Proceedings of the 14th International Conference on Mathematical Creativity and Giftedness (MCG 14)

Part of the Combined ECHA and igMCG Conference on Inclusion and Sustainability in Gifted Education

Elisabet Mellroth, Scott A. Chamberlin,
Yvonne Liljekvist, Linda Mattsson,
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Faculty of Health, Science and Technology

Mathematics

PROCEEDINGS | Karlstad University Studies | 2025:22

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WORDS FROM THE EDITOR

Elisabet Mellroth
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INTRODUCTION

It is a great pleasure and honor to present the *Proceedings of the Fourteenth International Group for Mathematical Creativity and Giftedness (igMCG) Conference*. After many months of dedicated work by both the contributors and the editorial board, we are proud to share the final result.

The conference took place in Karlstad, Sweden, and was held in collaboration with our sister organization, the European Council for High Ability (ECHA). We hope this joint event fostered valuable collaborations that will advance the field of gifted education worldwide. Attendees were free to attend sessions from both ECHA and igMCG programs, allowing them to tailor their experience to their specific interests.

Over the course of the three-day conference, four keynote speakers—Kirsi Tirri, Jan Kwietniewski, Jorryt van Bommel, and Tracy Reiley—shared their insightful perspectives and expertise with all participants.

Although the event was a joint conference, the submission processes for the two organizations differed: ECHA presenters submitted abstracts, while igMCG maintained its tradition of requiring full texts prepared according to specific guidelines. As such, this volume includes only the full-text contributions from the igMCG participants. Additionally, we have included the full text of Jorryt van Bommel's keynote, as her presentation specifically addressed mathematical giftedness. Abstracts of the other three keynote speeches are also included.

ACKNOWLEDGEMENTS

As Chair of the Editorial Board for the *Proceedings of the 14th International Group for Mathematical Creativity and Giftedness (igMCG) Conference* on inclusion and sustainability in gifted education, it is my honour and privilege to extend heartfelt acknowledgements to all those who have contributed to this publication.

Firstly, I would like to express my sincere gratitude to our presenters and keynote speakers. By sharing your research and practical experiences, you not only influence and inspire others in the field but also promote reflection and advancement in both research and practice. Your contributions are essential in enriching and expanding educational opportunities for gifted students.

A special thank you goes to Karlstad University and the City of Karlstad for their commitment and collaboration in co-organising this joint ECHA and igMCG conference. The generous support of Professor Valerie Margrain—both personally and through the GiftED doctoral school and the Nordic Network on Gifted Education (NNGE)—has been instrumental in making this event a reality. Without Professor Margrain's unwavering support and dedication to the joint conference, the igMCG component would not have been able to provide such a comprehensive and enriching experience for our attendees.

I am deeply grateful to the Editorial Board for their tireless efforts. Your hard work, keen attention to detail, and commitment have been the foundation of this publication.

I would also like to extend my appreciation to our esteemed panel of reviewers, who volunteered their time and expertise to the review process. Your thoughtful engagement and dedication to maintaining high academic standards have ensured the quality of the contributions presented in these proceedings.

Lastly, I must acknowledge the outstanding team at the Conference Department of Karlstad University. Without your unwavering support in both technical and practical matters, the successful implementation of this conference would not have been possible.

Elisabet Mellroth
Chair, Editorial board
14th MCG conference 2025
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REVIEW PROCESS

The International Group for Mathematical Creativity and Giftedness (MCG) was established in 1999 with the aim of bringing together professionals with a shared interest in nurturing and supporting the development of mathematical creativity and the realisation of mathematical potential. Since its founding, the group has held 13 international conferences. MCG14 was the first to be hosted in Sweden.

For this conference, we introduced a presentation format known as a *short communication*, which is commonly used in Swedish mathematics education conferences. A short communication provides an opportunity to present an idea for a research project, an early stage of research, or a specific aspect of a study, such as the research design or chosen methodology. It offers a platform for receiving constructive feedback from colleagues at various stages of a project. This format proved to be a popular submission option.

In total, 41 texts were submitted: 18 full papers, 13 short communications, 7 workshops, and 3 posters. In keeping with the mission of igMCG—to include both researchers and practitioners in the fields of mathematics, creativity, and giftedness—authors were invited to submit proposals grounded either in research or in practice. Those submitting research-based proposals were also asked to volunteer as peer reviewers.

All submissions were reviewed by two reviewers through an open review process. Reviewers were provided with clear guidelines to ensure consistency and fairness in their assessments, and were encouraged to give constructive and inclusive feedback while maintaining high academic standards.

The editorial board was divided into three teams, each responsible for summarising the reviews for a specific category: (1) research papers, (2) practice-oriented papers, and (3) short communications, workshops, and posters. Based on the reviewers' feedback, the editorial board made decisions regarding acceptance, resubmission, changes in submission category, or rejection. These decisions were then communicated to the authors. The majority of authors were asked to make revisions and resubmit their work. The editorial board also oversaw the re-review process and continued correspondence with authors throughout.

Upon completion of the review process, the following number of proposals were accepted: 15 papers, 11 short communications, 7 workshops, and 4 posters. However, due to global circumstances in 2025, a few authors were unable to attend the conference. As a result, a slightly reduced number of texts are included in these proceedings.

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KEYNOTES

CHALLENGING STUDENTS THROUGH PROBLEM SOLVING

Jorryt van Bommel
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This article explores how problem solving can serve as powerful tools to challenge and stimulate young learners. Drawing on a decade of research in Swedish early childhood education, the study presents classroom cases where 6-year-old students engage with mathematical content through non-written, collaborative tasks. While the project did not explicitly target gifted students, several such students became visible through their advanced reasoning and creativity. Three case studies, Linn, Kim, and Sara, illustrate how problem solving created opportunities for intellectual growth, strategic thinking, and meta-cognitive reflection. The analysis highlights how problem-solving tasks not only deepen students' mathematical understanding but also offer differentiated learning experiences. The article argues that these practices serve as educational catalysts, enabling all students to engage meaningfully at their own level, while offering gifted students the chance to be challenged and develop their potential within the differentiated classroom context.

Keywords: Problem Solving, Problem Posing, Early Childhood, Gifted Students, Mathematics Education

CASE 1: PROBLEM SOLVING

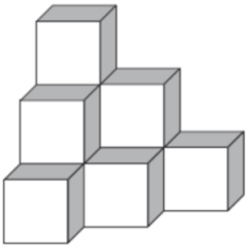

Figure 1 Problem solving task - Pyramid

Jorryt: Okay, so here is a picture of a pyramid made of cubes. How many cubes do you need if you want to build the pyramid?

Linn: Oh, that's easy, it must be 10 cubes.

Jorryt: How did you get that answer just by looking at the picture?

Linn: Well, the top one needs two extra and those two on the side need one extra each, so ten all together.

Jorryt: So, what if we would add another layer to the pyramid?

Linn: Hmm, okay let's see, three and two times two…no that strategy doesn't work – I have to think a bit now!

Linn solved the first task quite easily and could explain how she obtained the answer in a structured way. After she explained her answer, I decided to offer her another problem in which she had to change strategy. After a while, Linn came up with an answer for a four layered pyramid and started explaining how she derived at the answer 20:

Linn: Well, for the top layer we need only one cube, then three for the next layer, which is two more, then 6 on the third layer which is three more, and then four plus six on the fourth layer. I wonder if I could do this for any layer…

CASE 2: PROBLEM POSING

In another activity we started by letting the students, in groups of three, solve the following two problem solving tasks. *We have 15 cookies which we want to divide evenly between the three of you (i.e. 15/3).* Followed by the teacher joining the groups and says she also wants to have her share (i.e. *15/4*). After this problem-solving task, the students are asked to pose a similar problem.

Kim creates the problem to divide two whole cookies and four half cookies between two people. The teacher writes 6/2 and Kim says "not 6 divided by two, but four and four halves divided by two. And draws the lines to show how the cookies are to be divided (Figure 2).

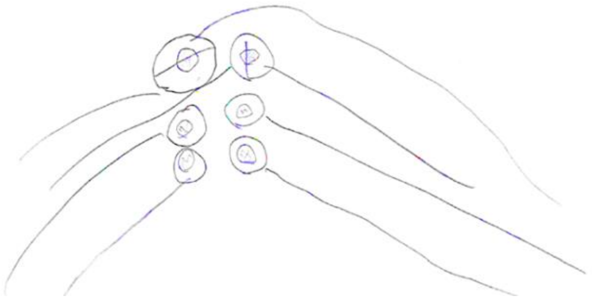

Figure 2 Kim's task: 4+(4 • ½)/2

CASE 3: DISCUSSING PROBLEM SOLVING

Sara was a bit skeptical when I arrived once a week with problem-solving tasks. After documenting the result of thirty draws of two balls at a time out of a bag with two red and two yellow balls, I gathered the students around me, putting some of the students' documentations in front of me. Just when I want to get started, Sara raises her hand and wants to share something with the rest of the class. Sara explains that she noticed the documentations differed:

> Like here you can recall each draw, one at a time [figure 3, left], but in this one it is easier to see which pair was drawn most often [figure3, middle]. And also, I drew two long lines (Figure 3, right), one for each ball, but in fact I only needed to draw one short tally mark, then it would have been much clearer.

Figure 3 Three different ways to document

THE CONTEXT

The examples above are taken from a project in which we designed and tested problem-solving activities to suit students at school start, 6-year-olds in Sweden. The problems had to be communicated orally or through pictures as not all students could be expected to be able to read and write at school start. The project aimed to study the opportunities for learning mathematics through problem solving and did not specifically focus on gifted students (see Palmér & van Bommel, 2023; van Bommel & Palmér, 2021).

PROBLEM SOLVING AND PROBLEM POSING

A mathematics task becomes a problem-solving task when the person who is to solve the task needs to develop new knowledge or new strategies to solve the task (Lesh & Zawojevski, 2007). Which tasks are problem-solving tasks is thus determined by the relationship between the student and the task in question. Niss and Højgaard (2019) argue that problem posing is a key component of problem solving. To become skilled problem-solvers, students must engage with tasks that require both the solving and posing of problems. Problem posing can be carried out before, during, or after problem solving, for the latter one the posed task is often to be connected to the problem-solving task at hand. When students are asked to pose a task for a given mathematical content or a given situation, this is referred to as problem posing in a semi-structured situation (Stoyanova & Ellerton, 1996).

Already in 1945, Polýa described four stages of problem solving (understand the problem, devise a plan, carry out the plan, review) (Polýa, 1945/1990). Recently, Cai and Rott (2024) identified four corresponding stages for problem posing: orientation, connection, generation, and reflection. The last stage in both models (review and reflection) is both critical and challenging, especially with younger students. A whole class discussion opens up for review and reflections, but for students' themselves to adopt such a meta perspective is unusual.

The process of problem solving (and posing) including all stages, encourages students to collaborate, to discuss, to be creative and to use varying strategies. Further, it offers students autonomy, especially when posing problems, and the opportunity to create authentic problems.

PROBLEM SOLVING AS A CATALYST

As stated before, when students engage in problem solving they develop new knowledge or strategies, meaning they are intellectually stimulated. In the collaborative part, they advance their verbal and mathematical skills, and in the problem solving & posing tasks, their creativity is called upon. These aspects resemble what Gagné (2021) would call for mental domains, stimulated by (in this case) a problem-solving task. The problem-solving task enriches, creates, and gives meaning to the content offered and can thus be seen as an educational catalyst. Thus, problem solving (and problem posing as an extension of problem solving) can serve as a catalyst for students in their development.

RETURNING TO LINN, KIM AND SARA

In the three cases described above, it became apparent that Linn, Kim and Sara excelled beyond their peers. Linn had newly arrived in Sweden and had just started to communicate with others in class. When offered a problem-solving task on 3D geometry, Linn instantly got the answer -unlike her peers. Moreover, she was able to articulate and explain her thinking. When offered a more difficult task, she started using a similar strategy, switched to another one and posed herself a question regarding generalization of the solution (I wonder if I could do this for any layer…). At the end of the lesson, I decided to gather the whole class and let Linn explain how she could go from a pyramid with three layers to a pyramid with four layers. The task seemed to stimulate Linn intellectually and gave an opportunity to make meaning to the content offered and served clearly as an educational catalyst (Gagné, 2021).

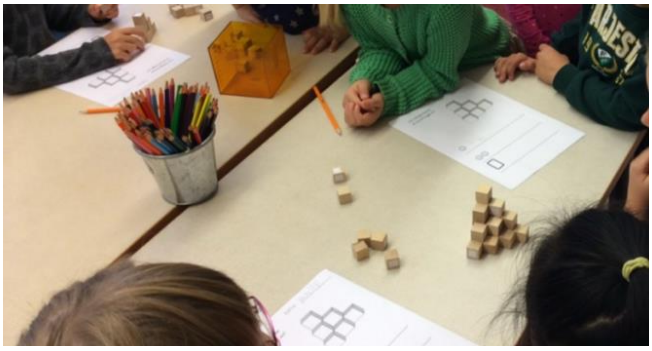

Figure 4 Linn and her four-layer pyramid

When asked to pose a question, Kim took the opportunity to adjust the difficulty of the task to a challenging one. It was not the solution of the task that was challenging, but the numbers used in the task and the way Kim choose to partition the number 6. Kim was clear in her communication to the teacher that the picture did not represent the task 6/2, but represented $4+(4 \cdot \frac{1}{2})/2$. For a 6-year-old this is an advanced way of partitioning the number 6 (in four wholes and four halves), more common would be 1+5 , 2+4 or 3+3. The previous tasks (15/3 and 15/4) did not offer Kim an opportunity to challenge oneself. Students like Kim can work with such tasks and will most likely show a nice and elegant solution rather quickly, but it will not give them the opportunity to be creative, to explore and to engage on their own mathematical level. The semi-structured situation of posing a similar task gave Kim the opportunity to expand the number range while still needing to keep track of the limits of divisibility inherent in the task.

Finally, Sara, sceptical as stated before. Sara exceeded her peers and worked in a textbook for year 2 instead. During the problem-solving sessions we decided she would join the rest of the class, something Sara was very reluctant to in the beginning. When documenting the thirty draws, the teacher first wanted to go to Sara and help her to create a more efficient way to document, but we decided to just let Sara have a go. Sara raising her hand and joining the discussion signalled a change in attitude. From distancing herself in previous discussions, she now got engaged. Elegantly, she picked several points I had planned to elicit in the discussion. The goal of the discussion was for the students to see the variances between the different ways of documenting and to reflect upon their own documentation strategies. Sara was able to articulate the shortcomings of her own way of documenting and adopted a meta-perspective in which she reviewed her method (phase 4, Polýa, 1945/1990).

CONCLUSION

Problem solving, problem posing and problem-solving discussions offer different opportunities for students who excel beyond their peers, to engage in a purposeful matter on their own intellectual level, while still being able to interact and collaborate with the others. For problem solving there is almost always another, similar problem to be formulated in which the student can be challenged. Problem posing provides students with the opportunity to be creative and test limits and thus to engage at their own mathematical level.The whole group discussion often aims to review solution methods and to compare different methods. Not seldom we see students, like Sara, who generalize solutions methods and connect these to other previously solved problems or to alternative new problems.

The setting of problem solving and problem posing seemed to offer learning opportunities where gifted students were stimulated and where they challenged themselves to their capacity.

FINALLY

Sara asked me after each session if I would come back again and sighed deeply each time after a 'yes' from my side. This time she asked the same question and after me replying ‘yes’ she did not sigh but answered, "okay, maybe it’ll be good". And maybe that’s what problem solving is – something that can turn out well!

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EMBEDDING THE PROMOTION OF TALENT AND GIFTEDNESS IN A SCHOOL ACTION PLAN

Psychologist Jan Kwietniewski
Head of the Counseling Center for Special Talents (BbB) at the State Institute for Teacher Training and School Development in Hamburg.

PRESENTATION OF ABSTRACT

The promotion of talent and the support of the gifted are two sides of the same goal: a broadly understood strategy in modern education to enable all pupils to develop their personal talents. Both society and education policy are in support of such a development-oriented approach in schools. (See LemaS Research Network in Germany 2020, Standing Conference of the Ministers of Education and Cultural Affairs of the Länder in the Federal Republic of Germany, Germany 2019).

On the one hand, schools should identify and promote the potential of all pupils as part of a broadly understood strategy. They should also compensate for possible disadvantages due to factors such as migration, social class or gender. At the same time, in the spirit of inclusion, they should address in a targeted and personalized way the development of those children whose potential deviates from the norm, e.g. due to giftedness, or whose successful academic development is at risk due to a combination of strengths and weaknesses. In practice, this is referred to as broad-based talent promotion and individualized gifted education.

How can these two goals be anchored in everyday school life? How can a realistic support concept be developed in a school in which all support modules are meaningfully anchored? What communication processes and structures are needed in the school to manage the tasks associated with these goals?

In order to answer these questions and to support schools in practice, an orientation system was developed in Hamburg and has been tested for several years. This system helps schools to analyze their current situation and to develop their goals for the promotion of all talents. It divides the overarching aims of gifted education in schools into 'five fields of action': identification, support, guidance, needs assessment and school development. This structure reduces the complexity of school development by first defining basic 'minimum standards' for each area of action, followed by further 'development opportunities'. In this way, schools are given a clear orientation in terms of standards and can plan a gradual development over time. The principle of evolution applies: the aim is for every school to go one step further!

The presentation begins by outlining the tasks of school development. Using the "Five Fields of Action" orientation system, possibilities are described that a school can work through step by step in order to arrive at a comprehensive support concept. Afterwards, possible applications and experiences of school counsellors with this tool will be presented.

SUSTAINING AND DEVELOPING THE PROFESSION'S GIFTS AND TALENTS: LESSONS LEARNED FROM TEACHER EDUCATION AND RESEARCH IN NEW ZEALAND

Tracy Riley
Professor of Educational work, Massey University New Zealand.

PRESENTATION OF ABSTRACT

Every teacher in New Zealand is considered a teacher of gifted students, a reflection of our inclusive education system. However, some teachers have developed specialist knowledge and skills through tertiary education and research. As one of the country's leading academic in gifted education since the mid-1990s, I have had the opportunity to grow and shape the profession through my teaching and supervision of university students, as well as my contributions to national advisory groups and professional associations. In this keynote, I will share my personal and professional experiences, alongside the history of gifted education in New Zealand, reflecting on what I have learned about how we can sustain gifted education research and praxis at local, national and international levels. I will also spend time reflecting on what I have learned in my role as Dean, which requires broad oversight and strategic development of the university's research community. In this address, I will share the highs and lows – what has worked and what hasn't worked, alongside the joys and delights, and maybe even a few regrets, underpinning my lessons with theory and research where available and applicable.

GIFTED STUDENTS IN INCLUSIVE EDUCATION: A FINNISH PERSPECTIVE

Professor Kirsi Tirri
University of Helsinki, Finland

PRESENTATION OF ABSTRACT

Finland is known as an exemplar country in education with high students' achievement in PISA studies and good teacher education. Inclusive education, curriculum integration, and digital learning are examples of current innovations in Finnish education. The Finnish educational system is built on inclusion, equality, and equity. Teachers have great freedom in schools to develop the curriculum for their students and use different teaching methods in their teaching. In this talk I will discuss how gifted students are recognized in Finnish inclusive education. I will give some examples how differentiation of education has been used in Finnish elementary and secondary education to provide challenges for gifted students. I will also discuss the issues related to non-native gifted students and to those gifted students who can't cope in inclusive education and need other educational options. The ethical nature of education is emphasized with the goal to develop transformational giftedness among gifted students and help them to combine excellence and ethics in their lives.

PAPERS

RESEARCH

MULTIDIMENSIONAL ASSESSMENT OF FLEXIBILITY AS A COMPONENT OF MATHEMATICAL CREATIVITY

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When assessing creativity, the component flexibility is often described as a numerical value of the diversity of solutions or solution paths. This is limited in the case of invention problems, in which the individual ideas differ with respect to several independent characteristics. In investigating primary students' creation of figural patterns, we developed a multidimensional analysis tool that enables a differentiated view of flexibility by generating flexibility profiles. Data from an interview study with third graders indicate that this tool provides a viable basis for building types of flexibility.

Keywords: mathematical giftedness, mathematical creativity, assessment of flexibility

INTRODUCTION

The starting point for this paper was a study on the mathematical creativity of mathematically gifted third graders compared to non-gifted but high-performing peers (Assmus & Fritzlar, 2022). The mathematical content chosen for the study was the creation of figural patterns which have a high mathematical richness due to the combination of arithmetic and geometric aspects, and thus diverse potentials for creative mathematical activity. Nevertheless, they are easily accessible to young students and require only a small knowledge base.

Based on Torrance (1974), creativity is usually measured in terms of fluency, flexibility and originality. For the operationalization of these components, various suggestions can be found in the literature most of which have in common the determination of a summarizing numerical value for each component (e.g., Kattou et al., 2013; Kwon et al., 2006; Leikin, 2009). At least for flexibility, this is accompanied by greater challenges because the diversity of solutions or approaches is not always easily assessable using a one-dimensional metric and often requires more than just a binary decision. In fact, for inventing figural patterns, the calculation of such a numerical flexibility score does not seem appropriate at all, due to the wide range of possible geometric and arithmetic variations.

To be able to describe and compare students' flexibility in creating figural patterns, the development of a suitable analysis tool was necessary. This paper will present this analysis tool and demonstrate its power for our research interest. Finally, its applicability to other tasks used in mathematical creativity tests will be discussed.

THEORETICAL FRAMEWORK

Flexibility as a component of creativity

For a characterization of mathematical creativity, common indicators from general creativity research are often specified namely fluency, flexibility, originality, and much more rarely, elaboration. Another indicator is meaningfulness or usefulness, which is often applied implicitly because only useful products or approaches will be considered for the previously mentioned indicators. However, fluency – defined according to the number of meaningful responses in a mathematical situation – has only limited significance for mathematical creativity (e.g., Akgul & Kahveci, 2016). For creativity in school mathematics, originality, which refers to new, unusual, or unique ideas and solution approaches, can be assessed only in comparison with a reference group which relativizes its significance as well. Based on this, flexibility seems to be a particularly important indicator and backbone of mathematical creativity in school age.

In mathematics, flexibility was investigated in detail for instance by Krutetzkii (1976). For him, it is the "ability for a rapid reconstruction of mental activity, for 'breaking' a just-established solution pattern and replacing it with a new one" (p. 277). In a slightly more general way according to Silver (1997), flexibility can be conceived as referring to "apparent shifts in approaches taken when generating responses to a prompt" (p. 76). However, identifying these shifts is not always easy.

In the context of problem solving, which is very often used to determine mathematical creativity, flexibility describes the variety of solutions and approaches based on different schools of thought or different perspectives for looking at the problem.

Assessing flexibility

An assessment of flexibility is complex and often challenging. On the one hand, it depends on the context or the tasks used in the particular study. On the other hand, the extent of flexibility can be operationalized quite differently.

For assessing creativity in the context of problem solving, open problems are usually used, which have several solution pathways and / or several solutions. If participants are explicitly asked to solve an open problem in different ways, for example Leikin (2009) and other colleagues speak of a 'multiple solution task' (MST). More recently, problem posing tasks have also been used to assess mathematical creativity.

Although a process-oriented view is emphasized in line with the general understanding of flexibility with the focus on different approaches, it is often also possible to reveal the diversity of approaches through the diversity of solutions. For this purpose, thoroughly different variants of counting have been established. For example, there are studies (e.g., Kwon et al., 2006) in which flexibility is determined by the absolute number of response categories each student produced. In contrast, in other studies a relative flexibility value is generated. This is derived either by the ratio of the absolute to the maximum total number of different types of solutions produced by one of the subjects (e.g., Kattou et al., 2013) or by the ratio to the total number of different possible solutions. For example, Leikin (2009) uses the expert solution space for this purpose.

Even though there are various ways of determining flexibility by counting, these have only limited explanatory power due to their one-dimensional characterization. The nature of the differences between the respective solutions and approaches or qualitative aspects of the diversity created can hardly be described in this way. Therefore, a multidimensional approach to characterizing flexibility could be particularly suitable, the development and potentials of which will be presented below using an example study. For reasons of space, the study itself can only be outlined in basic terms; for further details, see Assmus and Fritzlar (2022).

DEVELOPMENT OF A MULTIDIMENSIONAL ASSESSMENT OF FLEXIBILITY AS PART OF AN EMPIRICAL STUDY

Aims and details of the study

The aim of the empirical study was to investigate, how mathematical creativity in creating figural patterns differs among mathematically gifted and non-gifted third graders. To obtain particularly meaningful results, mathematically gifted versus non-gifted students with at least good achievements in the mathematics classroom should be included (cf. Assmus & Fritzlar, 2022). In order to be able to compare students' creativity, the development of an instrument for the differentiated description of flexibility was particularly necessary. Crucial research questions therefore included: *How can the flexibility of primary students in creating figural patterns be described in a differentiated way? Is it possible to construct different types of mathematical flexibility in this context?*

Data were collected using semi-standardized clinical interviews centered on the task of creating as many different figural patterns of wooden cubes as possible with important constraints: one cube should be used for the first figure and five cubes for the second figure. For each figural pattern, the first four figures should be arranged with concrete material. Additionally, students were asked to orally state an underlying rule, to show 'where you can see the rule in the pattern' and to record it in writing on a notepad. The participants were given a total of 30 minutes to accomplish this task.

Interviews were conducted with 24 third graders from different schools who had applied to participate in a fostering program. They were distinguished in mathematically gifted (N=14) and 'only' high achieving students (N=10) by means of a test.

Development of an assessment instrument

Based on the created figural patterns of two students (S23: Aaden, S13: Ben), difficulties in assessing their flexibility – as operationalized in many previous studies – will be pointed out and the new analysis tool will be presented.

As in other operationalization approaches (e.g., Kwon et al., 2006), we refer flexibility exclusively to correct outcomes, which in the presented study are termed 'valid patterns' and that fulfilled the following properties: considering 1 and 5 as numbers of cubes for the first two figures; considering a continuous regularity related to the numbers of cubes used from figure 1 or related to numbers and shape from figure 1 or 2. In this sense, all created patterns in Figure 1 and Figure 2 are valid. For the last pattern in Figure 1 and the last pattern in Figure 2 a regularity is shown from pattern figure 2, for all other patterns a regularity begins from pattern figure 1.

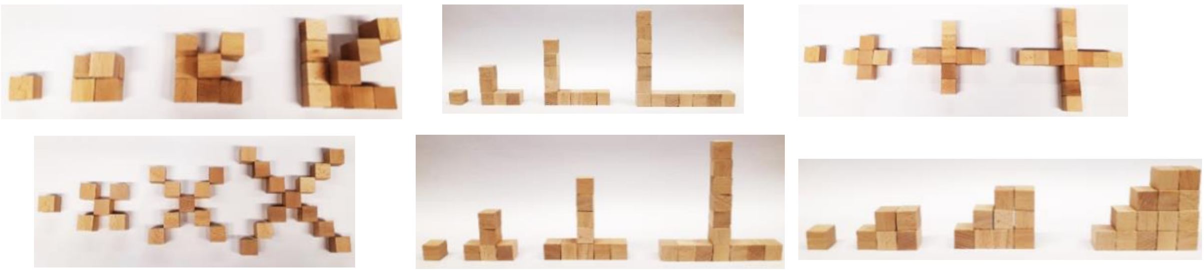

Figure 1: Figural patterns invented by Aaden

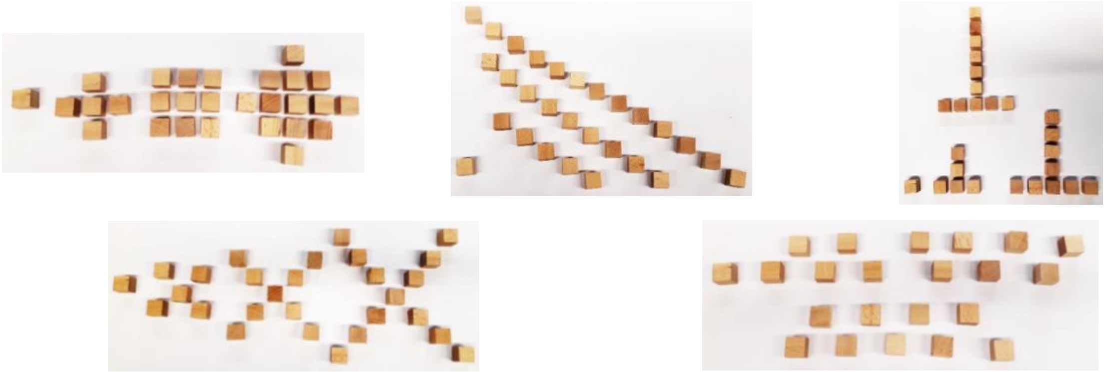

Figure 2: Figural patterns invented by Ben

How can the variety of figural patterns be assessed? It is obvious to look at the shape of the laid figures. Both students always used a new *shape* for each of their figural patterns, so all patterns must be different. Based on this, the flexibility would be shown solely by the number of patterns created. However, a detailed look reveals that both sets of invented patterns have different varieties. While Ben placed all cubes separately, Aaden used different *building principles*: Adjacent cubes touch each other at edges or surfaces, sometimes the cubes lie in one plane, sometimes in several. Also with respect to the *type of mathematical relation* used, the students' inventions differ gradually. While all of Ben's patterns are based on constant growth, the last pattern of Aaden has a uniformly changing increase. Are there further characteristics that describe the diversity of the patterns and thus the flexibility of the students?

