaching. The researchers encouraged the teachers to include the students' initiative in the planning of the activities, but this was not heeded. This can be considered one of the study's limitations. On the other hand, the results showed that during the implementation of the activities, the teachers clearly incorporated the students' input and allowed them to contribute to developing the mathematical content. This can be related to the previous research in extended education settings presented above in which none of the studies were explicitly planned and conducted to address students' expressed needs, initiatives, and interests. Adopting the student perspective as one's point of departure is a clearly defined goal in the curricula that govern the Swedish SAEC setting (Swedish National Agency for Education, 2022). We consider this approach fruitful for creating knowledge-promoting learning environments in which all students can develop their skills. We believe that it is an important part of supporting student motivation to engage in mathematical activities in general and in extended education settings more specifically.

Therefore, we emphasize the importance of stimulating students' interest and confidence in mathematics from their early years and in various educational settings. In this endeavor, the genuine engagements of both students and teachers create central points of departure, as Wallin (2022) argued in relation to the Swedish SAEC perspective. This

approach, which is similar to the one advocated by Marshall (2004), treats students as creators of mathematical ideas.

SAEC holds a complementary mandate in relation to the compulsory school system. Activities within SAEC may be deliberately designed to complement formal school curricula by drawing on the play-oriented pedagogy and practical-aesthetic methods that are distinctive of SAEC educational practice. This approach has the potential to support multisensory learning experiences for students.

Moreover, we emphasize the great potential of extended education when it comes to providing students with other ways of handling mathematics and with working methods other than those that they experience in compulsory schooling. Activities within SAEC may be deliberately designed to complement formal school curricula by drawing on the play-oriented pedagogy and practical-aesthetic methods that are distinctive of SAEC educational practice. This approach has the potential to support multisensory learning experiences for students.

This, in turn, should affect students' perceptions of what mathematics "is" and can be. As Razfar (2013) put it, it offers students ways of moving beyond static learning roles. We also emphasize the importance of taking advantage of pupils' free play to support their engagement in mathematical activities, as noted in Seo and Ginsburg's (2004) study. Like Lundbäck and Egerhag (2020), we also emphasize the value of cooperation between different learning contexts, as different teacher competences can contribute jointly to mathematics education for all students. We see a need for continued research on ways in which students could be involved in and exert influence on work with mathematics and on how cooperation between compulsory schooling and extended education could create the best conditions for fostering interest and confidence in mathematics, along with good mathematical knowledge for all students.

The researchers explicitly asked the teachers to take Bishop's (1991) theoretical framework into account when planning the two activities, but it turned out that they chose to consider it to a rather low degree. This can be considered another limitation of the study. However, in the teachers' reflections on the implementation during subsequent dialogue meetings, it became clear that Bishop's (1991) six mathematical activities helped the teachers capture and express the content when they noted that *playing*, *explaining*, *measuring*, and *counting* had all come into use in different ways in the activities. The teachers reflected that they had received tools enabling them to plan teaching to a greater degree than before, taking into account different dimensions in mathematics, and to carry out teaching directly aimed at specific mathematical activities defined by Bishop (1991) rather than at mathematics in general. This suggests that Bishop's activities could serve as a practical tool for teachers during the planning stage of mathematical tasks. By providing a shared conceptual framework, these activities help teachers more effectively identify and emphasize the mathematical content within the activity.

The study focuses on a small number of teachers and activities. Accordingly, we do not claim that the results can be generalized to the SAEC profession as a whole. A strategy for future larger studies could include a research design that combines digital components with in-person meetings with participating teachers, which would allow for the inclusion of a significantly larger group of participants. The challenges highlighted by the present study may serve to improve the conditions for involving students more explicitly from the planning stage and for making the learning objects of the activities more clearly defined. This ambition was indeed present and conveyed to the teachers during the initial planning phase; however, when the teachers proceeded to finalize the planning independently of the researchers, student involvement was not given a prominent position.

If the process had been able to continue for a longer period, the study's process validity could have been increased. It was clear that through the process, the teachers gained new insights into the planning, implementation, and evaluation of teaching. However, a longer process with more actions could have generated questions and answers that

would then have formed the basis for the reflective discussions. In this way, the catalytic validity could have been strengthened. This also relates to the study's outcome validity, by making visible whether, and if so how, the various components methodologically created conditions for deeper understanding and change in relation to the aims and objectives of action learning (Anderson et al., 1994).

We would like to conclude this section with an open question about how a teacher training program aimed at work in SAEC can best be arranged to promote the teacher students' ability to conduct teaching where leisure pedagogy meets a subject content, and where the student's initiatives, interests, and needs have a real impact on the teaching. The Swedish School Inspectorate (2018) found very few examples of teaching that stimulated students' mathematical thinking, which can be related to the results of this study which showed obvious difficulties for the teachers to formulate the purpose and goals of the planned activities as well as to connect it with a mathematical content. Therefore, student teachers need to practice those types of tasks during their teacher training in order to respond to the mission of the SAEC, not least when it comes to the complementary mission vis-à-vis the compulsory schooling.

14. Conclusion

To sum up, the challenges that emerged during the processes in which SAEC teachers, in collaboration with researchers, plan, implement, and evaluate mathematical activities were as follows: (a) the challenge of obtaining a sufficiently good picture of the teachers' knowledge and experience of working with mathematical activities to be able to provide the best possible support in the planning stage, (b) the challenge of creating a good balance between the teachers' own ideas and the degree of control from the researchers when planning a teaching session, and (c) the challenge of planning, implementing, and evaluating teaching in an educational setting covered only by a central content and lacking goals for students to achieve. Despite these challenges, the study shows the SAEC's potential to offer students teaching in a subject through practical aesthetic forms of expression and a play pedagogical basis. We also see potential in the fact that the SAEC context gives students the opportunity to learn and develop in an educational context where their performance is not assessed.

CRediT authorship contribution statement

Maria Hjalmarsson: Writing – review & editing, Writing – original draft, Validation, Supervision, Project administration, Methodology, Investigation, Conceptualization. **Maria Fahlgren:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Conceptualization. **Karin Våge:** Writing – original draft, Methodology, Investigation.

Declaration of the use of AI statement

AI was not used for any part of the work related to the manuscript submitted.

Data availability statement

The data consist of confidential audio recordings from teacher reflection sessions and cannot be shared publicly due to ethical and privacy considerations.

Ethical statement

This study was reviewed by Karlstad University ethics committee with the registration number HS 2023/563, April 23, 2024. The ethics committee did not require that the study be reviewed further by the Swedish Ethical Review Authority. The research adhere to the

Declaration of Helsinki, and informed consent was obtained from all participants.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

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