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Early childhood mathematics: a case study

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ABSTRACT

The aim is to investigate early childhood mathematics in a preschool unit in Sweden. A mixed-method research approach and a case study research design were used. Three preschool staff members and 19 children were enrolled. Data were collected by means of observations and a focus group interview. Numerical and thematic analyses were conducted. The mean value of ratings of early childhood mathematics in the preschool unit approached 'good' (m = 4.5) and can be described by means of six themes: *one to ten mathematics; a short question and a short answer; embedded mathematical concepts in everyday routines and play activities; whole-group mathematical circle times incorporating support for younger children and those who are struggling; accessible robots, materials and tools*, and *small group mathematics*. The study could form a basis for discussions on such matters as early childhood mathematics and differentiation in preschools, both in Sweden and in other countries.

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Differentiated instruction; early childhood mathematics; highly able children; preschool; special educational needs

Introduction

This study is about early childhood mathematics in a preschool unit in Sweden. Particular attention is given to the quality of early childhood mathematics, as assessed by a structured observation tool, and the notion of differentiation.

Currently, early childhood mathematics is valued (Gasteiger and Benz 2018) and considered important for children's educational success (Cross, Woods, and Schweingruber 2009). Gasteiger and Benz (2018) state: 'While early childhood mathematics education did not play an important role for many years, today the necessity to be attentive to children's early mathematics learning is no longer in doubt' (109).

In this study, early childhood mathematics refers to teaching mathematics and learning mathematics prior to compulsory school. It can be playful and based on children's questions and interests and is often more informal than mathematics in school. The quality of early childhood mathematics refers here to teaching mathematics in a high-quality manner as well as to rich learning opportunities for learning

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mathematics prior to compulsory school. Early childhood mathematics includes counting and the application of counting; reading and representing simple numbers; mathematical activities concerning shape and mathematical activities concerning sorting, matching and comparing (Sylva, Siraj-Blatchford, and Taggart 2011). Differentiation, or similar notions such as differentiated instruction (Willis 2009), refers to an effort to accommodate the various needs of children in the same preschool unit and also in the same educational activity, daily routine or play activity. In differentiated early childhood mathematics, preschool teachers and other preschool staff members strive to ensure that each child is given opportunities to grow and develop among other children, and in fellowship with friends. According to Björklund, Pramling Samuelsson, and Reis (2018), examples of important tasks in the Swedish preschool are the provision of an equally good start for all children, compensating for and complementing experiences from home and adapting teaching for each child.

Preschool education and care, which can also be termed early childhood education and care, is the first step in the Swedish education system (Swedish Code of Statues 2010:800). In Sweden, a majority of children (77% aged 1–3 years old; 95% aged 4–5 years old) attend voluntary preschool before they start compulsory school (Swedish National Agency for Education, SNAE 2017). These percentages include children with disabilities and special educational needs (Lundqvist 2016), as well as children who learn easily, know a lot and are highly able (Liljedahl 2018). In recent years, the educational mission of the Swedish preschool has been reinforced in matters such as mathematics. According to the legally binding national preschool curriculum (Swedish National Agency for Education, SNAE 2018), preschool teachers and other preschool staff members should provide an education that includes teaching, including in mathematics. Education in the preschool should 'give children the opportunity to use mathematics to investigate and describe their environment and solve everyday problems' (Swedish National Agency for Education, SNAE 2018, 10). The preschool should 'provide each child with the conditions to develop | ... | an understanding of space, time and form, and the basic properties of sets, patterns, quantities, order, numbers, measurement and change, and to reason mathematically about this,' as well as 'an ability to discern, express, investigate and use mathematical concepts and their interrelationships' (Swedish National Agency for Education, SNAE 2018, 14–15). In the Swedish preschool, a staff member who is a qualified preschool teacher 'is responsible for every child | ... | being challenged and stimulated in her or his development of language and communication, as well as mathematics, science and technology' (Swedish National Agency for Education, SNAE 2018, 16), and the preschool work team should 'challenge children's curiosity and understanding of language and communication, and also of mathematics, science and technology' (Swedish National Agency for Education, SNAE 2018, 16).

The children in a Swedish preschool unit, as in many other nations, have various types of knowledge and skills, differing abilities and needs, and are also of different ages (Franzén 2015a; Lundqvist 2016; Lundqvist, Westling Allodi, and Siljehag 2015; Sandall et al. 2008). Children do not learn in the same way or at the same speed; some need more time, and others understand immediately and want to proceed and learn more. Preschool teachers and other preschool staff members have to deal with such diversity. In Swedish preschools, preschool teachers need to differentiate their instruction, that is teaching, as well as to provide adequate support and stimulation/enrichment to each child in

educational activities, routines and play so that they are provided with good opportunities for learning, for example in mathematics. A unit refers to a group of staff members who work side by side, a group of children and their own spaces in a preschool.