To generate suitable dimensions, qualitative content analysis was used to identify as many distinguishing characteristics of figural patterns as possible, partly deductively and partly inductively. The second step was to identify those dimensions which describe the variety of the patterns in a differentiated and at the same time practicable way, considering the mathematically essential attributes of created figural patterns. Based on this, we developed the following description system:

- *Types of mathematical relations:* Which mathematical relation underlies (the regularity of) the figural pattern (e.g., constant increase, uniformly changing increase)?
- *Shapes:* What regularly changing shape emerges in the figural pattern (e.g., cross)?
- *Building principles:* How are the figures built and aligned to the top of the table (e.g., in multiple layers, lying in one layer, standing in one layer)? At which points do adjacent cubes touch each other? How are adjacent cubes aligned to each other?
- *Number of extension directions:* In how many directions are the figures extended from step to step (e.g., bar: 1, L: 2, cross: 4)?
- *Focus of the student's oral descriptions:* To which aspects does the orally formulated rule refer (e.g., number, shape, building process)?

Thereby, the *mathematical* features 'types of mathematical relations' and 'shapes' are rated as particularly important, and the focus of oral descriptions as least important due to their solely *communicative* function.

For each interview, the number of attributes was determined for each of the five dimensions. Based on these five values, a differentiated *flexibility profile* was created for all students and visualized in radar charts. The different importance of the dimensions was considered by scaling the axes in a ratio of 3:2:1.

The following radar charts resulted for the two students Aaden and Ben:

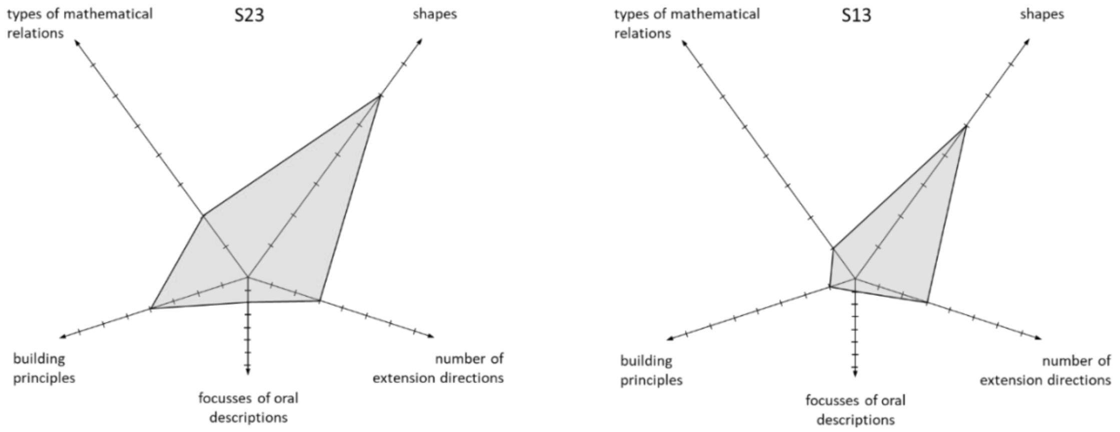

Figure 3: Radar charts of Aaden's and Ben's creation of figural patterns

The differences described above are clearly visible in the spanned areas. The area in the left diagram is larger, it is broader and its shape and position are less strongly oriented to the top right.

Type formation concerning flexibility

The representation of the flexibility profiles by radar charts allowed a visual, holistic comparing and contrasting of cases. For this, the five characteristics were used as 'attribute space'. The aim was to cluster the cases and to build different types of flexibility in creating figural patterns.

At first, the radar charts were grouped in regard to the approximate size of the polygon and its orientation. For a finer clustering, the other features were then also included. In this way, five types could be constructed: 1a - diversely varying; 1b - diversely varying with geometric focus; 2 - geometrically varying based on constant growth; 3 - arithmetically varying; 4 - sparsely varying based on constant growth; 5 - not or hardly varying. Aaden's flexibility profile is of type 1b. He used two types of mathematical relations and at least four different shapes, furthermore for all five dimensions the value is more than one. Ben's flexibility profile is of type 2 because he used only one mathematical relation (constant growth) but with at least four different shapes and two kinds of growing. The other two dimensions have the value one or higher.

Types 1a, 1b, 2, and 3 describe a creation of figural patterns that can be considered flexible with varying focus (arithmetic, geometric, or both) and to somewhat varying degrees. Types 4 and 5, on the other hand, characterize less flexible creation of figural patterns. For the study's research interest, it is important to note that the pattern creation of all 'only' high-achieving students belongs to types 4 or 5. In contrast, the pattern creation of almost all mathematically gifted students (three exceptions) was assigned to types 1 to 3. This clear difference between the two subgroups of participating students could only be brought to light through the described building of flexibility types of creating figural patterns.

SOME REFLECTIONS ON MULTIDIMENSIONAL ASSESSMENT OF FLEXIBILITY

Describing students' flexibility in creating figural patterns using several independent dimensions proved to be very fruitful. First, this was the only way to describe the variety of invented patterns in a sufficiently differentiated way. Based on this, the type formation led to unexpectedly significant results. This raises the question of whether this approach can also be applied to other mathematical content and tasks.

In our view, the possibility of a multidimensional description of the problem solvers' flexibility depends essentially on the structure of the 'solution space', which contains all possible solutions and solution pathways. Such a solution space can, for example, allow categorization in the sense of forming clearly different classes. Based on the number of categories or classes of solutions or approaches used by a person, a numerical characterization of her/his flexibility can easily be developed, but multidimensional description may not be suitable or even possible. In other cases, a differentiated description of the solution space or the flexibility of a problem solver could be achieved more easily or exclusively by referring to several dimensions that are applicable to all solutions and largely independent of each other. This approach could then also be used to describe originality more precisely by relating it to the particular dimensions. Possibly such a multidimensional approach is especially relevant for geometric invention problems like the coloring-problem below. Instead of a single flexibility score, this approach provides something like a flexibility vector. However, such a detailed characterization also has the disadvantage that results (flexibility vectors) for different problems can hardly be combined.

Coloring-Problem: Color a square in black and white so that half of it is black. Find as many solutions as possible. (cf. Kwon et al., 2006)

Examples:

Dimensions: Number of areas, symmetry of the whole figure, analogy between black and white areas, type of boundary lines, …

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KAKEYA NEEDLE PROBLEM FOR HIGH SCHOOL STUDENTS?

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In 1917, Sōichi Kakeya posed the following question: What is the smallest possible area of a convex region K in the plane within which a unit-length segment can be rotated by 180°? The problem becomes even more intriguing if we remove the requirement that the region be convex. Finding a complete solution is remarkably challenging and leads to surprising results. While high school students are unlikely to solve the entire problem, they can tackle several subproblems and even formulate a correct conjecture. This captivating question serves as a powerful motivator, inspiring students to engage with advanced mathematical concepts.

Keywords: Talent care, Pósa method, motivation, Kakeya needle problem.

RESEARCH QUESTION

In discovery-based learning (Artigue & Blomhøj, 2013), and specifically in the Pósa method, students are consistently given ample time to solve mathematical problems. Moreover, the problems are carefully selected to be both challenging and achievable. This study investigates whether it is sometimes beneficial to deviate from this framework by designing activities where students have little to no realistic chance of solving the entire problem, and even the solvable subtasks may be constrained by time. The research explores whether such an approach holds value when applied to mathematically gifted students.

BACKGROUND

We conducted two sessions on this topic with two distinct groups: 12th-grade students enrolled in a special mathematics curriculum (SMC) and 11th-grade students from the Flying School Program (FS)—a name inspired by Erich Kästner, though unrelated to aviation. The first session, attended by 32 students, lasted 45 minutes and was dedicated entirely to this topic. The second group explored the problem across two one-day programs, where it appeared as one of several mathematical problem threads (Artigue et al., 2020), with partial results discussed periodically. Eight students participated in the first of these sessions and ten in the second. SMC students follow an advanced mathematics curriculum, having gained admission through an entrance exam after the sixth grade and studied at an accelerated level for six years. In contrast, the Flying School sessions are designed for talented and motivated students who, for various reasons, do not have access to conventional mathematical talent development programs in Hungary. These students engage in intensive, full-day mathematics sessions once a month. This problem was introduced during the 15th and 16th sessions, by which point FS students had been attending the program for nearly two years. Both groups were taught using a discovery-based learning approach, specifically the Pósa method (Juhász, 2019). In regular sessions, students are given ample time to solve problems and often work on multiple tasks simultaneously, typically drawn from different topics. For the SMC students, this session was unusual, as they usually study mathematics in two separate groups of approximately 17 students each. However, due to the absence of one of the teachers, the groups were merged, resulting in a class of 32 students. As a result, the session deviated from the usual methodology: there was significantly more direct instruction, and students had less time for independent exploration.

FROM A STORY TO A PRECISE MATH PROBLEM

Imagine a worker standing in a courtyard, holding a long ladder under their arm. They want to turn around, meaning they wish to end in the same spot but face the opposite direction. How large must the courtyard be to allow them to do this?

Of course, this is not yet a well-defined mathematical problem. Our initial question to the students was how they could translate this scenario into a precise mathematical formulation. At this stage, students' prior knowledge becomes evident, as their ability to abstract the problem depends heavily on the mathematical tools they have at their disposal. The SMC students approached this task with significantly greater ease, quickly distilling the real-world scenario into a formal mathematical question. While many valid formulations exist, the FS students found this process noticeably more challenging. Although they, too, are taught using the Pósa method, their 14 previous one-day sessions provided far fewer opportunities to practice formulating meaningful mathematical questions (Pósa et al., 2024) compared to the SMC students, who have been continuously exposed to this type of instruction for over five years. Ultimately, both groups successfully arrived at a precise mathematical question:
Given a line segment of length 1 in the plane, what is the minimal area of a region within which the segment can continuously move and return to its original position with its endpoints swapped?

PROGRESSION IN THE PRECISE MATHEMATICAL PROBLEM

Once the problem was precisely formulated, students were encouraged to explore possible solutions. Rather than immediately seeking the optimal answer, they were advised to first identify any reasonable solution and then refine it. This approach was successful in both groups, as all students quickly recognized the most straightforward solution: a circle with a radius of $\frac{1}{2}$, yielding an area of $\frac{\pi}{4}$.

We then asked students whether they believed this was the optimal solution. Among the SMC students, nearly 60% (19 out of 32) suspected that a smaller point set might exist, whereas only 25% (2 out of 8) of the FS students shared this view.

At this stage, we revealed that a solution with a smaller area does exist and encouraged students to find one. The SMC students were given a few minutes to work on the problem, while the FS students had significantly more time but were also engaged with other tasks. The problem proved difficult—without assistance, only 2 SMC students, and none from the FS group, managed to construct a better solution than the circle within the allotted time. To guide them, we provided a hint: they were asked to find a triangle in which the segment could be reversed. With this prompt, 6–8 SMC students and 2 FS students successfully identified a suitable triangle. During the discussion, we revealed that in an equilateral triangle with a height of 1, the segment can indeed be rotated by 180°. Students were then assigned two tasks: first, to determine a valid motion to achieve the desired transformation, and second, to calculate the area of the triangle. Both tasks were accessible to the students. While we did not formally track results, many students in both groups independently discovered the required motion in roughly equal proportions. However, calculating the area of the triangle was significantly faster among the SMC students, though the FS students also arrived at the correct answer. The SMC students were generally familiar with the fact that scaling a point set by a factor of c results in its area changing by a factor of c^2, which allowed them to compute the solution more efficiently.

AN IMPORTANT SUBTASK

At this stage, further progress becomes difficult—improving upon the equilateral triangle is challenging with only high school-level knowledge. To facilitate advancement, we introduced a subtask: students were asked to move a segment of length 1 so that its endpoints, in the final position, formed a square with their original locations, while minimizing the area swept by the motion.

A square with a side length of 1 is an obvious solution, but students in both groups immediately recognized that a more efficient approach must exist. Some students quickly proposed potential improvements (see Figure 1).

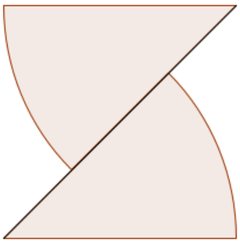

Figure 1: Translating the segment, a not optimal solution

The FS students struggled for a long time to refine their solution, but eventually, one of them discovered the optimal result: the translation can be performed while sweeping an arbitrarily small area. Among the SMC students, four managed to identify the optimal solution within the roughly 8 minutes available, though they did not arrive at the specific construction mentioned above.

BACK TO THE ORIGINAL PROBLEM

At this stage, we encouraged the SMC students to consider how their new insight might help simplify the problem. Specifically, we asked them to articulate why this discovery made further progress easier. However, this proved to be a highly challenging question, and only one student was able to express the key idea clearly.

Once we collectively formulated the insight—that the segment could now "jump" to a parallel position and that a finite number of such steps could be executed within an arbitrarily small area—more students recognized that the equilateral triangle could be improved upon. They saw that the equilateral triangle could be divided into two congruent triangles along one of its unit-length heights. These two triangles could then be slid on top of each other, reducing the area required for the segment's motion (see Figure 2). Consequently, the segment could be reversed within any region slightly larger than this reduced area.

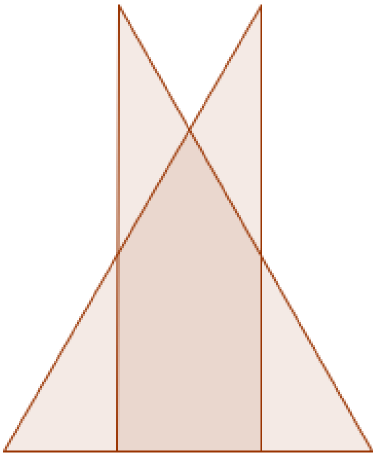

Figure 2: Improve the solution using the solution of the subtask

The SMC students had no difficulty calculating the area of this modified shape. Naturally, this led to the question of whether further improvements were possible. Some students suggested that increasing the overlap between the two triangles—by adjusting the amount of translation—might reduce the area even further, though there was no time to explore this idea in depth.

At this point, we returned to the original problem and introduced the concept of the deltoid curve (or Steiner curve), a shape whose interior and boundary allow a segment to turn. For a segment of length 1, the minimal enclosing deltoid has an area of approximately 0.3853.

We then asked each SMC student to estimate the smallest possible area required for a full reversal of the segment. Surprisingly, 7 out of 32 students conjectured that the segment could be reversed in an arbitrarily small area. When we revealed that this is indeed the case (Besicovitch, 1963), the majority of the other students were genuinely surprised.

To conclude the activity, we provided a non-rigorous introduction to Perron trees, which offer an elementary proof that a segment of length 1 can be turned within an arbitrarily small area. From this, students naturally inferred—articulating the idea themselves—that the same principle applies to any segment, regardless of its length: it, too, can be reversed within an arbitrarily small positive area.

FEEDBACK

A week after the activity with the SMC students, we distributed a feedback form consisting of six questions: 1) State the mathematical claim as precisely as possible. 2) How surprising do you find the claim? (1–7 scale) 3) Why did you choose the number in question 2? 4) What related question would you ask about the topic? 5) How much did you enjoy the program? 6) Additional comments.

Due to logistical constraints, we were only able to collect responses from half of the class, resulting in 17 completed forms available for analysis.

Among the responses to the first question, 12 were fully correct, though they expressed the theorem in different but equivalent ways. Examples included: "Any segment can be rotated by any angle within a point set of arbitrarily small area.", "A segment AB can be continuously moved into itself such that points A and B swap places, and the union of points touched by the segment during the motion has an arbitrarily small area."

Some students provided answers that were not entirely precise but still captured the essence of the statement. Others struggled with terminology, such as describing the result as "a segment can be rotated in an infinitely small but nonzero area," reflecting a conceptual misunderstanding.

Interestingly, four responses explicitly mentioned the word "ladder," suggesting that the original narrative remained more vivid in their minds than the subsequent mathematical abstraction. However, these responses were still mathematically valid when interpreted in planar terms with "segment" replacing "ladder."

Responses to question 2 yielded an average score of 5.41 with a standard deviation of 1.12, indicating that students found the theorem particularly surprising.

The responses to question 4 revealed that many students posed highly relevant and insightful follow-up questions, some of which were unexpected. A natural extension—what happens in higher dimensions? —was suggested by 9 students. However, a more unusual but intriguing question asked about the minimum required area if, instead of a segment, a point set with positive area had to be rotated by 180°. Many framed this in terms of finding the optimal motion for a "thin" rectangle.

Additionally, several students inquired about the minimum area required for flipping a segment under the constraint that the enclosing shape had a limited diameter. This demonstrates a strong understanding that minimizing the required area would likely involve a shape with a very large diameter. However, some students confused the concepts of "smallest" and "largest" in their phrasing. For example, one student asked about "the largest volume of a three-dimensional body in which a square with a side length of 1 could be flipped."

Responses to question 5 indicated an overwhelmingly positive reception, with an average rating of 6.59 and a standard deviation of 0.51. This suggests that even students already deeply engaged in mathematics appreciate the opportunity to explore exceptionally interesting and abstract mathematical topics, even when they extend beyond their current level of formal mathematical training.

CONCLUSIONS

This small experiment reinforces the idea that introducing high school students to advanced mathematical problems is highly worthwhile—even when solving them independently is beyond their reach or when merely understanding the solution pushes their limits. This approach provides several key benefits:

1) It becomes evident that even complex problems often rely on elementary ideas that are accessible to students. 2) Successful problem-solving requires a solid foundation of high school-level knowledge. 3) Students invest significant effort into achieving partial results, even if they cannot solve the entire problem. 4) They experience the excitement of research and exploration.

This problem requires proficiency across multiple mathematical domains, offering students diverse opportunities for success. While its foundation is geometric, it also demands an analytical perspective, bridging mathematical analysis and problem-solving techniques. Additionally, it highlights the critical distinction between zero and arbitrarily small positive values, reinforcing students' understanding of quantifiers and their ability to work with complex logical structures. The impact of the session extended beyond the classroom. In the two weeks following the activity, several students approached us with further questions. Some sought clarification on concepts they had not fully grasped, others inquired about details of proofs that had not been fully presented, and a few expressed curiosities about related problems that the session had inspired them to explore.

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FUTURE RESEARCH DIRECTIONS FOR MATHEMATICAL GIFTEDNESS

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In this paper, future research directions for mathematical giftedness are discussed. The first two authors are veterans of giftedness research with over 25 years of experience each and the third author is a top international graduate student with a research focus on giftedness in mathematics. Each author was asked to select on one of four areas of research from a list in mathematical giftedness and to provide another area of need. Following a general overview of what the field currently knows through extant research, areas of need are discussed with a particular emphasis on curricula and digital technology. The intent of this paper is to highlight areas of research need in subsequent years with an ultimate objective of providing direction to peer scholars and graduate students in mathematical giftedness.

Keywords: digital technology, mathematics curricula, mathematical giftedness, psychological constructs, project M^2 and project M^3

WHAT WE KNOW ABOUT MATHEMATICAL GIFTEDNESS RESEARCH

Early research on mathematical giftedness is often attributed to contributions by (Vadim) Krutetskii. His work, mostly performed in the 1950s and 1960s was delayed in being shared with western countries, as it is rumored that leaders in the former Soviet Union did not want such information shared with competing countries. In his empirical work, both quantitative and qualitative approaches were employed, and he studied mathematical abilities of schoolchildren, with a particular emphasis on students of advanced abilities. Likely, the most influential finding from his work was the actual abilities that advanced students exhibit and they are the following: ability to formalize mathematical material, ability to generalize mathematical material, ability to operate with numerals and other symbols, ability for sequential, properly segmented logical reasoning, ability to shorten the reasoning process, ability to reverse a mental process, flexibility of thought, (depth of) mathematical memory, and ability for spatial concepts (Krutetskii, 1976, p. 87-88). Much of this research is still employed today in empirical investigations.

Historically, overall intelligence was used as stand-alone criteria to ascertain (mathematical) giftedness, though ultimately the unidimensional metric was found to be lacking (Kießwetter, 1985). However, recent work has revealed that much more specific attributes serve as (better) indicators of mathematical advancement, though as Bicknell (2009) suggested, there is still much to learn. For instance, openness, fluid intelligence, and cognitive output are instrumental needs (Hansen et al., 2022). As Fritzlar (2013) suggested, logical thinking and problem solving must also be assessed in mathematical giftedness considerations.

In 2023, Bilgic and Bolağlu provided analysis on bibliometric data relevant to mathematical giftedness. Various observations are made about mathematical giftedness and in their analysis; Leikin and Stanley are each identified as major contributors to the research base. A similar analysis by Özdemir et al. (2024) provided an overview of synonyms used to describe giftedness, such as mathematically gifted, mathematically talented, mathematically precocious, exceptional mathematical promise, and high mathematical ability. In addition to a brief historical review of publications, the authors share extensive bibliometric data about such publications, including data about the most influential "journals, articles, countries, institutions, authors, and networks in the field of mathematical giftedness".

What remains to be known about mathematical giftedness is

(1) Are there other psychological constructs that might serve as (better) indicators of mathematical giftedness than those currently employed, and if so, what are they?
(2) What are the most effective four to five instructional approaches to serve students gifted in mathematics?
(3) How do learning facilitators know when to use the top selected instructional approaches?
(4) What can be done to instill a lifelong passion for mathematics learning?
(5) What psychological constructs are misunderstood (e.g., it has been postulated that a modicum of anxiety facilitates deep mathematical learning episodes)?

Curricula

In 1980, the National Council of Teachers of Mathematics (NCTM) in the United States noted,

> "The student most neglected, in terms of realizing full potential, is the gifted students of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world." (NCTM, 1980, p. 18).

This was followed in 1994 by an NCTM Task Force on Mathematically Promising Students (Sheffield et al., 1995). This task force defined students with mathematical promise as those who have the potential to become the leaders and problem solvers of the future and mathematical promise as a function of ability, motivation, belief and experience or opportunity, noting that these variables are not fixed and must be developed and maximized. Today, in an increasingly technological world with critical worldwide issues and problems, the need for developing the mathematical potential to ever-higher levels of increasing numbers of diverse students is even greater.

Any curricula for mathematically promising students should be designed to maximize students' engagement, enjoyment, expertise and innovation. That means that there is challenge for the highest-performing students that encourages continuous progress and ensures that they learn something new every day. In addition, the curricula should encourage and support many additional students across all geographic, demographic, and economic boundaries to reach those highest levels. In addition, there should be many other curricular and extra-curricular opportunities for students to engage with high-level mathematics, including mathematical circles, clubs and competitions.

Examples of two such award-winning programs are *Project M^2: Mentoring Young Mathematicians* <http://www.projectm2.org>, an advanced mathematics curriculum and research study for primary level students funded by the U.S. National Science Foundation, and *Project M^3: Mentoring Mathematical Minds* <http://www.projectm3.org>, a curriculum research project funded by the U.S. Department of Education Javits Gifted and Talented Students Education Act to nurture mathematical talent in elementary students. As one of the co-authors of these projects, we have found that curriculum that enhances the development of mathematically promising students: **Encourages perseverance and appropriate**

pacing. Posing more complex, challenging, interesting tasks that offer productive struggle is especially critical for our top students who often can answer math questions immediately using knowledge learned a year or more before. **Uses open and parallel tasks and scaffolds and assesses appropriately**. Curricula and assessment should reflect this through the inclusion of open tasks that encourage creative, in-depth solutions and multiple pathways for obtaining them and reward continuous progress for even their highest-performing students. **Promotes problem posing**. As a natural outgrowth of the previous point, students should ask (and answer) their own extended questions as well as solving those posed by the teacher or textbook. **Encourages oral and written discourse**. Mathematical discussions, both oral and written, including student journals, have been shown to increase student learning, motivation, and engagement as well as supporting teachers in understanding their students. **Is fun, challenging and engaging**. It is important for students to develop a passion for learning mathematics that will continue throughout their lifetime. **Is inclusive and engages families and community members**. Families and other community members can be most useful in supporting this development in students from all racial, ethnic, linguistic, gender, and socioeconomic groups. **Is supported by professional development**. All teachers should receive ongoing professional development in the content and the teaching of challenging, high-level, K-12 mathematics courses as well as in understanding students, those identified as mathematically promising or gifted and those without that designation, in order to prepare and support students in these courses. In the next section, attention is turned to research needs in mathematics and gifted education, using digital technology.

Advancing mathematical giftedness in the digital age

The integration of digital technology (DT), which refers to electronic devices, systems, and resources that process and communicate information, in education reflects its growing prevalence in everyday life (Sacristan et al., 2010). However, its influence on the development and nurturance of mathematical giftedness remains under-researched (Leikin, 2021; Özdemir et al., 2024). This gap in the research highlights both a challenge and opportunity for educators and researchers to explore the influence of digital technology (DT) on mathematically gifted students' ability to excel.

Nurturing mathematical giftedness in the 21st century

While there is no universally accepted definition, mathematically gifted students (MGS) often demonstrate these characteristics: memorization, understanding structures and patterns, making generalizations, logical reasoning, flexibility, seeking elegant solutions and a 'mathematical cast of mind' (Krutetskii, 1976). Providing appropriate support for the development of these skills in the 21st century could also involve ensuring that MGS become proficient DT users within the context of mathematics. This could be crucial for preparing them to respond to the evolving needs of society and their success in STEM fields (Özdemir et al., 2024).

DT provides new opportunities for engagement with mathematical concepts through immersive visualizations and simulations of abstract concepts in games and interactive exercises (Sacristán et al., 2010), which may promote real-world applications of mathematics for MGS. However, the success of DT in nurturing mathematical giftedness depends on how it effectively challenges mathematically gifted students' higher-order-thinking and problem-solving skills, as opposed to a mere supplement. By utilizing DT innovatively, educators can create engaging and enriching learning experiences that may help MGS reach their full potential.

Current research on the influence of DT in mathematical giftedness

Recent studies relevant to the impact of DT on MGS have revealed both potential benefits and challenges. For example, multiple studies have shown that DT facilitated enhanced engagement and encouraged creativity, supported problem-solving and promoted positive attitudes and motivation among MGS (Vargas-Montoya et al., 2023; Wang et al., 2016). Other researchers have found that DT can cater to the needs of gifted students in mathematics by accelerating their pace of learning, and thereby fostering their progress and motivation, which boosts their achievement (Tosunoğlu, 2021).

Although DT provides numerous benefits, several disadvantages remain that may hinder the development of mathematical giftedness for students. Generally, gifted students can experience uneven development in intellectual, social and emotional domains, known as developmental asynchrony (Singer et al., 2016), which may be exacerbated by the inappropriate use of DT. This misalignment can instigate anxiety, reduce motivation, and precipitate an overreliance on DT at the expense of developing independent problem-solving and social-interaction skills (Periathiruvadi & Rinn, 2012). Thus, the success of DT in nurturing mathematical giftedness relies on its effective use as a tool to support and enhance multifaceted learning goals, instead of replacing traditional teaching methods.

Implications for future educators and researchers

Findings from this review illuminate the significant implications of integrating DT into the research areas in MGS. For educators, it suggests the importance of continuously developing DT skills to successfully incorporate it into their teaching repertoire to benefit MGS. Conversely, researchers have the opportunity to investigate and explore how DT can influence the cognitive and social-emotional growth of MGS. Collaboration between educators and researchers can lead to the development of best practices and innovative approaches, which may benefit mathematically gifted students' education and career trajectories. In the previous section, needs in curriculum development are offered as a means to comprehensively develop the corpus of literature in mathematics education and gifted education.

In conclusion, DT in gifted mathematics education presents opportunities for enhancing the learning process and outcomes of MGS. However, its successful implementation in gifted mathematics education depends on its careful curriculum design and planning. Hence, greater understanding than currently exists is requisite in its implementation. Moreover, a systematic approach to curriculum development, as was employed by Sheffield, Gavin, and colleagues (2009) is requisite to advance learning and the research thereof of mathematically gifted students. Developing mathematical giftedness in a fast-paced digital age may require a careful balance between DT and promoting essential skills including, critical thinking, problem-solving, and social-emotional competencies. As DT continues to evolve, ongoing collaboration among educators and researchers may be crucial for fully developing the potential of MGS in preparation for achieving excellence in the 21st century.

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REVOLVING DOOR MODELS AS DIGITAL SUPPORT PROGRAM: AN EXAMPLE OF A SUPPORT, TEACHING AND RESEARCH PROJECT

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As teaching and learning are increasingly taking place under the conditions of digitality, the question arises as to how the support of mathematical gifted pupils can be further developed in this context. The paper outlines the concept of a current project called "Digitale Drehtür Wuppertal"; it is located in the study curriculum for student teachers as a project seminar and teaching-learning lab, and it aims to design mathematics courses for the individual support of gifted students at a distance.

Keywords: Mathematical giftedness, support program, revolving door model, digitality.

INTRODUCTION AND RATIONALE

In Germany, mathematical giftedness is a widely researched topic in the context of school education (examples in Germany are the nationwide initiatives "Digitale Drehtür", a roof of the project outlined in this paper, Benölken & Hoiboom, 2024, and "Leistung macht Schule", Benölken et al., 2019). In such contexts, giftedness is usually interpreted as domain-specific performance potential and as a dynamic phenomenon; its development is considered to be influenced by interpersonal and intrapersonal factors (e.g., Benölken et al., 2024). This implies the importance of suitable offers of individual support, as numerous scientific works have indicated (e.g., Szabo, 2017), especially for the school context (Pettersson, 2011), to avoid, for example, underchallenge, learning frustration or loss of interest in mathematical activities (Höhmann & Kroes-Tillmann, 2008). Among acceleration-oriented approaches, in particular enrichment-based formats in which learners work on rich and challenging tasks are seen as beneficial for the individual support of mathematical giftedness (Nolte & Pamperien, 2017). Revolving door models, which can address both approaches, offer an example of support programs that are deemed to have positive influences on the development of individual potentials (e.g., Auhagen, 2023). The question arises how such support might be organized from the perspective of learning that takes place under the conditions of digitality. This need became clear with the coronavirus pandemic at the latest. The concept presented here, which is still under development, attempts to provide initial answers to this need. Simultaneously, the question is obvious, which conceptual formats can possibly be adapted, in particular considering criteria of organizing digital learning settings (e.g., Kerres, 2018). The aim of this paper is to describe the current development status of a project "Digitale Drehtür Wuppertal" ([DDW]; "Digital Revolving Door Wuppertal") that attempts to combine approaches of revolving door models and teaching-learning labs to address the described desideratum. First, relevant theoretical backgrounds are outlined. Then, the project's concept is described, which is essentially determined by the fact that student teachers at the University of Wuppertal develop digital contents for pupils in a mandatory elective course of their academic studies. Therefore, and because the challenges as to the concrete activities of pupils are still under consideration, a concretization of the conceptual aspects is made mainly from the perspective of student teachers' education, including a sketch of an example of their work. Moreover, first impressions of student teachers' evaluation of the project are given. Finally, the current state of organization is subsumed, and next planned steps are identified.

OUTLINE OF THEORETICAL BACKGROUNDS

As to the framework of *mathematical giftedness*, the project's foundation follows the scientific consensus interpreting the construct as an above-average potential regarding certain mathematics-related criteria illuminated in different scientific works like abilities in remembering mathematical facts, in structuring mathematical facts, in transferring structures, abilities of intermodal transfers or of reversing thoughts, as well as sensitivity and fantasy; these criteria are seen in undividable interdependence with mathematics-related personality traits like high mental activity, inquisitiveness, willingness of efforts, joy at task solving, ability to concentrate, insistence, self-dependence, or skills of cooperation. As already mentioned, giftedness is understood as a dynamic phenomenon, assuming that the development of individual potentials to visible performance is influenced by intra- and interpersonal catalysts. Overall, in this manner, giftedness is interpreted as a holistic construct that can be applied to a broad variety of learners (Benölken & Veber, 2021), which seems to be senseful due to the possible wide range of learners who might participate in the project DDW. As to the support of giftedness, the frameworks focuses on typical criteria for rich learning task or open substantial problem fields like openness as to multiple discoveries, solutions and strategies, as well as to the choice of auxiliary materials and ways of presenting solutions (e.g., Nolte & Pamperien, 2017).

The roots of *revolving door models* can be found in the work of Renzulli and Reiss on the "Enrichment Triad Model" and the "Schoolwide Enrichment Model" (e.g., Renzulli, 1977; Reis & Renzulli, 2003). In summary, the aims of such formats are to give pupils access to new topics and fields, to foster the development of both subject-specific and interdisciplinary skills, and to engage with self-selected content, even beyond the regular curriculum, in a developmentally appropriate manner and at a high level. Positive impacts on the development of individual mathematical potentials are indicated for participation in revolving door models, for example with regard to the willingness to make efforts in mathematics, the curiosity to deal with mathematical questions or the motivation to be mathematically active in general (e.g., Auhagen, 2023). German schools use both (mainly) acceleration and (sometimes) enrichment variants of revolving door models for a wide range of subjects (Greiten, 2016). With regard specifically to the fostering of mathematical potentials, Auhagen (2023) distinguishes four different types of revolving door model support, which can be classified according to acceleration and enrichment as well as to the level of self-directed learning on the learner side. The Project DDW pursues an enrichment-oriented approach that requires a relatively high degree of self-directed learning due to a digital architecture that is organized asynchronously. Furthermore, the project concept follows Auhagen's (2023) understanding based on theoretical-analytical discussions: revolving door models mean school-oriented support activities in which learners leave regular lessons for a certain period of time in order to work on individual contexts. For the project DDW, this implies a direct connection to school structures as to pupils who participate in the program.

Following Brüning (2018), the main conceptual aspects of *teaching-learning labs* can be outlined as follows: teaching-learning labs are formats that combine learning and support processes for pupils with practical work of student teachers from a holistic perspective, connected to a specific thematic complex such as giftedness. Teaching-learning labs are characterized by a reduction in complexity due to the focus on a specific complex, which allows student teachers to develop sustainable skills in diagnosis and support at an early stage of their academic studies. In addition, iterative processes are characteristic of teaching-learning labs, for example for the further development of the support concept, continuous reflections on individual diagnostic impressions and considerations on the success of the support. The teaching-learning lab format, thus, combines three main objectives: firstly, the support of pupils; secondly, the professionalization of student teachers; and thirdly, research, for example on the content focus of a teaching-learning lab or on the effects of participation in such a program with student teachers or pupils. Complexity reduction in the DDW project is ensured by the focus on mathematical giftedness. Mainly, iterative processes are provided at the moment by development loops in respect of the composition of suitable learning environments.

Exemplarily, the project's framework refers to the work of Kerres (2018) who illuminates six *criteria of organizing digital learning settings*: (1) There must be a match between the media-supported learning setting, teachers and learners (e.g., in terms of experience, motivation or prior knowledge); (2) then, the media-supported learning setting has to match the focused educational objectives, didactic goals and teaching content; (3) it has to be adaptive and flexible, for example as to learning location, learning time or use of media; (4) it must stimulate active-constructive learning activities and communication supportive of learning, as well as (5) self-directed and cooperative learning activities; (6) technical, social, organizational and expert support have to be guaranteed. In principle, these criteria can be synthesized well with the framing of rich tasks for the student teachers' development work.

„DIGITALE DREHTÜR WUPPERTAL“: IMPORTANT CORNERSTONES OF THE CONCEPT

At the current stage of implementation, the project aims to help student teachers to acquire competencies in the support of mathematical potentials, particularly in digital settings. In the future, they should also acquire diagnostic skills through suitable digital interaction with pupils. Building on the theoretical foundations for organizing mathematical teaching-learning processes in general, on the construct of giftedness including adequate approaches for individual diagnostics and support as well as on media pedagogical postulates for the design of digital learning settings, the student teachers who participate in the project independently plan course fragments for pupils, and realize them to such an extent that those fragments can be accessed via a learning management system by pupils.

The integration as a mandatory elective course in the study curriculum takes place as a so-called project seminar. It is aimed at advanced student teachers in the studies of the Master of Education, both for secondary school teaching qualifications and for special education teaching qualifications (an extension to the primary school sector is being planned). The standards for project seminars of this type are mainly in the development of competencies in scientific analysis, project organization and inclusive lesson planning, presentation and communication (also of own project results) as well as diagnosis or assessment competencies. For example, student teachers should analyze mathematical topics autonomously, interpret and adapt those topics for a specific context and prepare them as a learning arrangement suitable for inclusive or potential-oriented support. The connections in the context of the DDW project are made in accordance with the theoretical focus on giftedness, for example with regard to criteria for rich tasks or open substantial problem fields as a typical format, especially from an enrichment perspective.

During a semester term, the project seminar is conducted in three phases: The first phase is an attendance block of three days of six hours each. On the one hand, organizational aspects are clarified here, including the goal of the theory-based development of a digital course fragment for pupils as a project product. On the other hand, introductions to the theoretical foundations are developed, especially with regard to relevant frameworks of giftedness support and the synthesis to criteria of organizing digital learning settings; in this context a best-practice example is discussed as well. The second phase is a six-week workshop in which the student teachers work independently in small groups on the development of a course fragment (see the example below; as already mentioned, a standard learning management system has proved useful for the basic technical implementation). Although the student teachers largely organize themselves in this phase, there are flexible discussion opportunities from the lecturers at fixed times, for example to discuss the suitability of suggestions for possible mathematical topics as to the required criteria and to initiate feedback loops. Finally, the third phase is the project close-out: the groups present, reflect on and discuss their development work. For pragmatic reasons, a digital format has proven successful for this, since the digital products can be better understood in this way. A final attendance session provides a forum for a final comparative analysis of all developed products as well as space both for direct feedback from the student teachers and for evaluation studies, for example by drawing learning maps. In accordance with the requirements of the study curricula, the student teachers must prepare a written elaboration of subject-specific and didactic analyses on their submitted product.

Current status of development and example of a course fragment

The project seminar was organized for the first time in the summer semester of 2024. Initially, it has been open only for student teachers, not for pupils. The aim was to pilot and subsequently evaluate its university teaching concept. After a further implementation in this form in the winter semester 2024/2025, the project seminar will also be open to pupils; it can be assumed that the development work of the student teachers will result in a wide range of course fragments being available after a total pre-development period of around one year.

A group of student teachers chose the topic “magic squares” as approach of designing a course fragment. As is well known, these are n × n squares (for natural numbers n) whose row, column and diagonal sums correspond, whereby the individual numbers within a magic square are necessarily disjoint. For example, for $n = 3$, the sum (also called the “magic number”) is 15 with the number 5 given in the center. The course fragment is embedded in the learning management system as a digitally created learning environment in interactive H5P Format. It starts with a welcome slide that introduces a learning agent in the form of a wizard who guides pupils as a narrative figure. Then, the “research aims” of the topic are presented transparently. The technical adaptation is implemented as an external linking to an educational app, which enables the design of interactive formats without, for example, advanced programming knowledge. This is followed by instructions on how to use this software and work in the learning environment itself, such as active (through direct contact with the teacher) or passive (through interactive buttons) assistance or integrated learning videos. This is followed by content-related impulses, starting with an examination of the numbers and their arrangement in a 3 x 3 and a 4 × 4 square, which can be worked on interactively. In the process, patterns and strategies can be discovered and they can be shared on a digital observation sheet. More substantial research impulses are then directed towards creating own magic squares and possibly other magical forms. In particular, asynchronous exchange is prepared through digital forums and upload folders for solution approaches.

FIRST IMPRESSIONS OF EVALUATION STUDIES

The university teaching concept of the project has been qualitatively evaluated from several perspectives. A comprehensive evaluation using learning maps is currently being conducted (similar to the approach of: Benölken, 2017). The impulse is as follows: *"Please reflect on your path through this project seminar and the creation of the learning environment and represent this path graphically. Consider, for example: What did you learn from the initial sessions? What content was particularly helpful for you? How did you feel about the design of the learning environment and what opportunities, challenges or stumbling blocks did you encounter?"* Learning maps of 14 student teachers have been produced; more extensive analyses of learning maps of around 30 student teachers expected by the summer semester using pedagogical-iconological interpretation methods are in preparation. One example (see Figure 1) will be outlined as a first insight, without going into detail about interpreting the senses of fact, expression and work indicated in the learning map (according to Benölken, 2017).

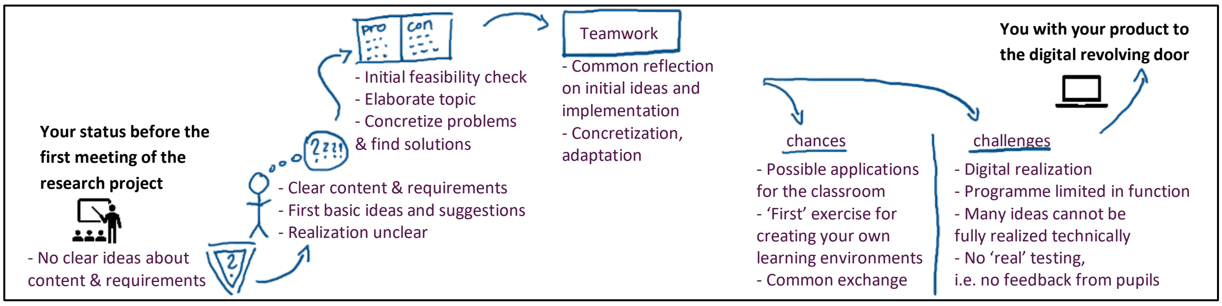

Figure 1: An exemplary selected learning map (facsimile of a German language original).

The connection between theory and practice focused on possibilities of individual (giftedness) support and differentiation options, including the corresponding task design, were seen as positive aspects, for example. The concrete handling of the digital medium, which was described as flexible and easy to implement, was also seen as an opportunity, while the positioning of the example at the end of the theory phase and the written elaboration were perceived as a challenge. Working with pupils was explicitly stated as a wish. The roughly outlined positive impressions correspond in principle with the accents of the learning maps drawn by other student teachers. With regard to negative aspects, the main additional points raised were that a concrete course fragment should be introduced as an example in the first project phase before the theoretical foundations are addressed, that there could be more time for the workshop phase, and that support from the lecturers could be more institutionalized.

Conclusion and perspectives

Against the background of the desideratum that individual support under conditions of digitality should be realized with mathematically gifted pupils (as with other groups it should be the case), connecting approaches of teaching-learning labs and of revolving door models seems to be promising. For the moment, this thesis is based on perspectives of university teaching, since such considerations are the first anchor of the conceptional work in the DDW project. Among deeper clarifying these impressions by evaluation studies, and according to whishes expressed by student teachers who participated in the project seminar, the most important next step can be seen in developing a format regarding an active involvement of pupils. The applied framework of giftedness implies holistic and process-oriented diagnostics – this will be necessary to consider when developing such structures. Furthermore, a challenge is given by the need of establishing networks of schools which institutionally support realizing an authentic organization of a revolving door model by the offered digital structure. As the project concept is mainly designed to be asynchronous (through a learning management system), there could appear limitations to collaborative learning as one of the central criteria of organizing digital learning settings, which should be compensated as far as possible (for example, through forums that can be used asynchronously or through synchronously organized conference days). If this was not the case, the architecture developed in the project would remain nothing more but a digital (even if interactive) design of a traditional "task circle".

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UNPACKING MATHEMATICAL CREATIVITY: THE INTERPLAY OF PERSONALITY, CREATIVITY, AND MATHEMATICAL ABILITY

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The aim of this study was twofold: first, to develop a theoretical model examine the impact of personality, general creativity, and mathematical ability on the emergence of mathematically creative behavior; and second, to compare this model across groups of students with varying levels of mathematical creativity. Participants included 476 students in grades 4-6 in Cyprus. Data were collected using a mathematical creativity test, a general creativity test, a mathematical ability test, and a self-report personality questionnaire. Structural equation modeling analysis revealed that mathematical ability and general creativity were stronger predictors of mathematical creativity than personality characteristics. However, comparisons of the model across the groups of students indicated that personality characteristics differentially impacted low-achieving students compared to their higher-achieving peers. Implications for teachers and educators are discussed.

Keywords: Mathematical creativity, personality traits, creative person, fluency, flexibility, originality.

INTRODUCTION AND THEORETICAL BACKGROUND

As educational systems prepare students for the complexities of the modern world, cultivating creativity is considered an essential skill for success in the 21st century. Therefore, educational systems should encourage students to navigate complexities, confront the unknown, and arrive at innovative solutions (Suherman & Vidakovich, 2024). Creative thinking has transcended traditional boundaries and is now emerging across diverse disciplines. In this context, mathematical creativity (MC) is defined across three dimensions: (a) fluency, which refers to the variety of possible solutions, (b) flexibility, which involves the use of diverse mathematical ideas, and (c) originality, which reflects the uniqueness of a solution (e.g. Kattou et al., 2013; Mann, 2005). Other definitions of MC incorporate the abilities of problem-solving, problem-posing, applying knowledge, and approaching situations innovatively (Silver, 1997; Suherman & Vidakovich, 2024).

While previous research has explored the relationship between MC and cognitive factors like mathematical ability and general creativity (e.g., Mann, 2005; Hong & Milgram, 2010), as well as the influence of personality traits on creative performance (e.g., Juriševič & Žerak 2024), these studies have examined these factors in isolation. This study addresses this limitation by examining the combined influence of cognitive and personality factors on MC. Specifically, it aims to develop and validate a model to identify the key factors impacting the manifestation of MC and to compare this model across students with varying levels of MC.

Cognitive Characteristics and their relation to MC

The effect of cognitive characteristics, such as content knowledge and general creativity, on the manifestation of MC has been widely examined, leading to mixed results. Some studies suggest that possessing mathematical knowledge enhances creative potential by facilitating information processing and concept connections (Kattou et al., 2016; Mann, 2005). However, others argue that familiarity with algorithms hinders the development of novel solutions

(Baran et al., 2011). This raises the question of whether a certain level of mastery is necessary before MC can flourish, or if excessive focus on procedures can stifle originality.

Similarly, conflicting findings have emerged regarding the generality versus specificity of creativity. While some studies indicate a causal effect of general creativity on MC (Kattou et al., 2016; Mann, 2005), others emphasize the domain-specific nature of creativity, citing low correlations between creative outcomes in different cognitive fields (Plucker & Zabelina, 2009). This study aims to shed light on these issues by examining the contributions of mathematical ability and general creativity to MC within a comprehensive model.

Personality Traits of creative individuals

In the question, "What are the characteristics of a creative person?" numerous traits have been proposed. In the context of the present study, the Investment Theory (Sternberg & Lubart, 1996) was adopted to organize the personality traits that define creative individuals. The Investment Theory likens creative individuals to successful investors who "buy low and sell high." In this analogy, creative individuals invest in ideas that may initially seem undervalued, but they transform these ideas into valuable and significant concepts (Sternberg & Lubart, 1996). According to the theory, the development of creativity requires six interrelated components: intellectual abilities, knowledge, cognitive style, personality, motivation and environment. Specifically, intellectual abilities such as synthetic ability (the generation of original ideas and novel approaches), analytical ability (the critical evaluation and recognition of valuable ideas), and practical ability (the application of theory to practice) are all essential for creativity to emerge. In addition, possessing knowledge is fundamental for contributing creatively to a field. The cognitive style—how an individual prefers to use their knowledge and skills—also plays a critical role in creativity. Sternberg and Lubart (1996) identified several personality traits that enhance creative behavior, including persistence, risk-taking, self-regulation, curiosity, and imagination. Another component of the Investment Theory is motivation. Creative individuals are driven by an intrinsic desire to engage in their work, rather than external rewards. Finally, a supportive environment is crucial for fostering creativity. Even if an individual possesses the necessary cognitive abilities, without the right support, their creative potential may remain unexpressed.

The way individuals perceive their personality traits seems to influence their creative abilities. Kattou et al. (2016) proposed that personality characteristics may impact students' MC, suggesting that individuals should understand their own creative potential to 'create.' Suherman and Vidakovich (2024) found that perceived creativity positively contributed to students' MC. Specifically, students who viewed themselves as creative were more likely to engage in creative thinking processes, driven by intrinsic motivation to find novel solutions and ideas. In contrast, their peers, who lacked confidence in their creative abilities, hesitated to act, assuming that the answers were already known.

Considering these controversial issues, the present study aims to develop and validate a model of MC, incorporating its indicators (fluency, flexibility, and originality) as well as its predictors (cognitive and personality characteristics). Using a quantitative approach, namely structural equation modeling analysis, we aim to determine how these factors contribute to variations in MC. Furthermore, building on the findings of Juriševič and Žerak (2024), which emphasize the importance of a more personalized approaches to fostering creativity, the second aim of the study is to compare the structure of this model across students with varying levels of MC, to uncover similarities and differences among them.

METHODOLOGY

Data collection

The study was conducted among 476 students of 4th, 5th and 6th grades by completing four tests: a mathematical creativity test (MCT), a general creative test (GCT), a mathematical test (MT), and a self-report questionnaire (SRQ). In particular, the MCT consisted of four multiple-solution mathematical tasks, requiring students to generate as many different and original answers as possible within a 40-minute timeframe. This approach aligns with the assessment method proposed by Kattou et al. (2013). Three scores were calculated for each task: (a) fluency, measured as the ratio of correct solutions provided by the student to the maximum number of correct solutions provided by any student in the sample; (b) flexibility, calculated as the ratio of different types of correct solutions provided by the student to the maximum number of different types of solutions provided by any student in the sample; and (c) originality, determined based on the frequency of the student's solutions compared to the solutions provided by all students. The final MCT score was obtained by summing the fluency, flexibility, and originality scores across the four tasks, with a maximum possible score of 3. An example task from the MCT is presented in Figure 1.

Make as many groups of numbers as you can, using the numbers: 2, 3, 7, 9, 13, 15, 17, 25, 36, 39, 49, 51, 60, 64, 91, 119, 121, 125, 136, 143, 150. Label each group with its characteristic.

Warnings: You can use each number in more than one group.
Each group should contain more than two numbers.

Figure 1: Example of tasks from the Mathematical Creativity Test.

The SRQ consisted of 15 statements describing behaviors related to mathematics, developed based on the Investment Theory (Sternberg & Lubart, 1996). Three statements were included for each aspect of the theory, except for "styles of thinking," which was not easily measurable through a self-perception questionnaire. Students responded to each statement on a 7-point Likert scale ranging from 1 (strongly disagree) to 7 (strongly agree). The GCT included two tasks adapted from the Torrance Tests of Creative Thinking (Torrance, 1974): an Unusual Uses task, which asked students to generate creative uses for a common object, and a Picture Completion task, where students were asked to complete a drawing in an original way. Students were encouraged to provide as many answers as possible within a specific time interval. Their responses were assessed based on fluency, flexibility, and originality, as described previously. The MT comprised five tasks designed to assess students' mathematical knowledge (e.g. problem-solving, numerical operations, patterns, graph interpretation, and the area and perimeter of complex figures).

Data analysis

The data analysis aimed to: (a) verify a theoretical model examining the relationships between MC, mathematical ability, general creativity, and personality; and (b) identify groups of students with varying levels of MC and investigate potential differences in the model's structure across these groups. To address the first objective, we employed Structural Equation Modeling using the SmartPLS. This analysis allows researchers to model and estimate complex relationships between multiple dependent and independent variables, and to visually display these relationships in diagrams (Hair et al., 2021). Specifically, we tested the validity of an a-priori theoretical model depicting the hypothesized relationships between the latent variables of MC (with indicators of fluency, flexibility, originality), and its predictors (mathematical ability, general creativity, personality). Regarding the model presented in Figure 2, the variables shown in ovals represent latent variables, which are not directly measurable, while those depicted as rectangles correspond to items and/or variables that are directly measured. The arrows connecting the variables illustrate the relationships between them. For the second objective, we categorized students into four groups based on their MC performance: Group 1 (lowest 15%), Group 2 (15%-50%), Group 3 (50%-85%), and Group 4 (highest 15%). We then conducted a multigroup analysis to determine whether the strength of the relationships between MC and its predictors varied across different levels of MC.

RESULTS

The Validation of the Model

Based on the theoretical background, we hypothesized that MC is composed of fluency, flexibility, and originality. Furthermore, we assumed that MC could be explained by mathematical ability, general creative ability, and personality traits. Figure 2 presents the structural equation model, including the latent variables and their corresponding indicators.

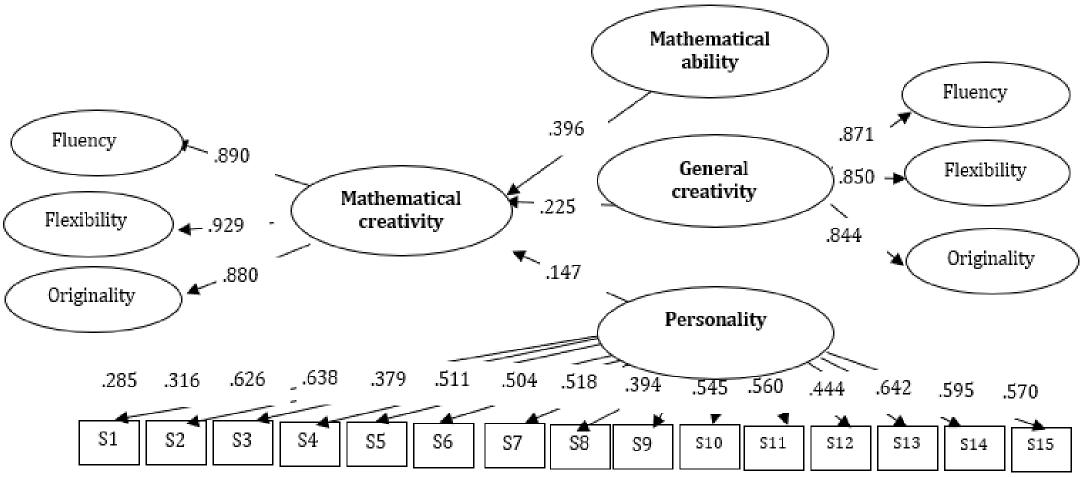

Figure 2: The structure of the proposed model.

The analysis revealed that the theoretical model is a good fit for the data set (NFI=0.715, SRMR=0.068). The analysis showed that the statistically significant loadings for fluency ($r=0.890$, $p < 0.05$), flexibility ($r=0.929$, $p < 0.05$), and originality ($r=0.880$, $p < 0.05$) formed a first-order factor representing MC. Additionally, MC was primarily predicted by mathematical ability ($r=0.396$, $p < 0.05$) and general creativity ($r=0.225$, $p < 0.05$), while personality characteristics ($r=0.147$, $p < 0.05$) were the weaker predictor. It is worth noting that this model explained 31.4% of the variance in MC.

Model variations across groups of students with different mathematical creativity

Four groups of students were identified based on their performance on the MCT, as shown in Table 1. This categorization enables us to investigate potential differences among them.

	N (%)	Mean (SD)	Fluency	Flexibility	Originality
Group 1	71 (14.92)	.563 (.12)	.19 (.057)	.17 (.035)	.21 (.065)
Group 2	168 (35.29)	.907 (.10)	.32 (.071)	.24 (.038)	.35 (.077)
Group 3	168 (35.29)	1.24 (.11)	.42 (.087)	.31 (.044)	.52 (.924)
Group 4	69 (14.50)	1.67 (.16)	.55 (.097)	.41 (.057)	.70 (.086)

Table 1: Descriptive statistics of the groups of students.

A follow-up multigroup analysis (see Table 2) revealed statistically significant differences in the effect of personality on MC between the groups ($p < .05$), except for the comparison between Groups 3 and 4. No statistically significant differences between the groups regarding mathematical or general creative ability were found. Notably, the personality factor loading for Group 1 was negative (-.456), suggesting that personality characteristics differentially impacted low-achieving students compared to their higher-achieving peers.

	Personality	Mathematical ability	General creativity
Group 1	-.456	.087	.344
Group 2	.126	.166	.315
Group 3	.286	.110	.378
Group 4	.355	.127	.263

Table 2: Factor loadings on the model's structure across students vary on MC.

DISCUSSION

"Every child has a certain level of creative potential, and this potential should be developed and nurtured in all children" (Juriševič & Žerak, 2024, p. 5). This study contributes to the goal of nurturing every child's creative potential by presenting a model that demonstrates the complex interplay of mathematical ability, general creativity, and personality traits in the development of MC. While our model provides valuable insights, it explains only 31% of the variance in MC. This suggests that other factors, such as motivation, learning environment, and specific teaching practices, may also play a significant role. Future research could explore these additional factors to develop a more comprehensive understanding of MC. Furthermore, longitudinal studies could provide a more dynamic perspective on how MC develops over time and how different factors interact to influence this development.

The present study suggests that to enhance students' MC, educators should focus on strengthening their mathematical background and fostering their general creative thinking skills. By strengthening students' mathematical knowledge, representations, concepts, and algorithms, teachers can help them apply relevant information more fluently, allowing flexible and original connections necessary for problem-solving (Kattou et al., 2016). Additionally, general creative ability equips students with the skills to combine ideas and explore alternative approaches with original perspectives (Hong & Milgram, 2010).

While personality traits were found to be statistically significant predictors of MC, their predictive power was lower than that of mathematical and general creative abilities. This suggests that personality, while important, is not the sole determinant of MC. However, our multigroup analysis revealed that personality traits differentially impacted low MC students compared to their higher-achieving peers. High MC students often exhibit greater self-awareness and self-confidence, traits that enhance MC (Kattou et al., 2016; Suherman & Vidakovich, 2024). Students who consider themselves creative are more likely to engage in creative thinking processes, driven by intrinsic motivation to find novel solutions. On the other hand, certain personality traits might hinder MC for students with low levels of MC. For instance, fear of failure or lack of perseverance might prevent these students from applying unconventional approaches or persisting through challenges. Furthermore, high levels of risk-taking without the necessary foundational knowledge could lead to unproductive exploration, while a lack of confidence might cause them to hesitate and stifle their creative impulses. As Juriševič and Žerak (2024) pointed out, a comprehensive profile of students will enable teachers to design and implement tailored creativity programs.

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A FIRST APPROACH TO MATHEMATICAL CREATIVITY AND CREATIVE SELF-EFFICACY IN GIFTED STUDENTS

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As one of the 21st Century Skills, creativity is presented as an important ability to enable processes of learning and innovation. To promote this skill in the next generation, creativity needs to be focused on in educational institutions. Because empirical research on creativity, especially in mathematics education, has been relatively rare in the last century, we investigate different factors influencing mathematical creativity. In this paper, we aim to answer the research question of the extent to which gifted students´ creative self-efficacy relate to their mathematical creativity. Our correlational study with 24 gifted students shows that creative self-efficacy is significantly correlated with mathematical creativity as measured by unstructured problem posing.

Keywords: Mathematical creativity, problem posing, creative self-efficacy, giftedness.