Previous research

There are several studies about early childhood mathematics where different aspects have been investigated which have relevance for this study.

The importance of early mathematics

The importance of early mathematics is reinforced by the fact that longitudinal studies (Bailey, Siegler, and Geary 2014; Claessen and Engel 2013; Watts et al. 2014) have shown a relationship between children's early mathematical knowledge and skills, and their later achievements. For example, Bailey, Siegler, and Geary (2014) have 'indicated that whole number magnitude knowledge in first grade predicted knowledge of fraction magnitudes in middle school' and that 'knowledge of whole number arithmetic in [the] first grade predicted knowledge of fraction arithmetic in middle school' (775). Claessen and Engel (2013) have found that 'early math skills [in kindergarten] predict reading, math, and science achievement as well as grade retention from kindergarten through [the] eighth grade' (129). Watts et al. (2014), who incorporated preschool-age children in their study, have presented results showing that children's mathematical development, between the age of 4½ years and the first grade, have great importance for the achievements of adolescent mathematics. They explained this as follows: 'Preschool mathematics ability predicts mathematics achievement through age 15' and wrote that their results 'demonstrate the importance of pre-kindergarten mathematics knowledge and early math learning for later achievement' (352). In line with these longitudinal studies, Doverborg and Pramling Samuelsson (2009) have claimed, based on their study with children from 38 preschools in Sweden, that the ages between one and three can be seen as a critical period for the development and understanding of mathematical concepts as well as the development of a mathematical interest. Clements and Sarama (2007) have investigated the effects of a preschool mathematics intervention for pre-K through grade 2 including research-based, technology-enhanced mathematics tools and materials. They have shown that 'early mathematical interventions help young children develop a foundation of informal mathematics knowledge, especially for children at risk for later school failure' (136).

The importance of a child-centred approach and play, as well as the role of teachers and the body

Björklund (2014) has conducted an investigation of how three teachers organise goal-oriented learning and child-centred practice around the concepts of half and double for preschool children aged four or five years old in Swedish preschools. The teachers introduced problem-solving activities where the learning object was focal, indirect and dealt with as a tool or embedded in a narrative (e.g. a familiar play or story of the children). Björklund shows that 'play is central' and that 'problem-solving as a means for mathematics education needs to be problematised and used with sensitivity to the children's intentions and perspectives' (380). Fisher, Hirsh-Pasek, and Golinkoff (2012) have also put

forth the value of a child-centred and playful learning approach in early childhood mathematics in their study from the United States. Moreover, Vogt et al. (2018) have conducted a study in Switzerland which enrolled several teachers and 6-year-old children. Their study indicated learning gains for play-based approaches in mathematics. They state:

> The study effectively demonstrates that innovative approaches to early maths can be successfully based on play, particularly on card and board games. | . . . | The results of this study highlight the importance of meeting individual children's diverse needs. | . . . | More instructional training programmes with a 'one size fits all' approach fail to challenge and empower every child. (Vogt et al. 2018, 599)

Bäckman (2015) has investigated mathematics in preschool children's activities and the content of early childhood mathematics in Sweden. These include 'volume, geometrical shapes, gravity, quantity, positioning | . . . | size, patterns, proportions, counting and the creation of pairs', for example (6) and give all the children opportunities to experience mathematical content. According to Bäckman, it is important that preschool teachers have a basic content knowledge of mathematics as well as knowledge of mathematics didactics. The staff's emotional attitudes towards mathematics, such as joy and interest in mathematics, also play a role. These can predict their sensitivity towards mathematical content in preschool activities such as play (Anders and Roβbach 2015). The study by Anders and Roβbach (2015) is from Germany. Björklund, Magnusson, and Palmér (2018) have investigated preschool teachers' involvement in children's mathematising in preschools in Sweden. Their analysis showed four categories: 'confirming direction of interest; providing strategies; situating known concepts; and challenging concept meaning' (Björklund, Magnusson, Palmér, 2018, 469).