INTRODUCTION

Creativity is a necessary ability to promote innovation skills in our society. As part of the learning and innovation skills from the 21st Century Skills (Pásztor et al., 2015), creativity has been put back in the spotlight. Especially in educational institutions, creativity should be encouraged from a young age because it helps people to maintain an interest in learning (Williams, 2002) which enables a self-determined and self-regulated way of working that is important in our modern society. That is why creativity is seen as a mental ability that is evident in all areas of our life (Conradty & Bogner, 2018). These might be some of the reasons why the latest PISA study measured creative thinking for the first time worldwide (OECD, 2023). However, due to the complexity of creativity and the multiplicity of its definitions, empirical research on creativity, especially in mathematics education, was relatively rare in the twentieth century, but has now increased significantly (Leikin & Sriraman, 2022). As we also want to contribute to an increase in research, especially in the area of mathematical creativity, we are investigating which different factors influence mathematical creativity. In this paper, we analyse the relationship between mathematical creativity and creative self-efficacy of gifted students in order to find out how to promote mathematical creativity in the educational context. Therefore, we aim to answer the following research question: To what extent does gifted students´ creative self-efficacy relate to their mathematical creativity?

THEORY

First of all, we need to look at creativity in general. Creativity consists of four components - fluency, flexibility, originality and elaboration - which can be used to measure general creativity (Torrance, 1974). Since many researchers understand creativity as a general ability that needs to be further concretised in different domains in order to adapt to the characteristics of this domain (Haylock, 1987), we focus on the mathematical domain of creativity. Regarding mathematical creativity, Leikin and Elgrably considered these four components of Torrance to be valid in mathematics as well (Leikin & Elgrably, 2022). However, when measuring (mathematical) creativity in practice, there seems to be a contradiction between the two components of fluency and elaboration: fluency is the ability to answer or pose as many problems as possible, whereas elaboration is the level of detail within a given answer or question (McCaffrey, 2018). Since a time limit seems inevitable when testing (mathematical) creativity, you cannot be careful about both components: writing down as many different answers or problems as possible and at the same time trying to be as detailed as possible. For us, when we focus on measuring mathematical creativity in practice, we have to leave out the elaboration component in the context of mathematics.

On the one hand, we see mathematical creativity as the creation of mathematical products that are creative according to the three components of fluency, flexibility and originality, and that include problem solving and problem posing, the latter having three categories: Structured, semi-structured and unstructured problem posing tasks (Stoyanova, 2000). Problem posing involves the generation of new problems and the reformulation of given problems, it can be used to measure mathematical creativity and is therefore a diagnostic or assessment tool (Silver, 1994). Also, problem posing can be used as a method to promote gifted students (Siller et al., 2024). On the other hand, we see mathematical creativity as a relative construct when evaluated in relation to students´ previous experiences and the performance of other students with similar educational histories (Leikin, 2009). Seeing creativity as relative refers to Plucker and Beghetto´s Four-C Model: considering the individual creativity of students, it belongs to the so-called Mini-C level, which is part of relative creativity (Plucker & Beghetto, 2004). Because there is still no universally accepted definition of mathematical creativity, we summarise our view of mathematical creativity by saying that we understand mathematical creativity as the ability to create new, original and diverse mathematical ideas or products within a (suitable) peer group, which may consist of finding solutions or solution paths, or (re)formulating mathematical problems.

As our comparison peer group consists only of highly gifted students, we take a look at giftedness and its relation to (mathematical) creativity. Gifted students are those who have a high level of ability, for example, in intellectual, creative, artistic, or leadership skills or in a particular academic field (Singer et al., 2017). Haavold found that gifted students can be divided into two groups: academically and creatively gifted (Haavold, 2013). There is empirical evidence that mathematical creativity implies mathematical giftedness, but not necessarily the other way around (Sriraman, 2005). That is probably one reason why most researchers agree that giftedness and creativity are positively related (Leikin, 2009).

Another construct that has a connection to creativity is creative self-efficacy which is defined as the perception of having the confidence and ability to be creative in work and produce creative results (Tierney & Farmer, 2002). Creative self-efficacy has its roots in the social cognitive theory, which first revealed that self-efficacy plays a motivational role in the process of creativity and innovation (Bandura, 1997). There is empirical evidence that creative self-efficacy predicts creative self-identity and achievement, as well as creative performance (e.g. Karwowski, 2014; Tierney & Farmer, 2002). Self-efficacy also influences science motivation which is actually related to creativity (Conradty & Bogner, 2019).

METHOD

With this theoretical background, we aim to answer our mentioned research question. For this, we created a paper-pencil test consisting of five items for measuring creative self-efficacy and of one item measuring mathematical creativity, specifically unstructured problem posing, as problem posing is the most overlooked part of mathematical creativity so far (Haavold, 2013). The creative self-efficacy items had to be answered on a five-point Likert scale ranging from ‘does not apply at all’ to ‘fully applies’ (Cronbach´s alpha of this scale: $\propto = 0.723$). These included the following questions:

> I have the confidence to… (1) …think of something new, (2) …realise a creative idea, (3) …tell others about my creative idea, (4) …be imaginative in a meaningful way in the classroom, (5) …be creative when solving math problems.

The problem posing item had to be completed within a time limit of five minutes and, prior to this item, participants were given written information that this task was about their mathematical creativity and that they should come up with as many questions as they could for one of the following games: Rock-Paper-Scissors and Ludo. The context of games we have chosen is extra-mathematical because it is a known, common and familiar area for students. We offered the choice between these two different games in order to support autonomy to increase self-determined motivation (Hagger et al., 2015). 10 students chose Rock-Paper-Scissors and 14 chose Ludo, but there was only little difference in the number of problems posed when looking at the two different groups of students.

All items were checked by an expert panel of eight experts. The described test manual was given to 24 high school students (11 girls, 13 boys, 4 attending academic year 10, 20 attending academic year 11) with an average age of 16.6 years. We consider all of them to be gifted students as they took part in a five-day programme at the University of Würzburg, for which they had to be recommended and registered by their mathematics teacher. Most students have a grade 1 in mathematics (the highest grade in Germany, their average is 1.04).

To measure mathematical creativity, we used qualitative content analysis according to Mayring (Mayring & Fenzl, 2019) to evaluate the problems posed. Here we gave each student a score using a scoring system adapted from Leikin (Leikin, 2009). First, for the fluency component, we counted the number of problems posed that were considered to be mathematically reasonable and solvable. We defined a question as mathematical if it required mathematical knowledge or skills to answer it. Each mathematical question was then scored as 10, 1 or 0.1 points for the flexibility component score, depending on the student´s use of different categories (e.g. counting, probabilities, percentage) – each new category scored 10 points and subsequent questions of the same category scored 1 or 0.1 points. Finally, we looked at each mathematical problem and gave credits (again 10, 1 or 0.1) according to its originality. The final score for the student´s mathematical creativity was calculated as in Leikin (2009).

In the following, we will give a student example with problems posed about Ludo:

Problems posed	Flu	Fle	Or
What is the minimum number of times you have to roll the dice to win?	✓	10	1
How far do you get on average per move (with bonus dice at 6)?	✓	1	10
Is it advantageous/disadvantageous to place the pieces that are already safely at the end as far forward as possible?	✓	10	1
On average, how often do you roll the dice without hitting?	✓	0.1	1

Table 1: Scoring example for fluency (Flu), flexibility (Fle) and originality (Or).

All four problems posed where considered to be mathematical. Questions 1, 2 and 4 are about counting – therefore these questions are scored with 10 (for the first occurred question), 1 (for the second) and 0.1 (for the third) for the flexibility component. Questions 1 and 4 were also asked by another student why their originality was rated 1. The second question was only asked once why it was given 10 points for originality. The third question covers a different category, namely strategy, so it was given 10 points in flexibility and, as it did not appear once, its originality score is 1. The sum of the student´s creativity score (Cr) is as follows: $Cr = \sum_{i=1}^{4} Fle_i \cdot Or_i = 10 \cdot 1 + 1 \cdot 10 + 10 \cdot 1 + 0.1 \cdot 1 = 30.01$

The analysis process was carried out by two researchers working separately, who then compared their scores and agreed on the final scores in a discussion. This secondary and consensus coding made the whole process reliable. After preparing the data, we carried out a correlational analysis using the Jamovi software.

RESULTS

In our correlation analysis, we found that there is a positive correlation between mathematical creativity and creative self-efficacy. Pearson´s correlational coefficient in our analysis is r=0.735 and is therefore highly significant according to Cohen (1988), which indicates that individuals with higher creative self-efficacy also had higher mathematical creativity scores. Looking at the different components of creativity, we can see that originality correlates very significantly with creative self-efficacy (r=0.737). There is also a strong correlation between fluency and self-efficacy (r=0.548) and a moderate one between flexibility and self-efficacy (r=0.452). From these results, it could be hypothesised that all three components of mathematical creativity are positively correlated with creative self-efficacy, although there are differences in the extent of their correlation.

	Fluency	Flexibility	Originality	Creativity
Pearson´s r	0.548	0.452	0.737	0.735
p-value	0.006	0.027	<.001	<.001

Table 2: Results of our correlation analysis.

DISCUSSION

In our results we can see that not every component of creativity correlates in a very significant way with creative self-efficacy. As the fluency component is based on the speed of processing, it should be noted that participants were instructed to produce as many questions as possible within a given time limit. This limitation in the realisation of our study could be an explanation for the extent of this correlation. Another limitation is certainly the small number of items in our test manual. That is why we need to expand the number of items in a next step, in order to get more information about the actual status of mathematical creativity in students, especially by not only focusing on the aspect of problem posing. Furthermore, the sample of n=24 needs to be expanded in order to make generalisable statements on this issue. However, the relative number of highly gifted individuals in the general population is reflected in the number of the students in our study sample. Once these limitations are removed, we can develop valid theories about how mathematical creativity develops and can be fostered through creative self-efficacy of which we only know that it predicts creative achievement and creative performance. Even if we can only express a vague tendency with our sample and test manual, the described result of our study about the link between self-efficacy and mathematical creativity is also confirmed in the literature (e.g. Meier et al., 2024). That this relationship also applies to gifted students is our contribution to a better understanding of the complex fields of mathematical creativity and giftedness. For the school context, this means that when promoting mathematical creativity and creative self-efficacy, it is not necessary to differentiate between pupils according to their performance level. As a result, these topics can be more easily incorporated into school lessons and thus contribute to the strengthening of our society.

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EVALUATING TWO STEM ENRICHMENT PROGRAMS ON COMPUTATIONAL THINKING: A PILOT STUDY

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In this paper, we outline the data collection of a pilot study conducted in the summer 2024. The main study, scheduled for 2025, aims at addressing the gap in the literature regarding the impact of STEM (Science, Technology, Engineering, and Mathematics) enrichment programs with a primary focus on computational thinking. While existing research has highlighted the effectiveness of such programs on cognitive and psychological factors, much of the evidence remains limited to context-specific studies. This project aims to compare two distinct enrichment programs one in Graubünden, Switzerland ("iCamps" at the University of Teacher Education of the Grisons) and the other in Thuringia, Germany ("Schülerforschungszentrum Jena"). The goal is to compare how these programs influence students' (both high-achieving and average-performing) psychological and cognitive outcomes. By providing insights into how enrichment programs (focusing on problem-solving skills connected to computational thinking) impact students across varied educational environments, this project fills a gap in comparative research within extracurricular enrichment programs.

Keywords: enrichment programs, psychology factors of giftedness, problem-solving skills

INTRODUCTION

In recent years, enrichment programs for STEM education have gained significant importance. There is increasing interest in literature investigating the impact of STEM enrichment programs on psychological factors, such as learners' attribution patterns, interests, self-efficacy, and self-concept, as well as their academic gains (e.g., Chiang et al., 2022).

In 2015 the "Ständige Konferenz der Kultusminister der Länder in der Bundesrepublik Deutschland" (KMK) issued a support strategy for high-achieving students. This document outlines measures in the areas of diagnostics, in-school and out-of-school support, as well as teacher training. It identifies four key strategies: enrichment, acceleration, grouping, and integrated support (KMK, 2015). The enrichment strategy focuses on offering additional, extracurricular courses in which students can expand and deepen their knowledge and skills in specific areas. This can be achieved through school clubs, study circles, competitions, or student academies. Enrichment programs, as outlined by KMK, are a key component in supporting high achieving and gifted students. In the following, the term giftedness is used based on the "Munich Model of Giftedness" (Heller, 1990), which distinguishes between giftedness and achievement. While every achievement is based on a corresponding potential, not every potential can be transformed into an actual achievement. The realization of potential into performance depends on numerous external variables, such as interest, motivation, or support (Geitel, 2020). This underlines the importance of focusing on psychological factors in this context. In the center stands the question: What should be the prerequisite for support?

Giftedness, as potential, cannot be directly measured and can only be assessed diagnostically through intelligence definitions using an IQ test. However, the established thresholds for (high) giftedness are arbitrarily chosen, and intelligence as the sole predictor of giftedness is questionable. Achievement can be measured reliably, but potential cannot always be translated into performance. Underachievers would be excluded in this case. Interest, on the other hand, while extensively discussed in theoretical literature (e.g., Hidi, 1990), has limited empirical research supporting its measurement and role in giftedness. Moreover, students can assess and recognize their own interests independently. Interest is often accompanied by intrinsic motivation and a willingness to perform, which typically correlates with high academic achievement in the corresponding subject area. The concepts of the two measures examined explicitly dedicate support to students with a clear interest in the subject (Geitel, 2020). The "Munich Model of Giftedness" (Heller, 1990) is based on five dimensions of giftedness. According to this model, giftedness can manifest in intelligence, creativity, social competence, musical ability, and psychomotor skills. These dimensions were later expanded to include two additional factors: artistic abilities and practical skills (Heller, 2001). These dimensions of giftedness are also referred to as predictors. Academic and extracurricular achievements can occur in various areas. The extent to which specific achievements develop from the individual factors of giftedness is influenced by moderators. These moderators include non-cognitive personality traits such as learners' attribution patterns, interests, selfefficacy, and self-concept, as well as their academic gains. Therefore, achievement is the sum of predictors and moderators (Heller, 1990; Geitel, 2020).

The main objective of this project, scheduled for summer 2025, is to empirically investigate the impact of enrichment programs, particularly those focusing on problem-solving skills connected to computational thinking, on students' psychological and cognitive constructs. Consistent with established literature (Zindel, 2022), computational thinking in this context is distinct from programming. It includes processes such as problem-solving, important in both mathematics and computer science education. Specifically, this project aims to examine two programs offered by the "MINT-Zentrum" of the University of Teacher Education of the the Grisons (PHGR; program "iCamps") and the "Schülerforschungszentrum Jena" (SFZ; program "Robot relay race"), each designed to foster computational thinking and engage students actively in mathematics and computer science. These programs are tailored to the educational contexts of the Canton Graubünden and the federal state of Thuringia, aligning with a broader definition of STEM enrichment program that includes digital tools and emphasizes computational thinking (e.g., Su & Yang, 2023). Considering the "iCamps" of the PHGR and the "Robot relay race" of the SFZ Jena as enrichment programs, the project aims to address three main research questions: (1) How do PHGR "iCamps" and SFZ Jena "Robot relay race" enrichment programs influence students' attribution patterns, interests, selfefficacy, and self-concept in mathematics and computer science? (2) How do the PHGR "iCamps" and SFZ Jena "Robot relay race" enrichment programs influence students' performance, particularly in computational thinking? (3) How do high-achieving and average-performing students differ in knowledge acquisition, self-efficacy, self-concept, attribution patterns, and interest when participating in STEM enrichment programs?

In this paper, we present the findings and observations from an initial validation pilot study conducted in Jena to assess the effectiveness of the instrument designed for a larger-scale data collection scheduled for 2025. The results offer valuable insights for refining the methodology, identifying potential areas for improvement to ensure the robustness of the upcoming study. The findings provide preliminary insights into children's problem-solving skills in computational thinking, as well. The content and pedagogical approaches of the "Robot relay race" enrichment program will be presented during the conference.

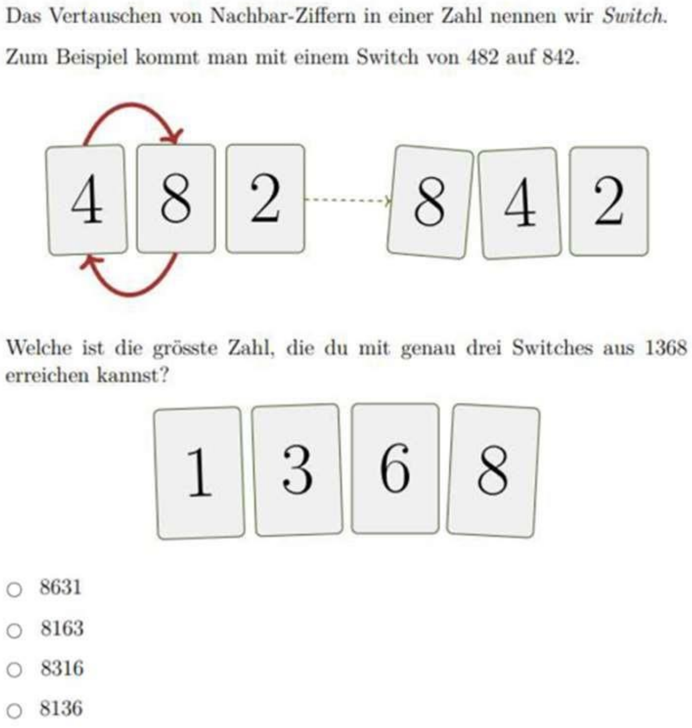
METHODOLOGY

Sample

The pilot study sample consists of 42 sixth to tenth graders participating in the " Robot relay race" program by SFZ Jena.

Materials

The study utilizes an instrument comprising a range of validated scales to assess the key constructs of interest (e.g., Benölken, 2013; Geitel, 2020). Participants responded to a total of 7 Likert scale and multiple-choice questions, covering the psychological constructs such as interest (2 questions), self-efficacy (1 question), self-concept (2 questions), and attribution patterns (2 questions). Additionally, participants answered 10 multiple-choice tasks (see Figure 1 for an example) designed to evaluate their competencies in computer science, with a focus on computational thinking, taken from Serafini (2025).



Das Vertauschen von Nachbar-Ziffern in einer Zahl nennen wir *Switch*.

Zum Beispiel kommt man mit einem Switch von 482 auf 842.

Welche ist die grösste Zahl, die du mit genau drei Switches aus 1368 erreichen kannst?

- ○ 8631
- ○ 8163
- ○ 8316
- ○ 8136

Figure 1: A multiple-choice task requiring students to swap adjacent digits in the number 1368 three times to achieve the largest possible number with exactly two switches.

Procedure

The pilot study, conducted in the summer 2024, assessed the instrument developed for the larger-scale data collection scheduled for 2025. This phase of the pilot study focused on establishing baseline measurements for all psychological constructs and subject knowledge.

DATA ANALYSES

This pilot study evaluates the instrument designed to assess informatics competence and psychological constructs for a planned larger-scale data collection. The analysis investigates the reliability of the instrument, including ten multiple-choice tasks designed to assess informatics competence, as well as seven Likert scale and multiple-choice questions measuring psychological constructs. Moreover, it examines how children performed on the informatics competence.

PRELIMINARY RESULTS

In the initial phase of the analysis, the internal consistency of the psychological constructs was assessed using Cronbach’s alpha for each set of items. The results demonstrated acceptable to excellent reliability, with Cronbach’s alphas of .70, .74, .87, .88, and .86 for the constructs of self-concept, interest, and self-efficacy. Notably, the items related to the construct of attribution patterns were not scaled, as they were presented in a multiplechoice format. These findings suggest that the items within each construct are sufficiently reliable for further analysis. Moreover, reliability for two constructs could be further improved by excluding subitems with negatively worded formulations that required recoding during the analysis, resulting in adjusted Cronbach’s alphas of .74, .74, .91, .88, and .86. Additionally, the 10 items assessing computational thinking yielded a Cronbach’s alpha of .83, indicating strong internal consistency.

Next, we analyzed children's performance on the 10 items assessing computational thinking. On average, children scored 74%, with a maximum score of 90.5% and a minimum of 33.3%. Two items emerged as particularly challenging. In one item, correctly solved by 64% of students, participants were asked to determine the largest number achievable by performing exactly three "switches," where a "switch" involves swapping the positions of two adjacent digits. Fifteen students struggled with this item, often treating it as a purely mathematical problem and simply rearranging the digits to form the largest number, disregarding the specified switching rules. This task aimed particularly to foster computational thinking skills such as algorithmic thinking, as students devise a step-by-step strategy to maximize the number’s value; logical reasoning, as they prioritize impactful swaps based on positional values; and optimization, as they explore possibilities to identify the most effective solution. The second item exemplifies how computational thinking can be assessed through the concept of "loops" in algorithms, but it proved to be particularly challenging, with only 33.3% of students solving it correctly. Participants were required to determine the outcome of an algorithm where an ice cream vendor repeatedly follows a specific sequence of steps: starting with an empty cone, adding the requested flavor and then repeating a predefined loop of adding chocolate and vanilla scoops. Nineteen students failed to apply the required number of iterations in the loop, highlighting a gap in understanding this fundamental concept in computer science. This task develops computational thinking skills by emphasizing the importance of understanding and applying loops systematically. It fosters algorithmic thinking as students analyze the sequence of operations, logical reasoning to predict outcomes, and abstraction to generalize the steps.

DISCUSSION

Overall, the designed instrument was found to be reliable. The analysis revealed that children generally demonstrate an adequate level of problem-solving skills, excelling in some questions more than others. To further enhance the instrument, the results suggest removing two subitems from the psychological construct scales to improve internal reliability. Moreover, the results indicate that the concepts of "switch" and "loop" present significant challenges for students. For instance, in mathematics, reordering the digits 1368 to achieve the maximum value of 8631 requires just two switches. However, when restricted to switching adjacent digits, six switches are needed. Similarly, many students struggled with the "loop" task, failing to apply the required number of iterations. This suggests a gap in their understanding of these foundational computer science concepts and emphasizes the need to strengthen both the practical applications and conceptual comprehension of "switches" and "loops" concepts in educational programs.

The assumption that the SFZ program "Robot relay race" is an enrichment program is clearly supported by the results of the pilot study. The average school grades of the participants for the subjects mathematics and computer science are 1,941 each. In Germany, the grading scale ranges from 1 to 6, with 1 being the top grade. The average grade is around 2.3, depending on the age and the region (KMK, 2024). While school grades are not a sufficient condition for determining giftedness (Heller, 2001), they provide a useful baseline for distinguishing between high-achieving and average-performing students. In this respect, the content-related look at problem-solving skills in connection with algorithmic thinking is worthwhile. Our sample in this pilot study was small, but we hypothesize that when considering a large sample, high-achieving students will show more confidence in their problem-solving abilities, demonstrating greater self-efficacy and interest in the subject matter. In contrast, average-performing students may exhibit a more variable self-concept and attribution patterns, particularly in tasks that challenge their existing skill levels.

From this point of view the results of the study underline the importance of using noncurricular bounded topics to identify strengths in problem-solving skills among students. This finding matches former research on this topic in a Scandinavian context (Mellroth, 2014). The programs, such as PHGR "iCamps" and SFZ Jena's "Robot relay race" are particularly valuable in providing insights into how these activities influence children's psychological constructs and computational thinking skills over time. Furthermore, this project plays a crucial role in addressing a significant gap in comparative STEM education research by exploring how enrichment programs impact students across diverse educational settings.

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INCLUDING CULTURAL AND CREATIVE PRACTICES IN MATHEMATICS

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In an increasingly diverse world, teachers could and should build connections between cultural practices to mathematical creativity. In this paper and presentation, we used established research on mathematical creativity and identification of mathematical practices from around the world to illustrate the seamless connection found in multicultural mathematical activity. For the paper, we identify a few examples of these connections. We use the paper and the subsequent presentation to invite discussion and collaboration on extending the examples to other cultures from around the world.

Keywords: Mathematical practices, Teacher learning, Mathematics and culture

As we move into the second quarter of the 21st century, with the increasing diversity of student populations across the globe, it is crucial for teachers to recognize the unique needs of the different students and cultures present in their classes and explicitly address heritage cultures (Turner et al., 2012). Students learn best when they take ownership of their own learning, so classrooms where teachers and teacher educators who identify, locate, and/or create activities in subject areas that connect to students' native cultures will improve student involvement in the learning process, which then improves student outcomes in the given content area as well. (Fredricks et al., 2004; Khatter et al., 2024; Pintrich, 2002).

Many people believe that mathematics is the "universal language" because numbers are the same in any language. However, as Zalman Usiskin points out, mathematics nor language is culture-free (Usiskin, 2024), thus requiring teachers to reject the Platonist belief that mathematics is decontextualized and purely objective (Aikenhead, 2018). This belief may even be taught to prospective teachers during their schooling, which has historically been perpetrated through the nineteenth century when public education was established in many countries. To move forward, mathematics teacher educators, teachers, and prospective teachers need to reinvent mathematics lessons into a dynamic and flexible, twenty-first century curriculum that can be individualized based on the needs of individual learners (Organisation for Economic Cooperation and Development, 2021). Such a curriculum would reposition schools as being a collaborative partner with communities and the environment that students and families live in. Such environmental stewardship generally is directly in harmony with many heritage cultures that students belong to (Kimmerer, 2013).

Drawing from a framework developed by Ogawa (1995), Sterenberg (Sterenberg & O'Connor, 2018) proposes a way of approaching mathematics that can be "defined and understood in many ways" (p. 181). Rather than just focusing on Platonist/Western views of mathematics, Indigenous/collective worldviews and personal observations/explanations that students bring to the classroom should also be integrated into the classroom. Lunney Borden (2018) agrees and includes themes that should be part of the conversation: 1) learning from heritage language(s) in the present in the local community; 2) teachers value all three ways of knowing and represent them in the classroom; and 3) helping students connect school mathematics with heritage mathematics. The key to this model is students making a meaningful personal mathematical connection.

CONNECTIONS TO ESTABLISHED RESEARCH

To identify, define, and implement effective tasks for classes containing students from heritage cultures, teachers should possess a level of creativity, spontaneity, and curiosity. These three characteristics help us to think about the intersection of mathematics and creativity.

Sawyer (2011) defines creativity through a sociocultural perspective: “generation of a product that is judged to be novel and also to be appropriate, useful, or valuable by a suitably knowledgeable social group” (p. 8). We have selected this definition because it of the sociocultural nature of the definition and it is domain-specific so can be applied to mathematics. Within mathematics, we often look for originality, fluency, flexibility, and elaboration of thinking, whether that of the student or of the teacher (Chamberlin et al., 2022; Leikin, 2009; Leikin & Lev, 2007; Lev-Zamir & Leikin, 2011; Mann et al., 2017). These ideas do not necessarily be new to the field (see the Four C theory; Kaufman & Beghetto, 2009), but if it is unique to the student, it is considered original and creative. An original mathematics teacher may have students engage students in discovery and exploration without a pre-conceived outcome and then communicate their mathematical discoveries, rather than just teaching from a textbook-supplied lesson. Fluency in mathematics is recognizing that there are multiple approaches to solve a problem-so a teacher who is strong in this area would be supportive of students generating multiple approaches for the same algorithm (Fox & Payne, 2025). Mathematical flexibility is having the ability to change directions while solving problems or having a variety of ideas on how to approach tasks. Thus, a flexible teacher is much more able to respond to changes in planning and address student needs while supporting the class. Lastly, elaboration is expanding on ideas and extending student contributions to tasks and activities. A teacher using elaboration in their classroom is likely encouraging students to extend their thinking to higher levels or dimensions.

EXAMPLES IN THE AMERICAS

While most of today’s world utilizes the Hindu-Arabic numerals (digits 0-9, in a base-10 system), the Iñupiaq and Yup’ik people of Alaska and northern Canada use a base-20, sub-base 5 numeral system that is directly tied to their cultural identity (Kisker et al., 2012; Tillinghast-Raby, 2023). Many native/First Nation languages are lost before they become transcribed, but the Iñupiaq people were fortunate: a teacher at a middle school in Kaktovik, Alaska, and his students took the oral numerals and created a written form, which are now known as the Kaktovik numbers (Chrisomalis, 2013). Invented in 1995, these cumulative-positional numbers use a nested place value system and allow for a fast, visual form of arithmetic. Students are able to quickly and intuitively compute any number of arithmetic calculations, including long division problems (Kisker et al., 2012; Tillinghast-Raby, 2023). Schools which began teaching with the Kaktovik numerals in mathematics classes saw a statistically significant increase in student performance on standardized testing for American Indian/Alaska Native students (Kisker et al., 2012; Lipka & Adams, 2004) and students working on projects involving Kaktovik numerals became excited and engaged – they were able to connect with their parents and community over their schooling and culture (Naiden, 2023). By integrating students’ heritage number system into classroom mathematics and projects, Iñupiaq and Yup’ik student performance statistically increased due to the use of mathematical creativity, which has the potential to benefit their whole community.

Four decades ago, Carraher et al. (1985) showed that schoolchildren in northeastern Brazil possessed ways of answering arithmetic questions within the context of commerce—selling fruit in the markets of their city—with more correct responses than answering the same questions in a formal classroom setting on a pencil-and-paper test. One takeaway from their study is the creative ways these children—even with limited formal education—could answer questions like 500-(2×40) correctly using clever and mathematically valid strategies but conflated many of the classroom algorithms needed to answer the same question on a pencil-and-paper test, such as treating the traditional addition algorithm like the traditional multiplication algorithm. By building on students' strengths—such as computing sums and products quickly—we as teachers and educators can build upon the mathematical creativity students already possess.

EXAMPLES IN OCEANIA

Meaney and colleagues' (2013) description of a school where teachers and other adults in the building respected the native culture and traditions of Aotearoa (New Zealand) makes references to having students being "academically stimulated and their cultural heritage…respected" (p. 236). What should be noted is that such aspirations are not unique to schools in this region of the world and should not be treated as some onerous addition to the school curriculum. Western researchers have been making claims to engage students through meaningful connects to students' backgrounds, cultures, and interests for years. In their description of a Knowledge of Content and Students, Ball et al. (2008) mention "teachers need to predict what students will find interesting and motivating" (p. 401). An approach that acknowledges the Maori tradition of many students in Aotearoa is, as Meaney and colleagues (2013) describe, "an integral part of both how and what mathematical understanding are taught" (p. 240). A recent example can be seen in the work of Hunter and Hunter (2023). Their use of connecting New Zealand's mathematics standards and curriculum to Pacific Islanders' constructions and cultures got students interested in the (formal) mathematics while maintaining necessary connections to their native heritage. By connecting student culture with mathematical creativity and formal learning, student performance improves (Gay, 2010).

In another historical example, Saxe's (1982) study of the Oksapmin peoples of Papua New Guinea illustrated how individuals use position of their bodies to communicate numerical values. His study pointed out a concern development: as Western mathematical and economic conventions became normal among the Oksapmin people, their ability to engage in their original mathematics declined. As Saxe observed, though, "the Oksapmins' use of these procedures constitutes novel cognitive adaptations" (p. 592). In another study where Saxe connected Oksapmin counting practices to Western mathematical content (Saxe & Moylan, 1982), Saxe and his colleague noted that the Oksapmins' "deeper level of measurement operations…may serve as the basis for the construction of new measurement concepts and procedures in the individual and in the social history of the cultural group" (p. 1248). Their observation rings true years later: incorporated established cultural practices can be beneficial for the mathematical development of the student, especially when valuing the contributions of previously underrepresented cultures. By re-engaging Oksapmin and other non-Western peoples with their cultural mathematical knowledge and supporting creative use of this wisdom while teaching mathematics empowers learners (Leonard et al., 2010).

CONCLUSION

Incorporating cultural practices into teacher preparation and professional development benefits both teachers and students. Where researchers have focused on the mathematical development of students using Kaktovik numbers, we have introduced the same numbers to prospective teachers (Fox & Payne, 2024) and numbers and operations to practicing teachers (Fox, 2024). Such introductions allow us, as teacher educators, to introduce creativity and originality in mathematical content, numbers, our prospective and practicing teachers are surprised to see creativity and originality incorporated in. Our additional research into incorporating cultures and practices beyond our own dominant cultures through interactions with other researchers and educators serve at least two additional purposes. When wanting to illustrate interdisciplinary connections with mathematics, using a variety of world cultures is a fantastic starting point that acknowledges and values what many students already know from their home cultures. From a prospective teacher development and professional learning prospective, the integration of cultural practices allows for the development of the flexibility and ingenuity that comprise mathematical creativity.

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MATHEMATICAL CREATIVITY BEYOND PROBLEM-SOLVING

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Researchers and educators interested in mathematical creativity and giftedness have focused predominantly on problem-solving and problem-posing. In this paper, we advocated for other forms of mathematical activity related to the creation of mathematical theory. Here, we revise some of the most common definitions and approaches to mathematical creativity in the literature, concluding that, while there is a consensus on the idea of engaging students in mathematical work that mimics the work of professionals, the main focus has been on solving and posing problems. Drawing from the fallibilist perspective on mathematics, we suggest that students could engage in contrasting different definitions for the same mathematical objects or concepts, exploring the implications of shifting meanings. We provide three examples that could be used for such an approach in the classroom.

Keywords: Theory development, Fallibility, Elementary classroom, Secondary classroom

Problem-solving has been a central focus for research on mathematical creativity and giftedness in education, leaving aside other aspects of mathematical thinking related to the creative work in mathematics, such as the development of mathematical theory. Mathematical objects are usually introduced to students through already-made, precise definitions, without room for critique and analysis of why the objects have been defined in such ways. Results, such as theorems, follow from given axioms and definitions that look artificial and complicated. Lakatos (1976) referred to this as the deductivist approach, claiming that: "Deductivist style hides the struggle, hides the adventure" (p.142). While this approach is understandable in terms of having a coherent curriculum with consistent definitions, it also deprives students of an opportunity to engage in the creative work of developing mathematical theory. In this manuscript, we advocate for the promotion of mathematical creativity beyond problem-solving by looking at other forms of mathematical thinking related to the definition of concepts and ideas in a similar way to how this happens in the field of mathematics. We start by revisiting definitions and approaches to creativity in mathematics education. Then, we discuss some examples that could be introduced in the elementary classroom.

MATHEMATICAL CREATIVITY

A brief review of common references to the definitions of mathematical creativity reveals the predominant focus on problem-solving. Such definitions distinguish the creative work of experts and prominent figures in the field from the mathematically creative acts that can happen in a school classroom.

Baer (1997) conceived of creativity as a continuum; that is, it is "not something that a person either has in abundance or lacks entirely." He claims that "we [are] all creative, to one degree or another, in many of the things we do" (p. 2). According to him, one's creativity varies from task to task and from moment to moment. Very close to Baer's conception of creativity is Kaufman and Beghetto's (2009) four-c model of creativity, comprising: (1) interpretative or mini-c creativity, (2) every-day or little-c creativity, (3) expert or pro-c creativity, and (4) legendary or big-c creativity. They refer to everyday creativity as the creative actions of the non-expert. Their “Pro-c” category is for the average professional. According to Beghetto and Kaufman (2011), little-c and mini-c are more appropriate in the classroom context. For example, “a youngster’s little-c idea for a science experiment can still be considered creative in the context of a particular [elementary classroom] even though it may not be considered creative in other contexts” (p. 98). The idea of little-c creativity is not unique to mathematics and has been applied to other subjects. According to Craft (2002), little-c creativity is a democratic notion; it can be manifested by anyone, and it refers to “an ability to route-find, successfully charting new courses through everyday challenges” (p. 56).

Silver (1997) suggested the use of inquiry-oriented mathematics instruction, which includes problem-solving and problem-posing tasks to foster mathematical creativity. According to him, the basic components of creativity—namely, fluency, flexibility, and novelty—are the same characteristics of creative activities evident in problem-posing and problem-solving. Silver further suggested the use of problem-posing and problem-solving activities to stimulate inquiry-based mathematics instruction, in which “the responsibility for problem formulation and solution is shared between teacher and students” (p. 77). Such problems may include, but are not limited to, ill-structured, open-ended problems that are stated in a manner that permits the generation of multiple specific goals and possibly multiple correct solutions, depending upon one’s interpretation.

Some scholars have related creativity not only to the work of mathematicians, but also at the center of mathematics education. According to Mann (2006), mathematical creativity is not just about solving problems but also about how the students deal with the problems and what they gain from trying to solve mathematical problems. Stimulating students to do what mathematicians do has the potential, according to Mann (2006), to develop their mathematical creativity in the classroom. This requires “creating authentic learning situations where students are thinking, feeling, and doing what practicing professionals do” (p. 241). In such situations, which may include problem-solving and problem-posing, “knowing of” is not enough. In addition, students need “knowing about”; i.e., “a way of knowing that is based on a continuity of experiences” (p. 242).

“Divergent thinking” and “thinking outside the box” are two of the most popular expressions used in the literature to describe creativity. In the field of mathematics education, one can easily recognize that the research on mathematical creativity frames creativity in these terms (e.g., Hashimoto, 1997; Leung, 1997; Pehkonen, 1997; Silver, 1997). It is possible that because mathematical creativity is associated with problem-solving abilities such as fluency, flexibility, elaboration, and originality, it is seen as a synonym for divergent thinking.

Sriraman, in conversation with Liljedahl, suggested defining mathematical creativity at the school level as “(1) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (2) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle” (Liljedahl & Sriraman, 2006, p. 19).

Most recently, Bicer (2021) developed a framework to identify types of mathematical tasks that promote mathematical creativity in middle school. The framework comprises six categories: open-ended problems, problem-posing, connections, visualization, extensions, and communication.

We can see that, while some of the references to creativity mimic the work of mathematicians, the definitions and approaches commonly used are strongly focused on problem-solving and problem-posing. In this paper, we are interested in the creative acts of developing mathematical theories at school levels.

CREATING MATHEMATICS

This section includes three examples that could be used to invite students to develop mathematical theory. This can happen through exploring competing definitions, or meanings, of mathematical objects commonly taken for granted. This approach, similar to approaches to mathematical creativity discussed in the previous section, can be considered as divergent thinking and expanding possibilities (see Aljarrah, 2020; Aljarrah & Towers, 2022). However, the focus is not on problem-solving or problem-posing. The goal of this approach is similar to the last level in van Hiele's (1986) model for geometric thinking: "Students can make comparisons of various deductive systems and explore different geometries based upon various systems of postulates" (Pegg, 2014, p. 613). This level is not commonly included at the school level. However, analyzing the implications of different definitions for the same mathematical objects, such as "numbers," is similar to exploring systems of postulates. Our first example comes from a Grade 3 class discussion on even numbers.

Is zero an even number?

In the monograph number 14 of the Journal for Research in Mathematics Education (Schoenfeld, 2008), a group of researchers studied the same Grade 3 lesson from several perspectives. The monograph includes a video in which one of the researchers, Deborah Ball, orchestrates a students' discussion regarding even numbers. The lesson was recorded in a video, which corresponds to the data for all of the papers in the monograph. The discussion involved different definitions for even numbers and, implicitly, different definitions of number. We stress two arguments about whether zero is an even number to contrast different definitions.

1. Even numbers are made of two equal things, but zero is made out of nothing. So, zero can't be an even number.
2. Zero is in between -1 and 1, which are odd numbers [the students pointed to a number line in the classroom]. So, zero is an even number.

The first argument not only involves a definition of even numbers but also involves, implicitly, a definition of numbers as a quantity, in which zero is nothing, and therefore, it can't be made up of two equal things. In the second argument, even numbers are characterized by an alternating pattern in the number line. Here, the meaning of zero is different as it refers to a position rather than the idea of emptiness (nothing). The contrast between these two definitions of numbers suggests that to make sense of zero as an even number, it may be necessary to shift from the meaning of numbers as quantity to the meaning of numbers as position.

In the lesson, students also discussed whether other numbers, such as six and ten, were even numbers. The teacher allowed students to defend their positions with arguments and discussions. The point here was not to solve a problem but to engage students in a mathematical discussion similar to what experts in the field do. Consistent with Craft's (2002) comment on creativity and democracy, Posner (2008) posited that such a lesson also addresses issues of equity in mathematics education. From the perspective of mathematical creativity, we can consider such conversation as a path toward defining mathematical concepts (even numbers in this case).

The shift of the meaning of numbers involved in this example has profound implications. The next example involves a similar shift in the context of the history of negative numbers.

Can you have something less than nothing?

The history of negative and complex numbers is an example of creative work in mathematics (see for instance Martinez, 2006). In fact, Mazur (2004) elaborated on the work of imagining something new, something that does not yet exist, making analogies between poetry and mathematics. Negative numbers are included in the curriculum at the elementary level in many countries. However, some mathematicians rejected the idea of negative numbers before they were widely accepted. For instance, William Frend (1796), in his *Principles of Algebra* book, claimed that:

> [A number] submits to be taken away from another number greater than itself but to attempt to take it away from a number less than itself is ridiculous. Yet this is attempted by algebraists, who talk of a number less than nothing, of multiplying a negative number into a negative number and thus producing a positive number … (p. x)

We could claim that the argument in this quotation is reasonable when “zero” is considered as “nothing,” and therefore a negative number should be something less than nothing, which does not make sense. However, if we conceive a number as a position in the number line, then zero is a reference point; positive numbers are on one side, usually to the right of zero; and negative numbers are on the other side. We could say that to make sense of negative numbers, we need to shift our definition of number from quantity to position. Something similar happens for complex numbers, in which case a number is still a position, but the idea is extended from the line to a plane. Mazur (2004) explained the role of the geometric representation in extending the conceptualization of numbers as positions. Numbers can be considered as points and arithmetic operations as geometric transformations. In this conceptualization, addition and subtraction correspond to translations and multiplication and division correspond to a mix of scaling (stretching or compressing) and rotating.

The shifted definition of number, however, is not arbitrary. Extending a definition requires that the new objects (or operations) behave in certain ways. For instance, when introducing negative numbers, we want well-defined arithmetic operations (Martinez, 2006) that satisfy the properties of the original definitions, such as commutativity, associativity and distributivity. Martinez explored two different ways of introducing negative numbers. One is assuming they exist and use the *desirable* properties to explore how these new numbers would behave. For instance, assuming the arithmetic properties, it is possible to conclude that the product of a positive number and a negative number should be negative. The other option is finding a representation, as proposed by Mazur (2004), in which the new operations make sense and satisfy the desirable properties. For instance, numbers can be considered as position in the number line and multiplication by a negative number involves a 180-degree rotation. We believe that school students could engage in discussions defining arithmetic operations with negative numbers and deducing their properties rather than learning such properties as rules.

The two previous examples involve a shift of the meaning (definition) of number from quantity to position. The third example deals with the definition of quadrilaterals.

Quadrilaterals

The mid-point theorem of quadrilaterals states that the middle points of the sides of a quadrilateral form a parallelogram. The definition of quadrilateral often refers to a planar shape. However, this theorem is still valid for geometric figures that would not count as quadrilaterals in common definitions, such as four-sided shapes made out of four non-planar points in the space, shapes with self-intersecting sides, or with three collinear vertices (which would look like a triangle with an extra vertex in one side).

For the sake of this theorem, we are not interested in polygons (quadrilaterals in this case) as surfaces or regions in the plane. Rather, we only need a (circular) sequence of four points, vertices, and segments joining consecutive points (the first and the last points in the circular sequence are considered consecutive). A definition of quadrilateral in this sense can be considered as contextualized in the context of the theorem rather than a general definition that applies to all cases.

A discussion on this theorem could involve analyzing multiple definitions of quadrilaterals rather than endorsing a specific one. This example could be considered as an extension of the theorem, and so it might be argued that this is, in fact, an example of problem-solving. There is, however, an important aspect here related to creating new mathematical objects, and even calling them quadrilaterals, shifting the common definition found in textbooks.

CONCLUDING REMARKS

Herein we argue in favour of focusing on other forms of mathematical thinking and practice beyond the predominant focus on problem-solving, including problem-posing, in the research on mathematical creativity and giftedness. Such an approach to mathematical creativity is consistent with the goal of relating mathematical creativity with mathematicians' practice but in a different way. For instance, Van Haile (1986) levels of geometric thinking assume that the work of analyzing axioms is at the upper level of thinking, something beyond regular school instruction. We believe that students can engage in mathematical creativity in terms of defining objects and analyzing the implications of such definitions, which could include definitions and axioms. Such an approach has been implemented in some sessions with groups of our preservice teachers, such as the discussions about even numbers, the history of negative and complex numbers, and the mid-point theorem of quadrilaterals. During these sessions, we noticed that student teachers' actions and doings were consistent with Aljarrah's (2020) descriptions of creativity as acts of expanding possibilities, divergent thinking, and assembling things in new ways. Such acts allowed the new and crucial to emerge and evolve. The activity "Is Zero an Even Number?" exemplifies how the definition, or meaning, of "number" is not fixed, even in the curriculum, although this is not often explicit in programs of studies. The "Can you Have Something Less Than Nothing?" activity also entails a shit of the concept, or definition, of "number." This second case also exemplifies the historical debate that gave rise to mathematical theories. In contrast to these cases, the "Quadrilaterals" activity involves definitions in geometry. In this last example, the definition of quadrilateral is extended by preserving some characteristics of polynomial, in terms of sequences of points and segments, and dismissing other characteristics such as surface area, which is not relevant for the mid-point theorem. We believe that our proposed approach is a fertile terrain for both individual and collective creativity in the classroom.

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FACTORS INFLUENCING THE DEVELOPMENT OF MATHEMATICAL GIFTEDNESS – WHAT DO PROSPECTIVE TEACHERS FOCUS ON AND WHAT SHOULD THEY PAY MORE ATTENTION TO?

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The literature indicates numerous aspects that can have a beneficial or inhibiting effect on the development of giftedness. In the pursuit of increasingly comprehensive individual diagnostics and support, it is important to identify such influencing factors. Based on complex case studies and holistic considerations, we will illuminate which aspects prospective teachers consider relevant for describing mathematical giftedness.

Keywords: Giftedness development, catalyst, influencing factors, participation, prospective teachers.

INTRODUCTION AND RATIONALE

With the aims of both identifying as many individuals as possible, and offering each individual appropriate support against the background of, for example, their co-cognitive or cognitive facilities and consecutive needs, a central effort of giftedness research is dedicated to modeling the phenomenon of giftedness to characterize a picture that is as differentiated and as realistic as possible. This applies, for instance, to processes of individual potential development, possible domain-specific cognitive profiles, or aspects that determine or influence the potentials' development, including intersectional considerations (e.g., Benölken & Veber, 2021). Such approaches exist both without direct references to specific domains (e.g., Perleth & Heller, 2007) as well as with such references (e.g., Fuchs & Käpnick, 2009). In attempts of modeling mathematical giftedness, the genesis of giftedness is depicted as a dynamic phenomenon by clearly separating individual potential from visible performance, which may differ considerably. Furthermore, both mathematics-related giftedness criteria (in the tradition of Kruteskii, 1976; see also Sheffield, 2003) and personality traits that support the development of giftedness (e.g., Käpnick, 1998) are considered. In addition, both intrapersonal and interpersonal aspects are identified as influencing factors that affect the development of giftedness determined by different facets of individual potential. In congruence with the outlined efforts for more differentiated modeling of the construct of giftedness, approaches have emerged that aim to strengthen individual approaches to diagnostics and support in order to imply a view of the construct of giftedness that is as comprehensive and participatory as possible from a potential-oriented perspective (e.g., within the "Leistung macht Schule" initiative in the German context; Benölken et al., 2019). Accordingly, giftedness is not interpreted as a phenomenon that only affects a comparatively small group of high-achieving or particularly gifted students, but as a phenomenon that might affect diverse students with a high potential who have not yet been identified or recognized (e.g., Renzulli & Gelbar, 2020). This perspective implies the need to adopt a sensitive and multifaceted view to both individual dispositions and developmental processes. Considering current postulates on the modeling of giftedness, combined with a comprehensive participatory perspective, the question arises as to which aspects can be further explored to improve individual diagnostics and support in the context of the complex construct of giftedness, particularly concerning aspects that determine or influence the genesis of individual giftedness potentials. For future support of students with high potential, the competencies and perspectives of prospective teachers are important. Accordingly, this paper focuses on exploring potential answers by clarifying dispositions and factors influencing the development of individual potentials, leading to the attribution of mathematical giftedness to students – and interpreting aspects that prospective teachers may not yet consider sufficiently comprehensive. A meta-analysis of 21 case studies, created by perspective teachers about children identified as mathematically gifted, serves as the basis for this exploration. First, we give a brief overview of the theoretical frameworks with respect to possible aspects that impact the development of individual potentials. After that, the study's methodology is outlined. Finally, the results are presented and discussed, especially with regard to the implications for teacher education programs to ensure comprehensive and multifaceted potential-oriented gifted support.

THEORETICAL BACKGROUNDS

As summarized by Benölken (2015), the scientific consensus views mathematical giftedness as a complex construct, interpreted as an above-average potential as to certain criteria (e.g., abilities in remembering mathematical facts, structuring mathematical facts, transferring structures, as well as sensitivity and fantasy) in interdependence with personality traits (e.g., high mental activity, willingness of efforts, ability to concentrate, or skills of cooperation). The development of individual potential is assumed to be based on individual prenatal, natal and postnatal dispositions and influenced by interpersonal and interpersonal factors (e.g., Aßmus, 2017; Fritzlar, 2015; Fuchs & Käpnick, 2009; Meyer, 2015; Nolte, 2018; Sjuts, 2017). According to Auhagen (2023), catalyst impacts are understood to be the impacts of inter- or intrapersonal factors that trigger a reaction to the development of a child's potential, personality or identity. However, these factors do not always have a beneficial effect; they can also inhibit development, including its foundational processes. Based on narrative literature reviews, Table 1 provides an overview of aspects related to relevant determinants influencing individual potentials (an attempt to synthesize the contributions of Brandl, 2011; Fuchs & Käpnick, 2009; Meyer, 2015; Nolte & Pamperien, 2014; Perleth & Heller, 2007; Renzulli et al., 2006; Sjuts, 2017; Ulm & Zehnder, 2020).

While these aspects are individually characterized, adopting a holistic approach to the construct of giftedness suggests the presence of intricate interdependencies between them. The summary of Table 1 can be understood as a framework of categories for clarifying factors that influence the development of individual potentials and for interpreting aspects that may not yet have been comprehensively considered. The category abbreviations defined in the square brackets are referenced in Table 2.

prenatal, natal and postnatal dispositions [disp]	intrapersonal factors [intra]	interpersonal factors [inter]
• creative predispositions [crea] • Physical constitution (e.g., brain structure, perceptual abilities, sensorimotor predispositions) [phys] • Spatial perception [sp] • Socio-affective predispositions, fundamental social abilities [soc] • Character traits [cha] • Linguistic potentials [ling] • Intellectual/general cognitive potentials [cogn] • Initial development of math-related skills, e.g. Number sense, Sense of structure [math]	• Psychological characteristics, e.g. motivation, willpower [psych cha] • Individual personality, identity, including mental and personality-defining skills (e.g., resilience, temperament, optimism, courage, empathy, curiosity) [pers] • Language [lang] • Physical skills and traits, including appearance, handicap, health [phys sk] • Psychological skills, e.g. self-management, volition, awareness [psych sk] • Social skills [soc] • Participation [part] • Cognitive skills [cog] • Experiences [exp] • Perceptions [per] • Passion for Domains or subjects [pass]	• Milieu [mil] • Cultural contexts [cul] • Important people (e.g., peers, teachers, family) [peop] • Institutional conditions (e.g., school system) [inst] • Interventions, support (e.g., in Kindergarten or school) [supp] • Physical environment [phys] • Teaching quality at school [teach qual] • Events [ev] • Coincidences [coin] • Media [med]

Table 1: Aspects influencing the development of individual potentials.

THE STUDY

The exploratory study reported in this paper is based on a meta-analysis of 21 case studies on children who are considered to have high mathematical potential. The methodologies of the case studies follow comparable approaches to identifying mathematical giftedness, often conducted in support projects in Germany (e.g., Bauersfeld & Kießwetter, 2006). Primarily, the diagnostics applied in the case studies were organized as a long-term process, conflating different tools into a synthesis to gain a differentiated picture of a child's potential and influencing aspects (e.g., by combing teacher nominations, "indicator tasks"-tests, observations on task solving using rating-sheets, interpretations of video documentations' transcripts or guided interviews; in single cases amending IQ-tests were conducted). In this way, interpretative methods were integrated alongside standardized methods. As summarized by Tiedke et al. (2022), the case study approach can be interpreted as an attempt to draw a multi-layered view of a topic intending to mirror realistic pictures of the social world applying different tools such as those mentioned above. The case studies included in the meta-analysis were produced as one bachelor's thesis and 20 master's theses by prospective teachers as part of the support programs "Think!" and "MiKaDu" at the University of Wuppertal (Beumann & Weber, 2022) and "Mathe für kleine Asse" at the Universität of Münster (Benölken, 2015). The case studies considered in the meta-review aimed at reconstructing both a child's profile of mathematical giftedness (regarding characteristics of criteria of mathematical giftedness as well as personality traits) and the factors with their catalyst impacts on the development of the student's individual mathematical potential. Beyond the students' individual giftedness profiles, the aim was to provide insights into the factors prospective teachers considered essential for influencing the development of children's mathematical potential. The analysis of the 21 case studies was conducted using Mayring's (2015) content-structuring content analysis method. Guided by the framework of categories for clarifying factors that influence the development of individual potentials and interpreting aspects (Tab. 1), the existing data material was coded deductively towards three main categories and with a total of 29 subcategories. The coding of the transcripts and the results of the case studies was carried out by the first author of this paper.

RESULTS

Table 2 presents an overview of the categories of influencing factors coded across the case studies, specifying in which and how many they appear. It is important to note that individual categories may occur multiple times within a single case study. Additionally, the table does not distinguish between supportive or inhibitive factors.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	Freq.	%
Disp crea	•	•	•	•		•		•		•		•		•	•		•		•		•	13	61,9
Disp phys	•	•	•	•		•	•	•	•	•	•	•		•	•	•	•	•	•	•	•	19	90,5
Disp sp		•		•		•		•	•	•		•	•		•		•		•	•	•	13	61,9
Disp soc	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	21	100,0
Disp cha	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	21	100,0
Disp ling	•	•	•	•	•	•	•	•		•	•	•		•	•	•	•	•	•	•	•	19	90,5
Disp cogn	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	21	100,0
Disp math	•		•	•	•	•	•	•		•	•			•	•	•	•	•	•	•	•	17	81,0
Intra psych char		•	•		•	•		•	•	•			•	•	•				•		•	12	57,1
Intra pers	•	•	•	•	•		•		•		•		•	•		•		•		•		13	61,9
Intra lang			•		•							•					•		•	•		6	28,6
Intra phys sk	•			•		•		•			•	•		•	•	•	•	•				11	52,4
Intra psych sk	•		•	•										•	•				•	•		7	33,3
Intra soc	•	•	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	20	95,2
Intra part	•		•		•		•				•			•	•			•		•		9	42,9
Intra cog			•	•				•		•				•			•	•	•	•	•	10	47,6
Intra exp																						0	0,0
Intra pers												•		•						•		3	14,3
Intra pass	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	21	100,0
Inter mil	•		•		•		•		•		•	•			•	•	•			•	•	12	57,1
Inter cul															•	•	•		•			4	19,0
Inter peop	•	•	•	•	•	•	•		•	•	•	•	•	•	•	•	•		•		•	18	85,7
Inter inst												•										1	4,8
Inter supp		•		•			•		•			•		•					•	•		8	38,1
Inter phys				•			•					•			•			•	•			6	28,6
Inter tech																						0	0,0
Inter ev	•											•	•									3	14,3
Inter coin		•				•											•					3	14,3
Inter med																						0	0,0

Table 2: Coded aspects of influencing factors per case study and their relative occurrence.

Regarding dispositions, it is notable that, with the exception of creativity and spatial perception, the prospective teachers focused on these traits in the analyzed case studies. In contrast, intrapersonal aspects—aside from social skills and passion for specific domains or subjects—were not comprehensively described. Furthermore, the potential influence of language, psychological skills, participation, and general cognitive abilities has not been sufficiently addressed by the prospective teachers. On the interpersonal level, only key individuals were identified as influencing factors; however, the prospective teachers gave little attention to other aspects, such as teaching quality, support, or cultural contexts in the analyzed case studies.

DISCUSSION

The paper examined the focus of prospective teachers on dispositions and factors influencing the development of individual potential in mathematical giftedness. Through a meta-analysis of 21 case studies, it provides insights into aspects that may not yet be adequately addressed in university teacher education programs: The focus of the analyzed case studies, conducted within the framework of support programs at two German universities, has primarily been on dispositions. While intrapersonal influencing factors are considered to some extent, certain non-mathematical intrapersonal aspects (e.g. psychological and personality-defining skills) and, in particular, interpersonal factors (e.g. teaching quality, contexts and conditions), have received little attention so far, despite their potentially supportive or inhibitive effects. It should be noted as a limitation that the process of analyzing results and transcripts and coding the deduced categories (based on a limited review of literature) inevitably involved some loss of information due to the necessary reductions. Nevertheless, the findings indicate the necessity for targeted enhancements in teacher training, enabling them to identify and support students in the classroom who demonstrate the potential for high achievement in mathematics. In light of the considerable number and significance of factors that influence the development of giftedness, the following questions, situated within the context of future teachers' professional competencies, serve as practical examples to facilitate comprehensive, multifaceted, and potential-oriented support for gifted students: What knowledge (e.g., about masking effect) and competencies are required of teachers in order that they may be able to understand, recognize and describe influencing factors as well as their supportive or inhibitive effect? Which perspectives (e.g., mathematics education, psychology, special education) are particularly significant for this task? In which learning situations and processes can influencing factors be observed, and how can they be selectively analyzed by teachers using appropriate intersectional tools and diagnostic approaches? How can teachers effectively leverage supportive factors or mitigate inhibitory influences to enhance the development of mathematical giftedness?

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PAPERS

THEORY IN PRACTICE

BRAIN EXERCISES AS TOOLS FOR FOSTERING MATHEMATICAL THINKING AND CREATIVITY

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This study explored the potential of brain exercises, like riddles and textual problems, on fostering creativity and critical thinking in mathematics education. Five classes of 10-11-year-old students participated in a one-week intervention using thinking-focused tasks delivered through active teaching methods, including play-based and experimental learning in individual, pair, and group settings. Achievement was assessed through knowledge tests and brief quizzes. The findings suggest that integrating brain exercises into daily mathematics lessons invigorates learning by challenging students to think creatively and independently. Carefully selected tasks not only enhance mathematical problem-solving skills but also stimulate intellectual reflection, fostering individual development and a deeper understanding.

Keywords: Brain exercises, riddles, textual tasks, student thinking, learning, creativity

INTRODUCTION

The adage "healthy body, healthy mind" underscores the importance of nurturing mental well-being. While physical activity is widely recognized as essential for development (Biddle et al., 2019; Hillman et al., 2008) what do we know about intellectual nourishment? How can we train the mind to encourage logical thinking and cultivate mathematical creativity?

Mathematical tasks are the "food" for the mind, particularly puzzles, logical problems, textual challenges, strategic tasks, and problems involving symmetry or tricks (Boaler, 2022; NCTM, 2000; Savić et al., 2023). These tasks develop thinking skills and maintain mental agility. Properly curated exercises can stimulate the thinking process, guide problem-solving strategies, and foster curiosity and enthusiasm for mathematics (Leikin & Pitta-Pantazi, 2013; Schoenfeld, 1992). Beyond this, they enhance understanding and the application of mathematical knowledge, creating lasting and meaningful learning experiences (Bransford et al., 2000; Savić et al., 2023).