Lundström (2015) has explored how preschool children use mathematics in communication with other children and staff in Swedish preschools. It is through 'linguistic expressions, semiotics, linguistic tools and bodily expressions' in activities when, for example, they compare and try to give descriptions of their surroundings. According to Lundström, the preschool spaces, materials and equipment stimulated communication in mathematics. Children's mathematical knowledge was also developed and deepened when the teachers were engaged and supported communication in mathematics. In addition, Delacour (2013) studied two preschool teachers working in Sweden with outdoor early childhood mathematics for children who were 4 or 5 years old. These were child-initiated or preschool-teacher-initiated and led. Child-initiated mathematics refers to mathematics apprehended through the body and active children who learn by doing, for example children who are walking, feeling and seeing. This always has children's interest as a starting point. Preschool-teacher-initiated and led mathematics refer to mathematics that is planned and that follows a process. Björklund (2017) has come to a similar conclusion in a pilot study of two preschools in Finland: opportunities to explore mathematics in preschools occur in both spontaneous and planned situations. Franzén (2014, 2015a, 2015b) has also highlighted the necessity for children, in particular younger children, to have an opportunity to use their bodies as a supporting tool in learning processes involving such matters as mathematics. For example, in a study by Franzén (2014), Swedish preschool teachers state that when it comes to older children in preschool, a preschool teacher or other staff member can explain with words and children

can often understand what they mean. With younger children in preschool this is not always possible; they have to investigate things with their bodies. They need to try to see if they can fit into a small doll's chair, for example (Franzén 2014, 2015a). Franzén (2015a) wrote: 'The results show that children often use their bodies as a tool for understanding mathematical concepts' (43).

Quality matters

Sheridan, Pramling Samuelsson, and Johansson (2009) have explained that 'much depends on the quality of the preschool' (6). This means that quality matters for the children. In the study conducted by Doverborg and Pramling Samuelsson (2009), involving 38 preschools in Sweden, early childhood mathematics in such matters as teaching logical reasoning and concepts was rated only 3.58 on a 7-point ECERS Likert scale, which was below the level labelled 'good'. Larsson et al. (2021) have also investigated the quality of early childhood mathematics in preschools (N = 153) in Sweden using The Early Childhood Environment Rating Scale-third edition (ECERS-3). They conclude that the quality, on such matters as mathematics materials and activities, mathematics in daily events and understanding written numbers, on a group level, is rated as 'minimal' or 'below minimal': 2.38, 3.08 and 2.03, respectively, on a 7-point Likert scale. They have stressed the importance of bridging gaps between national curriculum goals, expectations of quality and preschool practice to ensure good and equal conditions for all children's learning.

Theoretical frame

The bioecological model for human development encompassing the notion of proximal processes, also referred to as the process-person-context-time or PPCT-model (Bronfenbrenner and Morris 1998), is used as a frame. The theory explains how children's development is influenced by several systems (i.e. the biosystem, microsystem, mesosystem, exosystem, macrosystem and chronosystem) and proximal processes in the microsystem. Proximal processes can be defined as interactions between staff members and children in a preschool, and interactions between children in a preschool, and can include objects and symbols. These are 'enduring forms of interaction in the immediate environment' and engines for development (Bronfenbrenner and Morris 1998, 996). Thus, the preschool unit investigated in this study is understood as a microsystem, part of a nested bioecological system, and early childhood mathematics as proximal processes.

Aim and motive

The aim of this study is to investigate early childhood mathematics in a Swedish preschool unit (the case study). The questions posed are the following: What is the quality of early childhood mathematics in the preschool unit? What characterises the preschool unit in terms of early childhood mathematics? The present study is an important continuation of previous research and provides further knowledge about early childhood mathematics, differentiation, and assessments of the quality of early childhood mathematics.

Method

This study is part of a research project about early childhood mathematics in the context of a Swedish preschool. The project also includes a case-study of another preschool unit.

In the study, a mixed-method research approach (Bryman 2006) was taken and a case study design (Yin 2014) was conducted. A preschool unit, educating children from 3 to 5 years old, constituted the case. The phenomenon in focus was early childhood mathematics. The preschool unit was chosen by way of a convenience sample (Bryman 2006). The preschool contacted one of the authors (KF) and wished to take part in a study about early childhood mathematics. They met this author during a university course; the project was not advertised. The case of the study constitutes one of four units in a preschool located in a suburb and multicultural, socioeconomically disadvantaged area. Approximately, 80% of the children have immigrant parents. Nineteen children attend the unit and there are 3.75 (full-time equivalent) adults (a preschool teacher, childminders and other staff members) working in the unit. A special educator works in the preschool; the preschool also has a head teacher. In the unit, the preschool teacher, childminders and other staff members shared responsibilities. For example, both the preschool teacher and childminder conducted circle times. They were all present during data collection.