Developing thinking skills such as logic, adaptability, criticality, and creativity is a cornerstone of mathematics education (CCSSI, n.y.; Leikin & Pitta-Pantazi, 2013). These skills support intellectual and personal growth, enabling students to approach challenges with confidence (Dweck, 2006; Savić et al., 2023). A key psychological factor in this development is students' reflective awareness of their cognitive processes and potential (Flavell, 1979; Leikin & Pitta-Pantazi, 2013; Schoenfeld, 1992).

Mathematical creativity, in particular, challenges conventional methods, promoting exploration and innovation (Savić et al., 2023; Sriraman, 2005). The concept of "productive failure" reframes mistakes as opportunities for learning and discovery (Gogovska, & Malcevski, 2008; Kapur, 2008). Thus, mathematics education should extend beyond computation and algorithms to higher-order thinking, equipping students to tackle complex, real-world problems (OECD, 2013).

This study investigates the role of brain exercises in enhancing reflective thinking, creativity, and problem-solving among young learners. By engaging students in active experimentation, collaboration, and critical thinking, the study aims to demonstrate the value of mental math in fostering mathematical thinking and structural knowledge. I will give examples of tasks in which we will encourage students not only to think and to be creative, but also to want new mathematical challenges and new tasks.

METHODOLOGY

The study was conducted with five fourth-grade classes, consisting of students aged 10-11, over a one-week intervention. The exercises were designed to incorporate various thinking styles, with a strong emphasis on creativity, logical reasoning, and adaptability. To achieve this, we carefully selected a range of tasks—including riddles, textual problems, and puzzles—that stimulated different cognitive approaches.

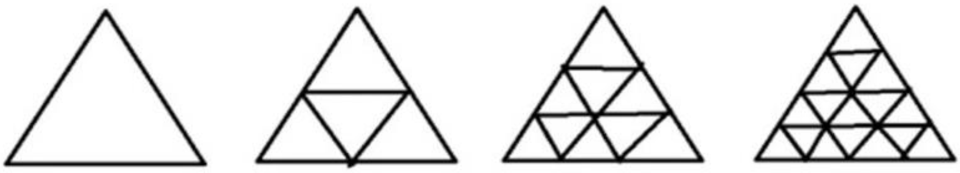
The intervention was delivered using active teaching methods, specifically play-based and experiential learning strategies, ensuring that students found the activities both engaging and accessible. Each day began with two carefully chosen tasks; The first task was completed individually, encouraging students to develop independent problem-solving skills and the second task was completed in pairs, fostering collaboration and peer learning. After completing the tasks, we facilitated a structured discussion where students analyzed their solutions, explored alternative approaches, and identified common mistakes. This reflective process reinforced learning and enhanced critical thinking skills. Students received a set of 10 tasks for homework but were required to complete only two of their choice. Any task previously solved in class was also included as an option, allowing for reinforcement and deeper engagement with the material. To measure student progress and engagement, we employed various assessment tools: Pre- and post-intervention knowledge tests to evaluate overall learning gains and two brief daily quizzes (lasting 5-6 minutes) to assess immediate comprehension and retention.

The results showed a significant improvement in students' cognitive skills and problem-solving abilities. However, the most surprising and impactful outcome was the students' enthusiasm and motivation. Many expressed a newfound interest in mathematics, requesting additional math lessons. As a result, we introduced two extra sessions dedicated solely to homework discussions. The students' excitement was evident—they actively sought more tasks and eagerly participated in extended intervention-based classes.

Examples of Tasks from the Lessons

The following problems are taken from tasks completed during lessons and are presented here as examples, based on a specially designed set of problems aimed at fostering mathematical talent and giftedness.

1. The image shows an equilateral triangle divided into 4, 9, 16... smaller equilateral triangles, arranged in 2, 3, 4 "levels." If this pattern continues, how many triangles will there be in the n-th level of the triangle? Which level of the triangle will have exactly 7 triangles? How did you arrive at the correct answer? Does a triangle exist with exactly 36 smaller triangles? Explain your answer.



This problem challenges students by: sparking curiosity to "see" the next element in the pattern and find a way to draw the structure; encouraging mathematical imagination; motivating a "creative spirit" according to one's own rules. The initial observation is that every second pattern contains an even number of triangles. Hence, the idea is to divide the sequence into two sub-patterns: one with an odd number of triangles and one with an even number of triangles. The second observation is that each subsequent pattern is larger than the previous one by an odd number of triangles. In other words, just as ordinal numbers alternate (odd, even), so do the numbers of triangles in the patterns. We came together the final step that: the number of triangles in a pattern is equal to the square of its sequence number; the number of triangles in the n-th pattern of the sequence is n^2; every square of a natural number can represent the number of triangles in some pattern of the sequence.

2. How much is 16% of 25? Instead of solving the previous problem, we can calculate 8% of 50 or 4% of 100. But is that the same as 25% of 16?

This problem challenges students by: Encouraging interest in manipulating factors without changing the product; stimulating mathematical thinking and "playing" with the properties of multiplication; motivating independent choices when simplifying calculations.

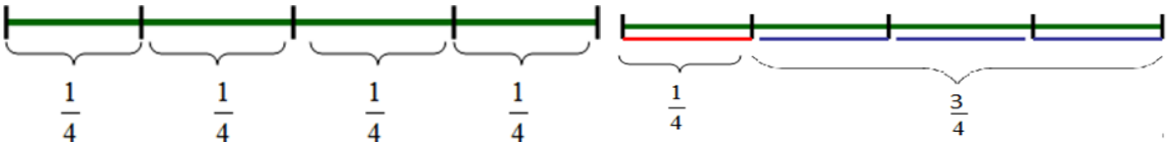
The initial observation is that percentages are fractions with the same denominator of 100, so the comparison of values of different percentages (parts) from different wholes can be made without using the denominator, that is, the comparison $\frac{16 \cdot 25}{100}$ with $\frac{8 \cdot 50}{100}$ is equivalent to the comparison $16 \cdot 25$ with $8 \cdot 50$. The next step in solving the problem is a challenge for manipulating the properties $6 \cdot 25 = 8 \cdot 2 \cdot 25 = 8 \cdot 50 = 4 \cdot 2 \cdot 50 = 4 \cdot 100 = 4 \cdot 5 \cdot 20 = 20 \cdot 20$

In general, equal products of two factors represent equal percentages. In this way, the parts: 25% of 16; 16% of 25; 8% of 50; 4% of 100; 20% of 20, and others, are equal to each other.

3. The number 48 needs to be divided into two parts so that one part is three times larger than the other.

This problem challenges students by: encouraging mathematical thinking and a "game" that follows two rules and satisfies two conditions; motivating students to make their own choices when selecting a method to represent and solve the problem; stimulating logical connections between mathematical facts and ideas to solve a problem under given conditions. The initial observation is that the division of the number must satisfy two conditions: the parts together make up the number 48, and the parts individually are in the ratio 1:3. The next step in solving the problem is choosing a way to represent the number. Here, the choice is to represent the number 48 as a single line segment:

Next, we think about how to divide the segment into equal parts that satisfy the given ratio: 1 segment for the smaller part of the number and 3 segments for the larger part of the number, making a total of 4 equal segments. Thus, each segment is 1/4 of the line segment.



From here, it follows that the smaller part is 1/4 of 48, and the larger part is 3/4 of 48, meaning the number 48 is divided into two equal parts, one of which is 12 (1/4 ⍰ 48 = 12), and the other part is 36 (3 ⍰ 12 = 36). In general, the task allows students to gain experience in different ways of representing and combining facts, conditions, and procedures in order to solve a given problem situation.

Five-minute Quiz 1

How many four-digit numbers have a sum of their digits equal to 4?

Five-minute Quiz 2

The area of a square is 36 cm^2. What is the area of a square that has a side twice as long?

Examples of Problems from the Assessments

1. A false "equality" is formed using matchsticks. In how many ways can exactly one matchstick be moved to create a correct equality?

2. The perimeter of a square is 88 cm. What is its area? (circle the correct answer.)

A) 400 cm2 B) 354 cm2 C) 484 cm2 D) 476 cm E) 476 cm2

3. The side lengths (dimensions) of a rectangle are natural numbers, and the area of the rectangle is 12 cm2. What is the perimeter of the rectangle?

The perimeter of the rectangle is ___________ cm.

4. Decipher the addition ТРИ + ТРИ = РЕМИ, where identical letters correspond to the same digits, and different letters correspond to different digits.

5. A rectangular bathroom with a length of 3 m and a width of 2 m needs to be tiled with square ceramic tiles, each with a side of 25 cm. How much money is needed to purchase the tiles, if one tile costs 100 dinars?

6. In a math test, there were 10 problems to solve. For each solved problem, a student earns 4 points, and for each unsolved problem, they lose 3 points. How many problems did the student solve correctly if they ended up with 19 points?

RESULTS

Analysis of assessment data revealed significant improvements in students' ability to reflect on their cognitive processes and apply lessons learned to new challenges. Students approached problems with increased creativity and flexibility, employing logical reasoning and adaptability even in unfamiliar scenarios. The findings emphasized the importance of supportive learning environments where mistakes are treated as opportunities for growth.

Students reported increased confidence in tackling complex tasks and greater enjoyment of mathematics lessons.

Results (Percentage of Correct Answers)		
Task number	First test	Second test
1	65%	80%
2	72%	85%
3	38%	55%
4	38%	65%
5	9%	28%
6	11.5%	33%

The second test results showed a consistent improvement across all tasks compared to the first test. The improvement suggests that students performed better due to increased familiarity, better preparation, or improved problem-solving strategies. However, with only 3.28% solving all tasks and 6.5% solving none, challenges remain. Notably, tasks with the lowest initial scores saw the greatest relative improvement, indicating students benefited most from additional practice on difficult problems. Assessment results highlighted notable progress. Knowledge tests showed a rise in problem-solving accuracy, while five-minute quizzes demonstrated improved performance, with scores increasing from initial attempts to later assessments. These findings suggest that the intervention had a positive impact on students' mathematical thinking and their overall approach to learning.

Discussion

The first task that encouraged students to think more deeply was the rebus puzzle CAR + CAR = KRAL. Students quickly discovered one solution through trial and error and eagerly raised their hands in excitement. However, none of them managed to find all four solutions (602 + 602 = 1204; 704 + 704 = 1408; 795 + 795 = 1590; and 897 + 897 = 1794) independently, without help or discussion. Nevertheless, after this task, the students approached the following problems with more attention, interest, and careful verification.

The findings underscore the transformative potential of integrating brain exercises into mathematics education. By fostering creativity and critical thinking, these activities enhance mathematical skills and contribute to overall intellectual development. Incorporating brain exercises as a regular component of mathematics curricula can sustain long-term engagement and skill development. Equipping educators with strategies to implement creative and reflective learning methods is essential for maximizing the impact of these interventions. Future research should investigate the long-term impacts of brain exercises on both academic and personal growth, providing further insights into their effectiveness.

CONCLUSION

Integrating brain exercises into mathematics education offers a promising approach to fostering creativity, critical thinking, and reflective awareness among students. These skills are vital for navigating modern complexities and achieving personal and academic success. By embracing innovative teaching methods and prioritizing intellectual growth, educators can inspire a new generation of learners to explore the limitless possibilities of mathematics.

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CHALLENGES OF TEACHING ALGEBRAIC IDENTITIES TO GIFTED CHILDREN

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Algebraic identities play an important role in the middle and high school classes. It doesn't stop with school curriculum but is used in many branches of science at the college level also. Teaching these identities is not difficult for a teacher if the students are average. But it becomes a challenge to teach this topic to gifted children. They go beyond these identities and expect the teacher to teach the applications and generalizations. The challenges faced by a teacher in teaching this topic are quite vast because algebraic identities play an important role in basic number theory also. If a positive integer 'n' is equal to the sum of two squares, will 2n also be? A gifted child relates this when $(a+b)^2$ *and* $(a-b)^2$ *expansions are taught. A group of 10 gifted children of age group 13 – 15 was taken. Using the identities* $a^2-b^2=(a-b)(a+b)$ *and the expansions of* $(a \pm b)^2$*, application problems were given and the interactions were recorded.*

Key word: Algebraic identities, gifted children, outcomes

INTRODUCTION

Teaching gifted children is a challenge for a teacher. Renata Michalak (P-164, 2022) says", Education work with a gifted student is not an easy challenge for any teacher, because it requires from them certain competences that particularly support the development of the students abilities in addition to the obvious competences". M. Katherine Gavin et al. (2009) says, "Mathematically talented students come to know and understand mathematics differently than other students". A group of 10 children were chosen from 30 children in the age group 13 – 15 by conducting a test containing problems which are not the usual text book problems. An example is, a, b, c are three natural numbers such that a < b < c and a + b + c = 6. What is the value of c? Such type of problems was not taught to them. The purpose of such questions in the test was to find out how logical the students are in talking new situations. These 10 children were not exposed to the algebraic identities before. So, this topic was chosen. Algebraic identities do not stop with mere results but play an important role in basic number theory problems. Singer, F., & Voica, C. (2017) say that "In the process of building a solution for a nonstandard problem, expertise and creativity support and mutually develop each other, enabling bridges to be unknown. This interaction leads also to an increase in expertise." Problem solving plays a major role in any mathematics program. Florence Michaela Singer et al. sights the reference of Sheffield on the teacher's approach, "Sheffield (1994) recommended that teachers of gifted and talented mathematics students should convey a sense of the beauty and wonder of mathematics in their enthusiasm for both mathematics and for teaching".

The reactions of gifted children on the identity $(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{a}+\boldsymbol{b})=\boldsymbol{a}^{2}-\boldsymbol{b}^{2}$

Three heading styles should suffice to structure your paper: MCG Heading 1 for the title, MCG Heading 2 for main sections, and MCG Heading 3 for subsections. Please do not number sections or sub-sections (as opposed to lists and footnotes).

The teacher taught this very basic algebraic identity to a set of 10 gifted students, (S_1, S_2 … S_{10}), of age group 13-15. It is further mentioned that each algebraic identity has a two-way implication. When $(a-b)$ and $(a+b)$ are multiplied then one gets a^2-b^2. When a^2-b^2 is factorized, the factors are $(a-b)$ and $(a+b)$. A very genuine question came from a student (S_1).

S_1	If $a^2-b^2=(a-b)(a+b)$, then $a-b$ also can be further factorized as $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$ and again $\sqrt{a}-\sqrt{b}$ also and so on. The factorization goes on and on, why to stop with the basic identity.
Teacher	A good observation and a genuine question. Haw many of you agree with S_1?

Almost all of them told they agree with S_1.

Teacher	As S_1 observed, this can be done. But every time we use the fact that the difference of the squares of two symbols (or numbers) can be factorized as above. But generally we stop with integral powers.

At this stage the teacher gave examples like.

1. Simplify: $(2a-3b)(2a+3b)$
2. Factorize: $16x^2-25y^2$

Clearly, the students expressed that such type of examples are too trivial.

S_1	Sir, do you mean that $16x^2-25y^2$ should be stopped with the factorization $(4x-5y)(4x+5y)$.

The teacher agreed with this.

S_3	What about $16x^4-625y^4$?
S_2	What about $16x^4-625y^4=(4x^2-25y^2)(4x^2+25y^2)$.
S_3	But $(4x^2-25y^2)$ can be further factorized as $(2x-5y)(2x+5y)$.

The other students also agreed with this. They made themselves clear that the factorization can be carried out till one gets the minimum positive integral power.

S_7	But what about $(4x^2-25y^2)$?

Some students are not sure about that the constants associated can have fractional powers.

The teacher had to clarify that $4x^2-3y^2=(2x-\sqrt{3}y)(2x+\sqrt{3}y)$.

The main difference between a gifted child and an average child is that the above-mentioned facts are arrived at by peer interaction or a gifted child raising a question, and for an average one the teacher has to clarify all these on his/her own.

At this stage, the gifted children no longer are interested in any type of the above problems. The teacher has to give some applications of the identity.

Teacher	Find natural numbers m, n such that the difference of their squares is 11.

There is no difficulty for the students to form equation $m^2-n^2=11$.

S_5	$m^2-n^2=11$ gives me $m=6$ and n= 5.

This will happen to many number theory problems.

Guessing the answer correctly for many such problems is a common feature.

The rest immediately calculated and agreed that S_5 is correct.

S_2 But, how do we know that this is the only solution?

This again is a notable characteristic of a gifted child. When one possibility is found, then analyzes whether that is the only possibility.

A small interaction happened among them. Some of them tried some more pair of numbers and failed.

S_3 $(m-n)(m+n)=11$ means $m-n$ and $m+n$ must be natural numbers whose product is 11.

This is a fairly a good step ahead for the solution.

S_7 So, the number 11 has to be factorized into two factors.

S_1 11 is a prime number. Therefore, the factors are 1 and 11 only.

They all started working on finding m, n out of the equations $m-n=1$ and $m+n=11$ and got $m=6$ and $n=5$. They are now convinced about uniqueness of the solution.

At this stage many had a doubt. In such a problem will there be only one solution?

The teacher gave the following problem.

Find two natural numbers m and n such that difference of their squares is 18.

There was no difficulty to proceed and to get

$m-n$: 1 2 3

$m+n$: 18 9 6

To their surprise there is no solution.

At this stage, they become quite interested to probe further with some more natural numbers.

S_6 $m^2-n^2=20$ gives only one solution (6, 4).

S_9 But $m^2-n^2=24$ gives two solutions (7, 5) and (5, 1).

Now there was a small buzz and each one is trying different natural numbers.

Some got only one solution, some no solution and some two solutions.

The teacher asked whether any one got more than two solutions.

S_{10} Sir, I got for $m^2-n^2=120$

$m-n$:	1	2	3	4	5	6	8	10
$m+n$:	120	60	40	30	24	20	15	12

Pairs (2, 60), (4, 30), (6, 20), (10, 12) give m = 31, n = 29; m = 17, n = 13; m = 13, n = 8 and m = 11, n = 1

Teacher: What are the conclusions?

S_1 When the difference of their squares is a prime number, then there will be only one solution.

S_7 The number of solutions depends on the number of ways the natural number is factorized.

S8 If the right-hand side number is capable of factorized in many ways, then also there may be no solution, one or more solutions.

Teacher made a remark on highly composite numbers (HCN). The students are given a project of collecting information about HCN and presenting it in the class next day.

The project presented by almost all the students (some a few points and some many points) comprised of:

i. There are infinitely many HCN.

ii. The number 5040 is a HCN because it is divisible by all natural numbers 1 to 12 except 11.

iii. A proper definition of a HCN is that it is a positive integer that has more divisors than any smaller positive integer.

iv. A related concepts is a largely composite number (LCN) which is a positive integer with at least as many divisors as all smaller positive integers.

v. Plato considered 5040 to be an “ideal number” for dividing things, such as citizens into cities or states.

vi. The highly composite numbers below 10,000 are 1, 2, 4, 6, 12, 24, 36, 48, 60, 72, 84, 120, 144, 180, 240, 360, 420, 504, 600, 720, 840, 1008, 1260, 1680, 2160, 2520, 3360, 5040, 7560.

A small problem led the students to go deeper into the realm of number theory.

It is really a challenge for the teacher to teach gifted children because the teacher must be aware of the deep knowledge associated with concept and results.

The teacher taught the expansions of $(a+b)^2$ and $(a-b)^2$. As mentioned above the students did not want to find the expansions of $(2a+3b)^2$ or $\left(\frac{a}{2}-5b\right)^2$ etc. They wanted the teacher to give a challenging question.

The teacher gave the following question.

If a positive integer ‘n’ is expressed as the sum of the squares of two natural numbers, will ‘2n’ also be?

The students have no difficulty in understanding the question.

Upon asking the meaning of the question a student answered.

S2 $n = 3^2 + 5^2$ (i.e) $n = 9 + 25 = 34$ will $2n(= 68)$ be expressed the sum of two squares?

They all started working and the first to report is S8.

S8 $68 = 2^2 + 8^2$

Then the teacher asked them to frame their own questions and find whether the statement is true for all such problem.

Many tried and convinced that the statement is true.

From among them the teacher picked three examples.

i. $n = 4^2 + 5^2 = 41, 2n = 82 = 1^2 + 9^2$

ii. $n = 2^2 + 6^2 = 40, 2n = 80 = 4^2 + 8^2$

iii. $n = 6^2 + 7^2 = 85, 2n = 170 = 1^2 + 13^2$

The teacher asked them to find a common idea in the above examples.

After some time (nearly 5 minutes) some of them realized that if $n = a^2 + b^2$ then $2n = (a-b)^2 + (a+b)^2$

Then they got the solution as, if $n = a^2 + b^2$ then $2n = 2a^2 + 2b^2 = (a-b)^2 + (a+b)^2$

Teacher If n is expressed as the sum of two squares then will 3n also be expressed the same Why or why not?

Some of them went for numerical examples which they failed. But some of them tried using the identity, if $n = a^2 + b^2$. Then can $3n = 3a^2 + 3b^2$ be expressed as the sum of two squares?

S3 This cannot be because $3a^2 + 3b^2 = \frac{3}{2}((a-b)^2 + (a+b)^2)$ and $\frac{3}{2}(a-b)^2$ is not a perfect square.

S7: If $n = a^2 + b^2$ then $4n = 4a^2 + 4b^2 = 2(a-b)^2 + 2(a+b)^2 d$, this also cannot be.

S7: If $n = a^2 + b^2$ then $8n = 8a^2 + 8b^2 = (2a-2b)^2 + (2a+2b)^2$

S10: $n = 3^2 + 3^2 = 25, 8n = 200 = 2^2 + 14^2$

At this juncture, most of them realized the method of framing such a question.

$n = a^2 + b^2, (3a-3b)^2 + (3a+3b)^2 = 18(a^2 + b^2) = 18n.$

Thus if $n = a^2 + b^2$, then $(4a-4b)^2 + (4a+4b)^2 = 32n$ etc.

Teacher Can you generalize this?

Many of them derived the following result.

$n = a^2 + b^2$, then $(\mathrm{m}a - \mathrm{m}b)^2 + (ma + \mathrm{m}b)^2 = 2\mathrm{m}^2(\mathrm{a}^2 + \mathrm{b}^2) = 2\mathrm{m}^2 n$, where $'\mathrm{m}'$ is a natural number.

Thus $(2\mathrm{m}^2)n$ can be expressed as the sum of two squares.

Conclusions:

The above are examples of two problems capable of extending and generalizing these kinds of problems and should be found and used by a teacher to teach gifted children. The following characteristics of the gifted children are verified by the above project.

1. A gifted child thinks logically and symbolically about abstract and quantitative relationships.
2. Enjoys solving problems, especially with numbers.
3. Learns basic skills quickly and with little practice.
4. Has advanced comprehension of abstract ideas.
5. Thinks creatively and intuitively.
6. Is highly curious.

How a teacher meets the need of gifted children?

A teacher should understand that all problems are not suitable to extend or probe into. An example is if a, b are odd, then $a^2 + b^2$ is not a square, as an even square is divisible by 4 ($(2a + 1)^2 + (2b + 1)^2 = 4(a^2 + b^2 + a + b) + 2$ is not divisible by 4). The expertise of a teacher lies in choosing an appropriate problem. Another important aspect is that the teacher must be patient enough to admit the errors made by the students and train them properly to go in the right direction. The teacher must refer to a lot of literature before giving a problem to gifted children because the knowledge bank is not only available for the teacher on the Internet but for the students also. When solving a good problem the child feels elated and needs appreciation from the teacher and the teacher must whole heartedly appreciate the students. To sum up, teachers of gifted mathematics students should:

1. Choose right type of problems which are capable of probing further.
2. Train students patiently in non-routine problem solving.
3. Know the latest knowledge connected to the topic.
4. Appreciate when the gifted child solves a problem and not blame them for not solving a problem.

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SHORT COMMUNICATION

DIGITAL INTERACTIVE MATHEMATICAL MAPS IN FOSTERING COURSES

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Keywords: digital interactive mathematical maps, fostering courses, competition tasks, history of mathematics

Creative and gifted pupils often favor learning environments that provide them with an ample amount of autonomy (Brandl & Szabo, 2019, 2024), a feature that traditional classroom settings can possibly only partially accommodate; one way to meet this need may be the use of digital online learning environments (Brandl & Szabo, 2024).

In this short communication, we present the status of a project conducted with a group of 40 pupils (grades 8-10) coached by 10 teachers from the Centre of Excellence in Sibiu, Romania, using the digital tool *Digital Interactive Mathematical Map(s)*[1] (DIMM). The DIMM has been developed at the University of Passau, Germany, and have the possibility to help students overcome the fragmentation of mathematical contents in the traditional curricula at higher secondary schools (Vinerean et al., 2023). The DIMM approach is rooted in a visualization of the historical evolution of various mathematical disciplines. It is partly further enriched through tasks derived from mathematical competitions (Brandl & Szabo, 2024) and supported by learning settings tied to the historical timelines, allowing students to engage with the material within relevant contextual frameworks.

During the project, the pupils dealt with the historical development of geometry using the DIMM in order to get a deeper understanding, as well as create an accurate picture of mathematics as an "evolving science". In the study, individual functions of the DIMM were tested on four work assignments and the functionalities were evaluated individually in short questionnaires for every assignment with regard to their usefulness, and with respect to how well the interactive digital tool has functioned as support in getting insights in new mathematics and in solving tasks from mathematical competitions. Results showed, for example, an agreement of over 80% with the statement “The content of the map helped me solve the proposed Olympiads problems". Concerning the usefulness of the DIMM an example answer was:

> The map helped me a lot. I easily learned the new content and found it easy to use for solving problems; I also understood why the theorems were developed and how they can be used.

The coaching teachers answered similar questionnaires where the focus was on students’ experiences from a teacher’s point of view.

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[1] The tool is freely available at the web address https://math-map.fim.uni-passau.de/.

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FUTURE RESEARCH DIRECTIONS FOR MATHEMATICAL CREATIVITY

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In this paper, future research directions for Mathematical Creativity (MC) are listed. The first two authors are veterans (professors) of creativity research with over 20 years of experience each and the second two authors have less than one year of experience in a tenure line position. The intent of this paper is to highlight areas of research need in subsequent years. In so doing, the purpose of the paper is to provide direction to the domain for advancement in research.

In this short paper, directions for future research in mathematical creativity are mentioned. Much is known about mathematical creativity (Bicer et al., 2021; Byers, 2024; Leikin & Sriraman, 2022; Saefudin et al., 2023), but issues remain unanswered regarding mathematical creativity and they must be addressed to advance understanding and efforts in the domain. Formalizing research through empirical data is necessary. In the interest of keeping the paper to 500 words total, this list has been compiled as a list with limited commentary.

Ongoing research continues to expand understanding of the various facets of creative mathematical thinking. Improved identification methods and effective teaching methodologies that encourage creativity while building the foundational tools and productive dispositions that provide a supportive environment are needed.

Insights from neuroscience on how students learn may lead to the development of tailored instructional methods that can significantly improve students' creative thinking in mathematics and as such requires research attention.

In discussing the paradigm shift, a child must navigate when their mathematical world evolves from working with whole numbers to rational numbers and ultimately into significantly more complex mathematical understandings. Insight is necessary in this area.

Most work in the mathematics field tries to identify specific instantiations of thinking (e.g., a student doing something personally novel) and documenting it as creative. However, a fundamental and unanswered critique of these descriptions of early childhood MC in specific is "*What distinguishes MC from high quality mathematical thinking?*"

Perhaps researchers should find other ways to discuss and consider MC in the early years; it must not be considered disparate from high quality mathematical thinking. Instead of trying to label student output as mathematically creative and attempt to discuss it as an interesting and distinct cognitive factor, they could use creative output as an indicator of high quality mathematical thinking, which is something desired from all students. This area too requires research attention.

Identifying components of MC in the classroom setting and classifying teaching practices to promote artificial intelligence in relation to mathematical creativity can help address the complexity of the construct of MC and the ecology of the learning environment, as well as provide future directions in MC research.

Additional evidence regarding what teaching practices promote mathematical creativity is necessary.

As well, the relationship between mathematical creativity and feelings, emotions, dispositions, attitudes, beliefs, and the like (affect) is necessary. This is the case so that researchers and teachers can be informed as to ideal preconditions for its emergence.

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CREATIVE MATHEMATICS IN AN INCLUSIVE ICELANDIC SCHOOL

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Keywords: creativity, inclusion, gifted students, mathematics, education, professional development, action research

With a growing focus on creativity in education worldwide, creativity is one of five guiding pillars in the national curriculum in Iceland (Ministry of Education, Science and Culture, 2013). Although mathematics is a creative subject, students often learn it by memorizing facts and following predetermined algorithms (Nadjafikhah et al., 2012). By focusing on creativity in mathematics, students can be supported in developing their own solutions and recognizing patterns while also supporting talent development (Sriraman, 2017, Nadjafikhah et al., 2012, Sternberg, 2017). Despite teachers often recognizing the importance of creative mathematics learning, many lack the means and ability to foster it in their classrooms (Sternberg, 2017). Although no universal consensus exists on a definition for creative mathematics (Sriraman, 2017), the focus for this research is on the creative learning process for all students.

This action research focused on a two-year whole-school professional development (PD) programme held in an Icelandic school with around 300 students in grades 1 to 10. The programme was based on seminars, held by the researcher, where 30 teachers partook in creative mathematics projects, discussed their learning, and collaboratively planned and reflected on lessons for their own classrooms. The researcher, who taught mathematics at the school, offered coaching to support teachers in fostering creativity in their inclusive heterogeneous classrooms. All the teachers, including the principal, gave written consent to participate in the research.

Data was gathered and analysed qualitatively to provide an understanding of how the teachers experienced the effects of the PD programme on their views, classroom practice and student learning. The data consisted of videos and notes from seminars, open surveys and purposefully selected interviews. No direct data was gathered from students, but teachers reported back on student learning. The analysis was cyclical, and the programme developed based on the results.

The results show that teachers experienced the PD as beneficial in learning about creative mathematics and implementing changes in their teaching. Collaborative support from co-workers and coaching from the researcher proved instrumental in bringing creativity into their mathematics classrooms. The teachers explained that the focus on creative mathematics helped their high-ability students develop their talents through making problems deeper and more meaningful. An example included students playing with higher multiplication to discover patterns and deepen understanding. The teachers saw conversations, hands-on learning and play as important components of creative mathematics education and experienced time and space as important factors in facilitating creative learning. The research adds to existing literature on how to support teachers with creative mathematics education as well as those in PD, teacher education and policy-making.

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THE ROLE OF GIFTEDNESS AND MATHEMATICAL CREATIVITY IN WORD PROBLEM SOLVING AMONG CHINESE CHILDREN

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Keywords: giftedness, problem solving, mathematical creativity, numerical relevance, non-routine problems, eye movement patterns, Chinese children

Understanding the interplay between giftedness, mathematical creativity, and problem-solving is essential for tailoring educational strategies to diverse learners, particularly in challenging or non-routine contexts. It is little understood how gifted individuals process numerical information in problem-solving, especially when numbers are relevant or irrelevant to the solution. Prior studies show that many children apply unsuccessful strategies in non-solvable word problems by overly focusing on numbers rather than the task's conceptual framework (e.g., Yoshida, Verschaffel, & De Corte, 1997). These tendencies may differ between gifted and non-gifted individuals, as novices often focus on surface features. Therefore, this study examines the relationship between giftedness, mathematical creativity, and problem-solving in solvable and non-solvable word problems, considering numerical relevance. A 2x2 within-subject design (solvable vs. non-solvable; number-relevant vs. number-non-relevant) includes a between-subject factor (gifted vs. non-gifted individuals). Participants included 79 gifted (47.02%) and 89 non-gifted (52.98%) 5th and 6th graders, including 66 native Chinese speakers. To explore mathematical creativity, three divergent thinking tasks inspired by Haylock (1987) were implemented. The first task asked participants to generate as many distinct mathematical operations as possible using specified numbers and symbols. In the second task, participants listed similarities between the numbers 16 and 36. The third task, challenged participants to solve the classic nine-dot puzzle by forming shapes of exactly two square centimeters within a four-centimeter grid. We hypothesize that mathematical creativity positively predicts performance, particularly in non-routine problems, where creative problem-solving is critical. Gifted participants are expected to demonstrate higher creativity.

Preliminary findings indicate that gifted participants generally complete tasks faster than non-gifted peers. In solvable problems, relevance affects performance for all participants, but the impact is less pronounced in gifted individuals due to their high baseline efficiency. In non-solvable problems, giftedness remains the dominant factor influencing faster task completion, regardless of relevance. As mathematical creativity increases, response time decreases, and it also interacts with solvability and giftedness.

Additionally, eye-movement data will be analyzed to compare solution strategies and attention allocation. Gifted participants are hypothesized to show more efficient eye movement patterns, with targeted fixations on relevant numerical elements and reduced focus on irrelevant items. Creativity's impact may also manifest in exploratory eye-movement behaviors, i.e. in different scanpath.

These findings have important educational implications. They suggest that curricula for gifted students should emphasize creativity and complex challenges to further enhance their problem-solving skills. For non-gifted students, interventions focusing on attention allocation and strategy refinement may improve efficiency and adaptability in solving diverse mathematical problems.

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A SUSTAINABILITY-FOCUSED STEM ACTIVITY DESIGNED BY PROSPECTIVE MATHEMATICS TEACHERS: GENERATE YOUR OWN ENERGY

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STEM-related schools have a relatively short history in Türkiye; however, Village Institutes from the Republican era offer a noteworthy precedent for interdisciplinary teacher education (Kaplan & Yılmaz, 2021). These institutes provided potential gifted rural children with comprehensive training, enabling them to contribute to regional and national education. The teacher training model of Village Institutes aligns with STEM principles, as noted in previous studies that draw parallels between their approaches and STEM education (Işıldak & Saylar, 2022; Yıldırım, 2018). STEM education fosters creativity by encouraging students to integrate knowledge from multiple disciplines to solve complex problems (Aguilera & Ortiz-Revilla, 2021; Henriksen, 2014). Thus, similar to various other components, STEM activities have the potential to enhance creativity within the classroom setting (Levenson, 2013). In this study, we adopt Amabile's (2013) definition of creativity as the production of novel, useful, and appropriate responses to open-ended problems. This study aims to explore how prospective mathematics teachers (PMTs) integrate creative elements into sustainability-focused STEM activity.

This qualitative study was conducted at a public university in Türkiye with 23 third-year PMTs enrolled in a Secondary Mathematics Education Program. A 12-week course plan was developed, incorporating creative learning environments and historical STEM activities inspired by Village Institutes. The course comprised two phases: introducing creativity in school contexts and adapting lesson plans from Village Institutes for real mathematics classrooms. This paper highlights a STEM activity titled "*Generate Your Own Energy*," designed by two participants to emphasize sustainability and creativity. Using Beghetto and Kaufman's (2014) framework, the activity was analyzed for creative elements such as choice, imagination, discovery, and motivating cues.

The findings demonstrate that sustainability-focused STEM activities can foster creativity, offering valuable insights for teacher training programs and curriculum development. This study underscores the potential of integrating historical educational practices with contemporary STEM approaches to enhance interdisciplinary learning.

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DESIGN FEATURES OF PROGRAMS FOR MATHEMATICALLY GIFTED STUDENTS

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Keywords: mathematically gifted, gifted education, learning environment, design features

The proportion of students in Germany with a high level of mathematical competence has decreased significantly since 2012. This may be related to inadequate or inappropriate support for mathematically gifted students. While there is a wide range of programs, there is insufficient empirical research on the design principles of such programs, especially at secondary level (Kaiser & Zehnder, 2025; Leikin, 2021). We address this research gap and examine a long-standing, successful gifted education program.

To realize students' gifts, teachers design learning environments and define tasks, content or methods for teaching and learning processes (Ulm & Zehnder, 2020). Prediger et al. (2022) identify five principles for high-quality mathematics teaching (conceptual focus, cognitive demand, student focus and adaptivity, longitudinal coherence, and enhanced communication) that can be used to design learning environments. It is unclear to what extent they can be transferred to gifted education.

We address the following questions: What features characterize the design of the program according to students and teachers? How do these design features correspond to principles for high-quality mathematics teaching?

We examine the design of a math course within the Hector Seminar. The Hector Seminar is a gifted education program that has been successfully supporting gifted students in the STEM fields for over 20 years. To identify design features of the math course, we conducted a semi-structured interview with thirteen students (grade 7) and two teachers. Both groups were asked about their perception of design features and differences in design compared to regular mathematics lessons. The study was approved by an ethics committee.

Key design features of the course will be identified through qualitative content analysis of the interview data. A comparison with the principles for high-quality mathematics teaching will reveal whether there is a considerable overlap between the design of high-quality (regular) mathematics lessons and gifted education.

A first unsystematic review of the interview data shows that open-ended and challenging tasks, homogeneous ability groups, and a focus on creative problem solving are perceived by teachers and students as important features of the program. These findings are at least partially consistent with the principles for high-quality mathematics teaching. However, there are program-specific emphases.

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FREQUENTLY CITED BUT SELDOMLY ACTUALLY MEASURED: THE MATHEMATICAL CAST OF MIND

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Keywords: mathematical cast of mind, Krutetskii, individual differences, mathematical creativity

Mathematical giftedness is a concept that suffers from different theoretical currents and different definitions. However, despite these discrepancies, many researchers agree that V.A. Krutetskii made a substantial contribution to the field (Campos Ferreira & Moreira, 2024). Specifically, he recognized a Mathematical Cast of Mind (MCoM) as one of the main components of mathematical giftedness. Individuals with a MCoM "see the world through mathematical eyes" (Krutetskii, 1976; p. 302), i.e., they perceive and interpret the environment through the lens of logical and mathematical categories. This way of thinking allows them to acquire mathematical knowledge more quickly and achieve higher mathematical performance levels. Although MCoM is often cited, there have been very few attempts to measure it in psychological studies. Therefore, the main goal of two consecutive studies was to assess whether there is a MCoM that can be measured using standardized and objective methods and whether MCoM can predict mathematical performance beyond intelligence. According to Krutetskii, MCoM is an integral system, a characteristic syndrome of mathematical aptitude that includes the perception and the processing of mathematical material, and motivational tendencies.

The first study focused on 82 university students in Austria. Perception of mathematical material was measured by the Attention to Numbers Task and the Spontaneous Focusing on Numerosity Task, processing of mathematical material focused on flexibility using two mathematical creativity tasks, and motivation was assessed with mathematical interest, need for cognition, and self-concept. Using EFA, we found no single factor of MCoM, but instead, the three theoretically proposed factors which showed a good model fit and could explain 54% of variance. However, only motivation was a significant predictor of mathematical achievement, while perception of mathematical material was a non-significant predictor and also showed the lowest correlations with the other two factors. However, the two tasks we used focused only on numerosity and not on other aspects of mathematics.

Therefore, a follow-up study is being prepared with data collection in early 2025. This study will extend the perception of mathematical material from numerosity alone to statistical regularities and the automatic processing of equations. Furthermore, the processing of mathematical material will be extended to flexible switching between different mathematical representations. In addition, an aspect not included in the first study due to constraints but mentioned by Krutetskii will be included in this study. Namely, mathematical memory, which is a generalized memory for mathematical relations, characteristic types, argument schemes, and proofs.

Together, these two studies will shed light on the concept of MCoM and provide evidence as to whether it is actually something that can be measured using objective, quantitative methods and can predict individual differences in mathematics, or whether it is just a phrase that people like to use.

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EXPLORING MATHEMATICAL CREATIVITY DISPLAYED BY GIFTED STUDENTS DURING TEAMWORK

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Keywords: creativity, flexibility, originality, teamwork, Olympiad, mathematics

INTRODUCTION AND THEORETICAL FRAMEWORK

Mathematical creativity—composed of flexibility, fluency, and originality (Leikin, 2018)—can be regarded as a key characteristic observed in mathematically gifted (MG) students. It is even highlighted as one of the critical traits that can help in identifying MG (Pitta-Pantazi et al., 2011). A student with a strong aptitude and better command of mathematics is more likely to apply it in diverse and creative ways. Open-ended problems are recommended for working with MG students, as they encourage creativity (Levenson, Swisa, & Tabach, 2018).

Teamwork has been identified as a favorable strategy for mathematics in general and particularly for MG students (Trpin, 2024). Team collaboration fosters the development of skills such as communication and argumentation. Research by Diezmann and Watters (2001) underscores the importance of investigating the behavior of MG students when collaboratively solving challenging tasks. In this study, we focus on observing how teamwork enables students to demonstrate aspects of mathematical creativity during their interactions while solving an open-ended problem.

Creativity is a recognized characteristic of MG students; however, it is not always easy to identify in their written work. Therefore, further research and tools are needed to detect its presence in student output. This study aims to determine whether teamwork facilitates the identification of evidence of this ability and to illustrate its concrete manifestations in student responses using the indicators proposed by Mora (2024). These indicators provide analytical tools for researchers interested in the topic, offering a way to demonstrate creativity and a means to encourage its expression.

METHODOLOGY

We selected 11–12-year-old students from Spanish-speaking countries, winners of national mathematics Olympiads.

The problem chosen meets characteristics highlighted as conducive to fostering mathematical creativity. Challenging, innovative, distinct from those typically used in classrooms, requiring new approaches and lack a predetermined resolution strategy (Mora, 2024).

The problem was posed in OLIMPRI Olympiad. We analyzed the work of teams of three from nine countries (Uruguay, Paraguay, Colombia, Costa Rica, Chile, Ecuador, Mexico, Honduras, and Venezuela), totaling 27 students. The interactions during this time were analyzed through video recordings and written responses. The aim was to identify qualitative evidence of flexibility descriptors (Mora, 2024), originality, and the ways in which teamwork interactions promote mathematical creativity.

RESULTS

As part of our results, we identified concrete examples of mathematical flexibility indicators in students' responses, based on Mora's (2024) theoretical framework, as well as instances of originality in some answers.

Additionally, we provide insights into the analysis of interaction patterns among MG students during teamwork and discuss the implications of these interactions for fostering and developing their talents. Evidence is presented on how teamwork facilitated the observation of these indicators and how engaging with challenging problems contributed to a balanced team dynamic.

Our findings highlight key considerations for future research and methodological proposals that utilize teamwork as a means to develop creativity in MG students. Specifically, we emphasize the importance of forming homogeneous groups to ensure balance in teamwork, as well as the use of carefully designed problems that promote argumentation, reasoning, strategic thinking, and verification.

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FOSTERING CREATIVITY IN DISTANCE EDUCATION CALCULUS COURSE THROUGH PROBLEM-BASED APPROACH

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While distance education has become an integral part of teaching and learning, it still poses certain challenges. Fostering creativity in distance education courses is one of them. The purpose of this short communication is to share the results of informal observations in the online college Calculus I classroom.

Keywords: distance education, critical thinking, creativity, problem-based teaching, communications in online classroom, digital tools.

ABOUT THE INFORMAL STUDY

This paper presents informal observations from two semesters of teaching a college Calculus I course online. This course was recently moved to an online format and is a work in progress. The observation by Ramadhani et al. (2021) is that "the important role of technology in learning affects…the quality of education" (p. 1239). Furthermore, Ramadhani et al. assert that mathematics distance learning contains abstract and computational material and, therefore, "is a challenge for both lecturers and students" (p. 1240).

A challenge of this modality is fostering creativity in students. Distance education requires faculty members to rethink the approach to promoting creativity in online classrooms, especially ones which are as diverse as those at a community college. Students represent various age groups (high school graduates to adults) from different backgrounds. The informal study explores the effect of posing problems for discussion in bi-weekly forums. These problems require verbal interpretation of the information provided rather than solution through an established algorithm. They reinforce connections between previously learned and current concepts. The forums provide opportunities to discuss unique ways to approach solutions. According to Lubna et al. (2023), Problem-Based Distance Learning (PBDL) includes contextual, constructive, and collaborative problem solving. The study described by Lubna et al. "indicates that PBDL is a successful approach in enhancing the Critical Thinking…skills of STEM students, irrespective of their cognitive types" (p. 557). This finding is significant for a study that is conducted in a diverse group of students. Yanuari and Turmudi (2023) attest that "mathematics is a subject that requires critical thinking skills to analyze and solve complex problems (p. 191). The hypothesis of the informal study is that, with systematic participation in problem-solving forums, students will demonstrate improvement of critical thinking skills needed to solve open-ended problems.

The formal qualitative study is planned for Spring 2026 to explore how problem-based learning can be used to identify exceptionally creative students in a diverse online classroom.

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DETECTING, INSPIRING, AND CHALLENGING HIGHLY ABLE STUDENTS IN UNIVERSITY MATHEMATICS

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In Sweden, it is unusual with extra-curricular interventions for highly able students studying mathematics on university level. In this short communication we will present a project, that since 2019 has been conducted for students at the engineering program at Karlstad University.

The project is based upon the thoughts of Hiebert and Grouws (2007); that all students need to be challenged, and struggle, to develop mathematical knowledge. We also foreground the notion by Reis et al. (2021) that participation in enrichment activities should be interest-based, therefore, it was fully voluntarily for students to engage in the activities. The aim is to reach students who find mathematics joyful, but find university mathematics easy, and by the intervention nurture their motivation and interest in mathematics.

Students participating in the first calculus course in the engineering program (250<n<300), are offered an extra-curricular challenge. To join the challenge, students must complete all recommended course tasks, achieve high scores on the first pre-exam, and demonstrate ongoing engagement in lectures and interest in extra-curricular topics. So far, 63 students have participated in the project, and they could choose one of two paths:

- A theory path focusing deeper knowledge that involves understanding proof and to partly invent new proofs. In addition, students are asked to deeply explore existence and plausibility and to find counterexamples of solutions.
- The problem-solving path is for those who enjoy non-standard tasks. The problems demand creativity and capacity to find useful theories for explicit problems and to adapt and combine theories to handle the problem.

Either the students could choose to do 8–10 problems from the theoretical path, or s/he could choose at least two problems from a list of 12 problems with very high complexity. Passing both parts can give up to three extra marks on student's final exam, maximum 50 marks, but these extra marks can only be given on top on the passing level.

Quality in the student's achievement is assessed through both written solutions and oral presentations at seminars. Students are expected to engage in the seminars through asking questions, comment or show other possible solutions. The lecturer acts as support for the participating students, asking questions, giving suggestions, provoke discussions and so on.

The project is ongoing and has been assessed by course evaluations and semi-structured interviews, in accordance with the ethical guidelines of Karlstad University. The preliminary results show that the students grow mathematically and have higher motivation to continue their studies.

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ACCELERATED TEACHING FOR HIGH ACHIEVERS IN YEAR 7: A MATHEMATICS TEACHER'S PERSPECTIVE

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In relation to the many positive effects associated with accelerated learning in mathematics for high achievers (Szabo, 2017), this approach seems under-implemented in Swedish schools. In practice, the teacher is often responsible for providing students with the opportunity to tackle subject matter at a faster pace. Since the teacher is in closest contact with both high achievers and other students, the teacher's view on accelerated learning becomes particularly interesting. In heterogeneous groups based on students' age, high achievers may possess competence that far exceeds what is required in their classes. Consequently, their opportunities to develop new knowledge are limited. Academic acceleration, the educational practice that allows students to access a curriculum earlier, or progress through it at a faster rate compared to their same-age peers (Southern & Jones, 2004), is a possible way to address this issue. However, creating acceleration for a few students in a heterogeneous group is a challenging task. Therefore, teaching high achievers in more homogeneous groups has been suggested (Szabo, 2017).

By serendipity, we found a teacher who planned to accelerate a group of high-achieving students in mathematics during Year 7. The group consisted of 26 students (aged 12–13) who had chosen to attend for its special focus on mathematics and science. Such groups had been offered at the school for several years. However, while in previous years the compulsory mathematics course for Years 7–9 was taught over two years, followed by upper secondary mathematics, the teacher now planned to complete the compulsory course in one year. The current pilot study was initiated with the purpose of describing this specific case of accelerated teaching from the mathematics teacher's perspective. Two interviews were conducted with the teacher during the first semester: the first focused on the motives for the acceleration, and the hopes and fears for the coming year; the second focused on experiences gathered during the first months, as well as on students' interests and potential in mathematics. Moreover, for continuous updates, a journal was co-created by the teacher and the two involved researchers. In this journal, the teacher described teaching experiences and reflected on assessments, responses from students, etc., while the two researchers commented and asked questions for clarification. A content analysis was carried out on transcripts from the two interviews and the journal. In the session, we will present and discuss some preliminary results, emphasizing constraints and affordances experienced by the teacher in relation to the accelerated teaching of this specific group.

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WORKSHOPS

ASSESSING MATHEMATICAL CREATIVITY USING A STANDARDS-BASED CRITERION-REFERENCED APPROACH

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In this workshop, we propose and discuss the use of a standards-based criterion-referenced approach (CRA) in assessing originality with a specific focus on teaching for creativity. Given the importance of using proper assessment methods in fostering and monitoring learning, this approach may have significant implications for teaching practices and future research. By adopting appropriate assessment methods, teachers of mathematics can better foster and monitor creativity, driving meaningful changes in teaching practices and future research.

Keywords: originality, criterion-referenced approach, mathematical creativity, mathematical problem solving

ASSESSING CREATIVITY IN MATHEMATICAL PROBLEM SOLVING

Encouraging creativity among schoolchildren is essential for preparing them to navigate the complexities of a rapidly evolving world (Sternberg, 2015). Creative thinking enables students to address problems innovatively, adapt to uncertain situations, and develop new ideas. These abilities are indispensable in an era defined by rapid technological advancements, global interconnectedness, and changing societal demands (Maker et al., 2021). Emphasizing creativity in learning does not only empower learners to achieve academically but also to contribute innovative solutions to their communities. However, fostering creativity in classrooms remains challenging (Bahar, 2022). There are few teachers who can successfully integrate creativity into their teaching due to structural barriers. (Bullard & Bahar, 2023). Assessment poses a significant challenge, as originality is subjective and doesn't align with standardized testing formats. Math teachers hesitate to prioritize creativity due to the lack of universally accepted assessment methods, raising concerns about fairness and practicality. The focus on quantifiable outcomes further sidelines mathematical creativity, leaving educators unprepared to assess or implement it effectively. Therefore, we need assessment strategies that strike a balance—structured enough to be useful but flexible enough to reflect the true nature of original thinking.

A Standards-Based Criterion Referenced Approach

A standards-based CRA is an evaluation of originality based on specific, pre-established criteria that were guided by teachers and curriculum standards, rather than comparing responses to a norm group or relying on expert consensus. When a scoring technique such as this is guided by academic standards in the curriculum, such as Common Core Standards, one can identify this method as 'teacher guided standards-referenced scoring'. The method means that teachers assess students' work by making comparisons with the pre-established criteria. Perhaps the most significant advantage of standards-based CRA is that it opens doors for teaching for creativity because it allows assessment of originality in everyday learning in any standards-based setting. Moreover, standards-based CRA bridges standards-based teaching and teaching for creativity, which have been considered two opposite directions in learning sciences.

An example of Standards-Based Criterion Referenced Approach

The example is a learning standard from an elementary mathematics curriculum:

Standard: CCSS.MATH.CONTENT.1.OA.D.8: Determine the unknown whole number in an equation relating three whole numbers (For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11). This standard is taken from U.S. Common Core Standards (Grade 1 » Operations & Algebraic Thinking: Work with addition and subtraction equations)

Criterion: Use of addition or subtraction operations to solve problems.

A Sample Problem to Assess Learning: Use the numbers (2, 4, 6) to write correct equations.

Given that the learning standard listed above is focused on working with addition and subtraction equations, student responses that use these operations [e.g., 2+4=6, 4+2=6, 6-2=4, 6-4=2] can be considered correct but not original. A solution including a novel operation, representation, or algorithm that deviates from the learning standard (in this case it is subtraction and addition), can qualify for originality [e.g., $4 \div 2 = 6 - 4$, 24 = 6 + 6 + 4].

CONCLUSION

Adopting a standards-based CRA to assess originality may be a valuable shift in educational practice. Unlike traditional methods, including norm-referenced, sample-based, and expert-referenced techniques, this technique allows for individualized evaluation that emphasizes students' individual creative potential rather than comparison with peers. By setting standard-based benchmarks for originality, educators can effectively identify and nurture creative abilities and can develop their unique ideas. Additionally, this approach is aligned with models that promote innovation and critical thinking in 21st-century education, with a goal to equip students with skills that are essential for success in an increasingly complex world (Maker et al., 2023).

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MATHEMATICAL CREATIVITY: EXPLORING EXISTING TOOLS AND CO-CREATING NEW APPROACHES

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This workshop on "Measuring and Diagnosing Mathematical Creativity" tackles the challenges of assessing mathematical creativity, a key aspect of mathematical giftedness. Despite various theoretical approaches, there is no consensus on the best way to measure it. The workshop explores existing tools and collaboratively develops new methods. In two parts, participants will first examine current models and assess tools through hands-on activities, then brainstorm and design new assessment methods in small, interdisciplinary groups. The workshop promotes dialogue, critical thinking, and collaboration in creating tools to measure mathematical creativity.

Keywords: mathematical creativity, assessment, existing tools, co-creation

MEASURING AND DIAGNOSING MATHEMATICAL CREATIVITY

Mathematical giftedness is a concept that is characterized by different theoretical streams and different definitions. However, despite these discrepancies, many researchers agree that mathematical creativity is a central aspect of it (e.g., Singer et al., 2017). Thus, the assessment of mathematical creativity is often used to identify mathematically gifted children and adolescents (e.g., Sriraman et al., 2013). However, like the concept of mathematical giftedness, mathematical creativity is a complex construct with no consensus on the best way to measure it (Suherman & Vidákovich, 2022). Additionally, fostering creativity can be a valuable approach to promoting inclusive education, ensuring that diverse learners have opportunities to develop their mathematical potential. Therefore, we want to use this workshop to explore existing tools and co-create new approaches with the help of the interdisciplinary participants of this conference. This 90-minute interactive workshop will be split into two parts. In the first part, participants will get a short introduction to some current definitions and models of creativity. We will also shortly mention how creativity was assessed in 2022 for the first time in PISA (OECD, 2024), as we think this is an exciting development. Afterward, participants will explore and critically evaluate existing measures of mathematical creativity through hands-on activities. The second part will focus on brainstorming and collaborating to develop a new measure, drawing on participants' expertise and creative thinking. This will be done in small groups of four participants each, who are ideally also from different backgrounds. Overall, we will limit the workshop to a maximum of 20 participants to still allow fruitful discussions in the big group. The workshop is designed for researchers, educators, and practitioners interested in creativity and mathematics. The session aims to foster dialogue about the strengths and limitations of current measurement approaches, followed by an opportunity for participants to collaboratively design an innovative tool to assess mathematical creativity. The goal is to engage in critical thinking about how mathematical creativity is currently diagnosed and assessed.

In the workshop, we will use a collaborative approach to develop a measurement tool for mathematical creativity in two 45 minutes parts. In small groups, participants will work together to create prototypes. Each prototype will be shared and refined through feedback and peer review. This interactive and iterative process will foster co-creation, ensuring the final tool effectively captures the multifaceted nature of mathematical creativity. In our workshop, we will use the online tool by Miro and Padlet to collaboratively develop a measurement tool for mathematical creativity. Miro's digital whiteboard allows real-time collaboration, making it ideal for creative group processes. In the following, we describe the two parts:

Part 1: Exploring Existing Measures of Mathematical Creativity (45 minutes)

This section includes a 5-minute welcome, an introduction, and an overview of the workshop objectives. The goals are: 1) To understand the construct of mathematical creativity (including its definition, its relation to PISA, and an introduction to general creativity models), and 2) to explore existing tools for measuring mathematical creativity. The session will provide an overview of established tools and assessment methods, followed by a discussion of the strengths and limitations of these tools. Participants will engage in hands-on activities using existing measures from fields such as psychology and mathematics education. This part will conclude with a 20-minute discussion in the form of a *fishbowl conversation* to identify gaps and challenges in measuring and diagnosing mathematical creativity.

Part 2: Brainstorming and Creating a New Measure (45 minutes)

We will begin with a brief 5-minute brainstorming session on new approaches and ideas for measuring mathematical creativity. This will be followed by a 30-minute interactive creation session, where participants will work in small groups to co-create features and criteria for an ideal mathematical creativity assessment tool. The objective is to collaboratively develop a new approach. Part 2 will end with a 10-minute discussion, summarizing key takeaways and discussing potential next steps and ongoing collaboration.

This workshop outline balances informative sessions, interactive discussions, and collaborative activities, that engage participants in exploring how to measure and diagnose mathematical creativity, with the secondary goal to build interdisciplinary networks.

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ENHANCING THE EXPERIENCE OF ABLE MATHEMATICIANS THROUGH CREATIVE RE-DESIGN OF PROCEDURAL TASKS

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International research has shown that the classroom experience is fundamental in developing students' mathematical understanding, motivation, and identity. In countries such as Ireland, which have a high-stakes examination system, students are more vulnerable to pedagogy that prioritises procedural mathematics over the conceptual understanding of the topics. One danger of this type of pedagogy is that able mathematicians may not experience tasks that maximise their creativity and engagement. This workshop explores tasks to convince teachers of the merits of employing more creative tasks that can effectively provide differentiation in a diverse classroom.

Keywords: creativity, collaboration, uncertainty, inclusion, diverse classrooms

INTRODUCTION

The research underpinning the workshop used a theoretical framework based on Sriraman's five principles (5Ps) to maximise creativity and Vygotsky's Sociocultural Theory of Cognitive Development. Sriraman labelled the 5Ps that should be incorporated into teaching to maximise creativity: the Gestalt Principle, Uncertainty Principle, Aesthetic Principle, Free-Market Principle, and Scholarly Principle. Exposure to uncertainty and having opportunities to take risks, embodied by Sriraman as the Free-Market principle, are fundamental characteristics of the framework and are often absent from classroom pedagogy. The research aimed to create a 'collaborative ZPD', an inclusive learning environment which distinguishes between an expert-novice relationship and one of equal status collaboration (Goos, et al., 2002). In a collaborative ZPD, students of similar abilities can learn from each other by providing scaffolds and justifying each other's conjectures. This type of interaction requires students to be equally involved in the process of problem-solving and to engage in risk-taking dialogue with each other. Incorporating Sriraman's 5Ps into a 'collaborative ZPD' can significantly affect student motivation, enjoyment and perseverance when solving unfamiliar tasks, thus providing more opportunities for challenge and independent thinking. The type of task selected and appropriate teacher intervention are important to ensure that the students engage in constructive dialogue, including refuting or challenging each other's ideas. This supports Sriraman's proposition that creating an inclusive classroom environment that supports different thinking styles and encourages risk-taking is necessary for student creativity to flourish.

RATIONALE & PURPOSE

Providing creative opportunities for able mathematicians in diverse classrooms can be challenging for teachers at all levels. Teachers have often cited the lack of guidance and resources as a reason they do not use differentiated instruction for high-ability students. The national syllabus may recommend differentiated teaching, but the level of integration into the classroom depends on individual schools and teachers. The workshop aims to assist teachers in exploring tasks that can increase student motivation and understanding of mathematical concepts more than procedural tasks, such as those found in the textbook. The research literature advises that when selecting tasks to foster relative creativity in the classroom, open-ended, multi-solution tasks (MST) are a suitable mechanism for exploring students' mathematical potential (Leikin & Lev, 2013). The workshop aims to convince teachers who feel they lack resources that they should be encouraged by the potential to adapt textbook tasks to transform them into MST for developing creative skills in students.

DESCRIPTION OF THE WORKSHOP

The workshop will be divided into two sections: the first will be an engagement with problem-solving, and the second will be an engagement with problem-posing. The workshop will explore how the framework designed around Sriraman's 5Ps can be used in diverse classrooms to adapt traditional tasks based on procedural pedagogy and enhance students' engagement and creativity. In the absence of traditional teacher authority, student participation with peers develops independent problem-solving skills that help them make sense of the mathematics together. The tasks selected for the first section of the workshop have been specifically chosen because of their opportunities for challenge, creativity, collaboration, enjoyment and risk-taking. In groups of 3, participants will be given a multi-solution task designed to support inclusive education by providing the freedom for collaboration, individual learning styles and levels of challenge. Each group will explore a variety of solutions and discuss the benefits of such tasks for fostering creativity in diverse classrooms.

The problem-posing section of the workshop focuses on finding ways to encourage teachers to adapt tasks, traditionally taught and examined using routine questioning, to incorporate multi-solutions that are more suitable for challenging students in diverse classrooms. In the same collaborative, peer-learning manner as above, participants will be given a selection of procedural tasks to discuss how they could be adapted to enhance student conceptual understanding, creativity and engagement.

The tasks are suitable for 15-16-year-olds and will be related to mathematical content in algebra, geometry, and pre-calculus.

The workshops are most suited to 3 or 4 groups of 3 participants.

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THEME DAYS ON WALLPAPER GROUPS

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The Workshop will provide an example of a Theme Day for 12-14 year-olds, inspiring discussions on 1) how to teach mathematically gifted teens hypothesis forming, counter examples and argumentation and 2) how to plan a Theme Day as structured by a complex question, concept, or method. The Workshop will present activities from a Theme Day on Wallpaper Groups, i.e., symmetry groups of the plane. During the Workshop, the participants will experience a condensed version of the entire Theme Day by being asked to do 4 tasks, presented to the learners on the Theme Day, all using manipulatives to throw light on mathematical questions.

Keywords: wallpaper Groups, inquiry based learning, manipulatives, design principles for Theme Days

Audience and duration of the workshop

The Workshop is intended for at maximum number of 25 participants and a duration of 90 minutes. Its intended audience is educators interested in ways to make abstract ideas accessible gifted students and in planning teaching with the students in the role of researchers.

Math students as researchers in their own school

We want gifted mathematics students to learn not only mathematical facts, but competencies relevant to research, such as forming hypotheses, making arguments and using counterexamples and proof-like reasoning. To do so, we need the students to explore and discuss mathematical questions with (to them) unknown answers, and not only for a lesson, but for a prolonged period, in order to train staying focused and putting together complex answers from numerous smaller pieces of information. Theme Day on Wallpaper Groups (Nielsen, in press) is designed to do exactly this. Wallpaper Groups are symmetry groups of the plane and as such lay down the limitations for which two-dimensional patterns are possible, using translations, reflections, rotations and glide reflections. Using the question "How many Wallpaper Groups are there?" with the surprising answer "17!" as a vehicle for discussions about shape and symmetry, the Theme Day lasts for 6 hours and puts mathematically gifted 12–14-year-olds in researchers' shoes at their own school. In this workshop, the participants will take the part of the students, trying out short versions of the activities and discussing the why and how of the structure and instruments of the Theme Day.

By conducting the Theme Days on students' own schools, we invite students to participate who are reluctant to travel Science Talenter's facilities in Sorø and to eat, sleep and be taught together with peers unknown to them. In this way, Theme Days are inclusive in that they have a broader appeal than the Camps that are Science Talenter's main activities.

Designing a Theme Day

The Theme Day on Wallpaper Groups is designed according to these guidelines

- It starts with an interesting question with an unknown answer (Harlen, 2013).
- Students work in groups, necessitating clear communication (Liljedahl, 2020).

- Activities are centred on manipulatives (Bondurant, 2015).
- The teacher does not answer questions concerning right or wrong (Lampert, 1990).

During the workshop, these principles will be showcased and put out for discussion.

Structure of the Theme Day and the Workshop

The workshop emulates a condensed version of the Theme Day. The fact that there are 17 Wallpaper Groups is normally demonstrated through Group Theory (Schwarzenberger, 1974), which is clearly out of reach for 12-14 year-olds in a day. Thus, the Day is structured around tasks illustrating factors that limit the number of Wallpaper Groups possible.

The Theme Day as a whole starts with the question "How many 2-dimensional patterns could you make?" in everyday language, which is translated into the mathematical question which concludes each phase of the Theme Day:"How many Wallpaper Groups are there?" Since Wallpaper Groups are pictured as tessellations made up of a single shape, we move on to study shapes that tessellate. Next, having classified which regular polygons may tessellate (leaving aside the question of how to transform any tessellation into a tessellation with regular polygons), we explore the relationship between shapes and their internal symmetries. Lastly, we investigate the classifications of Wallpaper Groups, in order to solidify and make public the knowledge obtained from the various activities. All the phases rely heavily on group work, manipulatives and dialogue.

After about 60 minutes of working with the activities of a Theme Day on Wallpaper Groups, I will present observations of student behaviour and reactions from 25 Theme Days on Wallpaper Groups, held at 23 different schools with a view to discuss students' view of mathematics and what it is to be thinking like a mathematician.

Immersion is furthered by a story, inquiry and the use of manipulatives

Rich questions with unknown (to the learners) answers that invites to exploring in groups using manipulatives make for excellent opportunities to design a Theme Day.

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SYMMETRY AND TASKS ON ETHNIC ORNAMENTS AT THE MATHEMATICAL CIRCLE FOR YOUNG-GRADE STUDENTS

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Visualisation of mathematical ideas can be an important means of raising students' interest in mathematics. Latvian ethnic ornaments contain rich information about geometric figures, ways of their placement, and mutual relations. The process of investigation the properties of ornamental figures shapes students' spatial and abstract thinking, which is an important prerequisite for creative problem-solving. The authors would like to present the tasks discussed in the Mathematics Circle for Younger Students to show how the properties of ornamental figures relate to combinatorial and geometrical problems.

Keywords: mathematical circle, ethnic ornaments, symmetry

WORKSHOP APPLICATION

Mathematical extracurricular activities such as maths clubs, mathematical circles, different types of contests and Olympiads are very popular nowadays. Mathematical circles are organised to motivate students to engage in mathematics, to solve problems that are different from the usual school tasks, to learn new ways of solving problems and to develop their creativity. Circle participants can be of different ages, from different social groups, and with different levels of mathematical knowledge (McCullough & Davis, 2013; Kennedy & Smolinsky, 2016). Therefore, the work of a mathematical circle leader is exciting, creating the lessons that are interesting and suitable for the appropriate audience.

Many schools in Latvia organize mathematical circles to prepare students for mathematical Olympiads. Mathematics School by Correspondence at the University of Latvia, Vidzeme University of Applied Sciences and Daugavpils University offer every interested student the opportunity to participate in events such as the Mathematics School, the Little Mathematics University, the School of Young Mathematicians and others.

Here, the presenters would like to share information about the Mathematical circle for young grade students (from grade 4 to 8) at the Mathematics School by Correspondence. The idea to start such an event arose because circles for primary school children are rare. The main aim of the circle is to introduce students to basic problem-solving methods that are useful for solving Olympic tasks. Students learn to research properties of objects, analyse them, and find regularities.

After reviewing the results of the Open Mathematical Olympiad for several years, we find that geometry problems are among the most difficult. Solving geometry problems requires not only the knowledge of various geometric facts, but also the application of various methods, mental visualisation and intuition. A number of these problems require additional constructions to be added to the given figure in order to find the solution. In some problems, the solution can be made easier by adding symmetric constructions to a given figure. Students can learn the concept of symmetry by practising to recognize symmetrical structures and distinguishing them from non-symmetrical ones, and drawing symmetrical figures (Ng & Sinclair, 2015). The method of symmetry can be applied in different branches of mathematics, not only in geometry (Leikin, 2019, Savas & Yavuzsoy Köse, 2023). Therefore it is necessary to strengthen students' comprehension of symmetry and how to apply it. The lessons of our mathematical circle discuss properties of axial and central symmetry for solving problems of combinatorial geometry, problems about simple geometric figures, combinatorics problems, and for finding winning strategies in two-player games.

During the session, the lecturers will present the activities for the workshop participants and lead the discussion:

1) Investigation of symmetry in Latvian ethnic ornaments, where axial and central symmetry must be seen; detection of different axes of symmetry and centres of symmetry.
2) Investigation of tessellation of the plane by ethnic ornaments.
3) Construction of symmetric figures using given shapes.
4) Solving of tasks on combinatorial geometry.
5) Solving geometry problems complementing given figures with symmetric constructions.
6) Searching winning strategy for two-player game.

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PROMOTING MATHEMATICAL CHALLENGE AND CREATIVITY DURING A SUMMER LITERACY INTERVENTION CAMP

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Based on a framework of equitable practices for all students to experience opportunities for mathematical creativity and challenge, a series of tasks was implemented during a summer camp for rising third graders. This workshop will describe the tasks, why we chose them, and how we gathered data from the teacher and students. Teachers, coaches, and coordinators will learn the importance of these experiences regardless of a student's literacy needs. Materials for participants (max. 25) to work through the tasks will be provided, and student work and audio clips from the teacher will be shared.

LITERACY INTERVENTION AND MATHEMATICALLY CHALLENGING TASKS

As math researchers and practitioners, we were curious to see how students identified as struggling readers would experience challenging tasks vs. low-level tasks often given to students considered at-risk in reading. We also wondered how the literacy specialist would engage with the tasks as part of her overall instruction for the camp. We told the students good math problems take time to solve; though many believe they should be solved “in a snap” (Schoenfeld, 2013). In addition to learning to stick with a problem over time and building stamina, mathematically challenging tasks are also needed for students to nurture curiosity and develop creativity (Manuel & Freiman, 2017; Singer et al., 2016). The literacy specialist launched two tasks weekly, providing support and scaffolding strategies. The students grappled with the problems, but the teacher encouraged them to stay with it. This productive struggle was intentional and is necessary to deepen students' conceptual mathematical knowledge and to increase stamina for solving problems over time (Huinker & Bill, 2017; National Council of Teachers of Mathematics, 2014).

Why Use Mathematically Challenging Tasks?

For the tasks, we selected a series of six problems from a University of Cambridge (n.y.) website called *NRICH*. We chose tasks focused on grade-level skills, different math topics, and which used unique materials like dominoes. Further, these tasks are designed to nurture students’ natural curiosity and require strong mathematical reasoning. The three levels of challenge and extension opportunities were also critical in choosing the problems. Rich tasks have a range of characteristics that offer opportunities to meet the different needs of learners. A task is not rich; it is how it is used in the classroom that may make it rich (*NRICH,* n.d.). We also believed, as the *NRICH* team did, that all students have the right to shine, and all have the right to struggle.

To promote mathematical creativity and challenge, we chose open-ended tasks considered “favorites”, with multiple solutions and concepts they had yet to encounter in their math classroom. “Mathematical challenge – defined as mathematical difficulty that a person is able and motivated (interested) to overcome at a particular moment – is an integral component of high-quality mathematical instruction” (Leikin, 2018, p. 394). Considering the students’ literacy challenges, we believed that to encourage mathematical creativity, one must change the teacher's conception of who exactly is capable of being creative (Moore-Russo & Demler, 2018).

Exploring the tasks

Task 1: The Tall Tower: On the first day of camp, the teacher presented the *Tall Tower* problem (Figure 1) immediately capturing the students' interest by positioning them in an imaginative story:

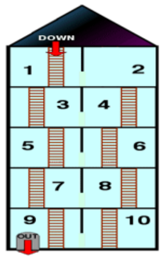

Figure 1 Tall Tower

You have been imprisoned at the top of the Tall Tower by the Wicked Magician. You can get out by climbing down the ladders. As you come down, you collect useful spells. You can go down the ladders and through the doorways into an adjoining room, but you cannot go into the same room twice or climb up the ladders. The numbers in the rooms show how many spells there are in each one. Which route would allow you to collect the most spells? The least number of spells? Exactly 35 spells?

Task 2: Arranging Cubes required students to arrange eight cubes of four different colors based on eight different clue cards (shown at the workshop). Unifix cubes were provided to build the shape; they could also choose to draw it with different colored pencils. This task also required students to collaborate, with a student reading one clue at a time, until all clues were read.

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CHALLENGING ADVANCED LEARNERS IN A MIXED-ABILITY CLASS

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This workshop aims to help educators develop an understanding of the characteristics of tasks that potentially are able to provide mathematical challenge for advanced learners in a mixed-ability class. The workshop is framed using the categories for adapting mathematics content for gifted mathematics learners developed by Gallegher & Gallagher (1994). Both the enrichment model as well as acceleration model in challenging advanced learners will be addressed. The workshop will be an interactive one during which participants will be engaged in primary-level mathematical tasks as well as in small and whole group discussion. Further examples will be used to summarise the main discussion points developed during the workshop.

Keywords: acceleration, enrichment, inclusion, mathematical challenge

INTRODUCTION

Participants will learn characteristics of tasks that are suitable for all learners yet are able to enrich as well as to accelerate the learning of mathematically advanced learners. They will be immersed in the experience of being challenged even as they solve problems that are primarily used for all leaners to (a) learn a new idea and (b) consolidate a recently learnt idea.

WORKSHOP OUTLINE

Overview

A brief description of an on-going research project (Yeap, 2021) from which the tasks used in this workshop are taken from serves as an introduction.

The on-going research employs lesson study (Yoshida, 1999) to investigate characteristics of mathematics tasks that are suitable in mixed-ability classes and yet able to provide both enrichment as well as acceleration (Gallegher & Gallagher, 1994) for advanced learners. The data collected in the research includes how students respond to tasks.

Experiencing Learning

Two tasks from primary levels will be used. Participants will have a chance to work on the tasks in collaborative setting and the workshop leaders will model teacher moves.

The workshop leaders will use participants' sharing to synthesize key learning points.

Task 1

The first task is for students to learn a new concept – in this case the idea of making ten in learning addition facts.

The task is presented using a short video.

There are three vases. In one vase, there are 7 flowers. In a second vase, there are 3 flowers. In the third vase, there are 2 flowers. Think of ways to find the total number of flowers.

Students are provided with ten frames and counters.

Task 2

The second task is for students to consolidate a recently learnt skill – in this case multiplying a two-digit number with a single-digit number.

In AB × C = DE, A, B, C, D and E are different non-zero digits. Think of possible equations.

Students are engaged in pattern observation after the class has generated enough solutions.

Small-Group Discussion

This session provides opportunities for participants to share with each other their experiences and struggles in providing challenge for their advanced learners in ways that can include all learners.

Conclusion

Further examples are used to illustrate the key learning points that the workshop leaders will synthesize using insights gleaned from the workshop participants.

KEY IDEAS TO BE INCLUDED IN THE DISCUSSION

Among others, participants will be presented with the enrichment model as well as acceleration model (Gallegher & Gallagher, 1994) in challenging advanced learners. We will also curate data from large-scale international comparative studies such as TIMSS 2023 (von Davier et al., 2024) to show that in some education systems, most learners – not just advanced learners – are able to access tasks that promote mathematical creativity.

Additional information

Summaries of this workshop will be posted at www.facebook.com/BanHarMaths

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POSTERS

TEAM PROBLEM SOLVING FOR MATHEMATICALLY GIFTED AND OTHER STUDENTS

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We describe an example from our practice illustrating mathematical problem-solving games that have been engaging and benefiting generations of students, both gifted and average, for about 40 years. We discuss four important features of this experience, and what it takes to implement it successfully.

Keywords: problem solving, mixed abilities team, gifted students, average students

Problem-solving (PS) is an important part of educational settings that go beyond direct application of ready-to-use methods. PS relies on the solver's knowledge, creativity and ability to communicate and collaborate. For students gifted in mathematics, problem-solving activities oftentimes increase their initiative, motivation, engagement, and enjoyment (Felmer et al., 2016). How can this approach benefit other students? We discuss a possible scenario based on a long-term practice taking place in one Canadian province.

The province of Newfoundland and Labrador (NL) is characterized by primarily rural settings with half a million population traditionally specialized in fishery. Like in many places, grade school students differ in their mathematical abilities, including those who possess greater interest and talents. Memorial University (MUN) is a major post-secondary institution in the province. The Department of Mathematics and Statistics (DMS) at MUN has embarked on various enrichment activities to identify, support, and recruit mathematically inclined high school students. These include annual country-wide and local contests and a math camp that embraces many collaborative PS activities such as problem sessions, *Mathletics* and *Paper Chase*. In addition, *Math League* team PS games support a high level of participation from female students (Kondratieva, in press).

These activities are an example of a fruitful collaboration between members of DMS and secondary school teachers. The former compose problems for the local contest, math camp, and the *Math League*, the latter help with organization and coaching student-participants. This collaboration is a great mechanism for DMS to convey a message to high schools about what is important in mathematical education, as well as receive feedback from students and teachers and stay informed of the latest developments in the secondary school system.

The outreach activities described above are available for all interested students in the province of NL. I have been a coordinator of the math camp since 2006 and a long-term supporter of the Math League along with several like-minded colleagues. I have had the pleasure of witnessing students' continued interest in these events that have been offered since the beginning of 1980th. They have influenced many students' decisions concerning their post-secondary education. Considering the endurance and vibrancy of these outreach activities and their potential impact on student lives, one might call it a great success.

Let us reflect on this experience using a scientific framework and highlight its most important features. (1) A network of PS games (Math Leagues, Mathletics, and Paper chase) with various characteristics is available during the calendar year so students can practice with PS regularly. (2) Problems vary in difficulty and require, beside remembering mathematical fact, the understanding of ideas, the application of methods, the analysis of problem situations, and the evaluation of proposed solutions (Booth et al., 1995). (3)

Students can engage in these cognitive processes through discussions and presentations of their solutions to the whole group at the games, as well as during the games' preparation sessions. (4) Organization of PS in teams allows students with a range of mathematical abilities to collaborate with each other in an inclusive setting.

We notice that gifted students participating in team PS benefit in the following ways. By explaining their solutions to others, they deepen their own understanding through articulation of their thought processes and potentially uncovering new insights. By taking on leadership roles within the team, they develop communication and interpersonal skills. By working in an inclusive environment, they are exposed to different perspectives, which may foster their creativity and flexibility of thinking. At the same time average students benefit from team PS as well. They observe and learn strategies and habits of their gifted peers. Working in a team towards common goals may increase their motivation and engagement. Gifted students may provide scaffolding and explain required concepts in concrete situations. This inclusive environment while simultaneously supplying challenge and support, ultimately motivates all students to develop their abilities. In the team PS settings this is achieved by providing level-appropriate questions and opportunities to communicate with each other. Careful selection of problems offering optimal challenge allows students to develop and demonstrate their competence. Problems with multiple possible methods of finding the answer allow students to make such choices and support their sense of autonomy. Communicating and discussing own ideas improves their sense of connection to the subject. This is consistent with the self-determination theory, which proposes that human needs for competence, relatedness, and autonomy are essential for students' motivation to learn (Ryan & Deci, 2000). Finally, we observe that maintaining this network of PS activities requires from the organizers their time, expertise, and considerable effort. It is largely due to their dedication that the program continues to thrive despite of ongoing economic challenges. Besides technical training these PS activities offer to our students a social, cultural, and aesthetic experience. While this belief is supported by evidence, it requires from organizers the initiative and recognition of selves as "historical and political beings" making this "collective space better for all" (Radford, 2012, p. 111) in order to ensure that good intentions are not caught up in unfavourable conditions of constrains. Success results from the synergy of enthusiasts committed to the ideas listed above and devoted to the intellectual growth of children through team PS in mathematics.

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SWEDISH MATHEMATICS TEACHER EDUCATORS' CONCEPTIONS OF PREPARING PRE-SERVICE TEACHERS FOR TEACHING MATHEMATICALLY PROMISING STUDENTS

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In Sweden, there is a lack of teacher preparation and teacher knowledge of mathematically promising students (MPS), even though most MPS are taught in regular classrooms. This poster presents an exploratory case study with the aim to investigate how pre-service teachers (PSTs) in Sweden are prepared for teaching MPS, from the perspective of mathematics teacher educators (MTEs). Five focus groups were conducted, and a thematic analysis revealed both opportunities to expand existing teacher education practices and introduce new content on giftedness. However, MTEs also identified challenges, including PSTs' limited mathematical knowledge and policy constraints in teacher education.

Keywords: Mathematical promising students, Teacher educators, Teacher education

BACKGROUND

Teacher education has an important role in preparing all PSTS to acquire the necessary knowledge and skills for meeting the needs of MPS (Singer et al., 2016). In Sweden, MPS are often taught in regular classrooms consisting of students with various mathematical abilities due to lack of recognition (Mattsson, 2013). For MPS to thrive, it is essential that all teachers are knowledgeable about giftedness. However, there is a lack of teacher preparation and teachers' knowledge of promising students in Sweden (Mattsson & Bengmark, 2011).

All Swedish teacher education programs offer special pedagogy, however few addresses gifted students and giftedness, particularly in the context of mathematics (Mattsson & Bengmark, 2011). As a topic, mathematical giftedness would preferably take place in mathematics specific courses. Based on this and considering teacher educators as key actors of teacher education (Lunenberg et al., 2007), this study aims to explore how PSTs are prepared for meeting the needs of MPS from the perspective of MTEs in Sweden. The study also aims to understand what MTEs experience influences Swedish teacher education in relation to MPS with following research questions: *(1) What conceptions do Swedish mathematics teacher educators have of how pre-service teachers can be prepared for mathematical promising students and their needs at teacher education?* And *(2) What possibilities and restraints do Swedish mathematics teacher educators experience influence the preparation of pre-service teachers regarding mathematical promise at teacher education?*

METHOD

This study is an exploratory qualitative case study, consisting of five focus group interviews with a total of 18 participating MTEs (3-4 in each group) from K-12 teacher education programs from various Swedish universities. The interviews were conducted during the fall semester 2023 for 60-90 minutes via online video meetings, which were recorded. Before the interview, each participant signed a consent form regarding ethical considerations and handling of personal data. Questions were posed following an interview protocol in line with the research aim and questions. The interviews were transcribed by the author and a thematic analysis inspired by Braun and Clarke (2022) was conducted for each research question.

PRELIMINARY RESUALTS

The preliminary results are based on one of the five focus groups and outlined in five themes for each research question. For the poster presentation the ambition is to present results from all groups.

Themes RQ1: A broadening perspective to mathematics, Development of PSTs' knowledge, competence and attitudes in mathematics teaching, Implicit approaches to mathematical promise, Inclusion and differentiation in mathematics teaching, Increased awareness of mathematical promise.

Themes RQ2: Attitudes towards mathematics and mathematical promise in society, Policy and framework governing teacher education, Opportunities for innovation at teacher education, Teacher educators as role models, Variation in PSTs' mathematical abilities and backgrounds.

DISCUSSION

Preliminary results indicate that MTEs are willing to incorporate more content on MPS in teacher education. They propose video vignettes and mathematical solutions by MPS, as well as the explicit inclusion of MPS in content in existing parts, such as viewing mathematics from a broader perspective, inclusion and differentiation. A necessity for teachers to orchestrate teaching for MPS is to have knowledge to recognize individual differences, but also to strengthen their own mathematical content knowledge (Singer et al., 2016). The MTEs experience that PSTs' lack of mathematical knowledge and the limited credits of mathematics at teacher education contributes to deprioritizing of MPS. They also acknowledge that societal and cultural factors influence what is included in teacher education both explicitly and implicitly in terms of policy documents and media reports.

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ANALYZING CREATIVE PERFORMANCE OF MATHEMATICALLY GIFTED STUDENTS USING FRAMEWORK OF PROBLEM-SOLVING

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This study explores mathematically gifted students' creativity observed in their independent mathematical activities using framework of problem-solving. Through the case study with two high school students, unique manners and their cognitive need related to their creativity were identified.

Keywords: creativity, iconoclasm, problem-solving, problem-posing, strategy

INTRODUCTION

Creativity is recognized as an important topic in the field of giftedness and gifted education in mathematics. In previous studies, considering problem-solving as "an essential attribute of creative mathematical activity" (Leikin, 2009, p. 399), several approaches related to mathematical problem-solving to foster or assess students' creativity have been suggested (e.g. Kozlowski et al., 2019). Still, it is pointed out that empirical studies focusing on mathematical giftedness and creativity within a framework of mathematics education is limited (Sriraman & Haavold, 2017). Especially, there is a lack of empirical data on creative characteristics of mathematically gifted students exhibited in their independent mathematical activities. This study explores how mathematically gifted students behave in their daily mathematics lessons through the lens of creativity within the framework of problem-solving. Therefore, this study has following two research questions; 1) In what way can mathematically gifted students' activities observed in such a complex situation as daily mathematics lessons be analyzed to see their creative performance? and 2) How do mathematically gifted students exhibit creativity during their independent mathematical activities?

ANALYTICAL FRAMEWORK

Although the concept of creativity in mathematics is elusive, it is widely accepted that its construct includes fluency, flexibility, originality, and sometimes, elaboration, and more recently, iconoclasm (Chamberlin & Mann, 2014). Previous studies focusing on characteristics of mathematically gifted students exhibited during solving given problems use cognitive categories in problem-solving process as an analytical framework "orientation, organization, execution, and verification" (Sriraman, 2003, p. 152). When it comes to their independent activities, however, by definitions of such components above (e.g., Chamberlin & Mann, 2014), gifted students, who are recognized as creative students, are assumed to work on not only given problems but also their original ones, sometimes not explicitly, and switch their way of thinking quickly using more than one strategy. Therefore, this study focuses on problem-solving strategy they bring into play and use three categories to see their creative performance: a) choosing a problem-solving strategy, b) executing the strategy and c) evaluating the strategy. By closely looking at the strategy, problems which they are working on in each moment is identified as well.

METHOD

This is a case study conducted at one of the most academically advanced high schools in Japan. The school is designated as Super Science High school (SSH) and has its own mathematics curriculum to welcome and foster students' creativity. Data was collected through observations of three consecutive mathematics lessons and interviews with some students and with their teacher. This study focuses on two students, who were identified as mathematically gifted students based on their intensive engagement during the observed lessons, a teacher's recommendation and their past experiences in the field of mathematics. Their activities were classified according to three categories above and analyzed with the methodology of constant comparisons (Corbin & Strauss, 2014).

RESULTS AND DISCUSSION

Problem-solving strategies which two students chose throughout the lessons were almost always different from what the teacher had suggested to the class so far. These strategies were based on their own curiosity or cognitive need, which exemplified originality and iconoclasm. One of two students showed iconoclasm even more explicitly by evaluating strategies based on not only whether it worked well or not, but also whether it satisfied his cognitive need for extending mathematical topic. When it turned out not to satisfy him, he posed a problem exploring for satisfying one, which was shared to the other student and generated active discussion between them. Also, they tend to stop executing strategy or even the problem-solving before deriving solutions to move on to other problems using similar strategies. Repeating that way, they made many attempts, which means they exhibited fluency but not flexibility. These findings are expected to move forward the discussion of how to recognize and foster students' creativity in mathematics classrooms.

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Proceedings of the 14th International Conference on Mathematical Creativity and Giftedness (MCG 14)

The Proceedings of the Fourteenth International Group for Mathematical Creativity and Giftedness (igMCG) Conference encapsulate the collaborative and scholarly efforts of researchers and practitioners in the field of mathematical giftedness. Held in Karlstad, Sweden, in partnership with the European Council for High Ability (ECHA), this joint conference allowed attendees to customize their experience by accessing sessions from both organizations. Four keynote speakers – Kirsi Tirri, Jan Kwietniewski, Jorryt van Bommel, and Tracy Reiley – shared diverse insights, with van Bommel's full keynote included in the proceedings due to its specific focus on mathematical giftedness.

The igMCG maintained its tradition of requiring full texts, distinguishing its contributions from ECHA's abstract-based submissions. As a result, this volume includes only full texts from igMCG participants, in addition to keynote materials.

The editorial team, led by Chair Elisabet Mellroth, expressed deep gratitude to contributors, reviewers, and organizing partners. Special thanks were given to Karlstad University, the City of Karlstad, Professor Valerie Margrain, and the Conference Department of Karlstad University for their crucial support. The editorial board's dedicated work, along with the reviewers' thoughtful assessments, ensured high academic standards and meaningful content.

This conference introduced a new presentation type–short communications–widely used in Sweden, allowing presenters to share early-stage or design-focused research. In total, 41 submissions were received across various formats: 18 full papers, 13 short communications, 7 workshops, and 3 posters. All submissions underwent an open peer-review process, with feedback used to determine acceptance and guide revisions. Ultimately, 15 papers, 11 short communications, 7 workshops, and 4 posters were accepted, though some could not be included in the proceedings due to unforeseen global circumstances in 2025. The conference marked a milestone as the first igMCG conference hosted in Sweden and emphasized inclusivity, collaboration, and academic rigor in gifted education.

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