Good research practice was followed in the study. Before data collection, the head teacher, the preschool teacher and other preschool staff members of the preschool unit ($N = 4$), as well as parents of the children ($N = 19$) attending the unit (both mothers and fathers), were informed about the project, and asked to sign a form to indicate their informed consent. In the information provided, they were informed about the project, its related studies and aims, the processes of data collection, use of results and their rights as participants, for example that they were guaranteed anonymity and secure storage of data, and they were given the opportunity to withdraw their consent without explanation. All consented to take part. The preschool teacher and other staff members were the focus of the observations.

The data on early childhood mathematics were collected by means of a structured observation rating scale, the Early Childhood Environment Rating Scale – Extension (ECERS-E, Sylva, Siraj-Blatchford, and Taggart 2011). The ECERS-E is an instrument for measuring quality in numeracy in preschool. The following items in the scale were examined: Counting and the Application of Counting (item 7); Reading and Representing Simple Numbers (item 8); Mathematical Activities: Shape (item 9a) and Mathematical Activities: Sorting, Matching, and Comparing (item 9b). In this study, both 9a and 9b were assessed. According to Sylva, Siraj-Blatchford, and Taggart (2011), either item 9a or 9b may be selected for evidence of quality. The items were rated on a 7-point Likert scale. A rating of 1 refers to inadequate quality, a rating of 3 to 'minimal' quality, a rating of 5 to 'good' quality and a rating of 7 to 'excellent' quality. Three observations were made (of each item) and mean values were calculated. These were conducted by one observer (author JL). It should be noted that the results of these observations are related to the quality of early childhood mathematics in the preschool unit; the values are not related to the overall quality of the preschool unit. Data were also collected by means of semi-structured observation of educational activities, daily routines and play situations in which early childhood mathematics became apparent. Paper and pen were used; no film or voice recordings or consistency protocols were made. Both types of observations

occurred over three whole days and included educational activities in small and whole groups (such as staff-initiated whole-group circle times), daily routines (such as meal time and snack time), free-choice activities and play situations (such as indoor and outdoor solitary and group play). These observations were conducted by two observers (author KF and author ACM). In addition, data were collected using a focus group interview with a preschool teacher and another preschool staff member (i.e. a childminder). In the focus group interview, conducted by one researcher (JL), one preschool teacher and one childminder were asked to describe and reflect upon early childhood mathematics. They chose to be interviewed together. The data from semi-structured observations and the focus group interview were analysed by means of a well-known thematic analysis technique (Braun and Clarke 2006). First, we familiarised ourselves with the data that were qualitative in nature, noted interesting features, and then searched for themes in the data. Thereafter, we reviewed the themes and defined these.

Results

The results begin with a description and analysis of the preschool unit's level of quality in terms of early childhood mathematics. Thereafter, the themes that emerged are presented. These two relate to the different data sources.

Early childhood mathematics and related proximal processes in the preschool unit – level of quality

The quality of early childhood mathematics and related proximal processes in the preschool unit at a microsystem level is presented in Table 1.

The highest scores and mean values

The highest scores and mean values are related to the item on Counting and the Application of Counting, and to the item on Mathematical Activities: Shape. As regards these two items, the preschool unit has 'good' quality. Indicators for 'good' quality under item 7 (i.e. Counting and the Application of Counting) are the following (Sylva, Siraj-Blatchford, and Taggart 2011, 32): 'Number activities such as songs, rhymes, counting books and/or games are often used with the children'; 'Children are encouraged to count objects and to associate the spoken numbers with the numerical concepts'; 'Adults use ordinal numbers | . . . | when working with the children'. Indicators for 'good' quality under item 9a (i.e. Mathematical Activities: Shape), are the following (Sylva, Siraj-Blatchford, and

Table 1. Mathematics items in the ECERS-E, scores from observations 1–3 and mean value.

Item	Score Observation 1	Score Observation 2	Score Observation 3	Mean (*m*)
Counting and the Application of Counting (7)	4	6	5	5
Reading and Representing Simple Numbers (8)	4	2	5	3.7
Mathematical Activities: Shape (9a)	6	5	5	5.3
Mathematical Activities: Sorting, Matching, and Comparing (9b)	4	4	4	4
Mean (m)				4.5

Note. These scores were collected by means of a structured observation rating scale, the ECERS-E (Sylva, Siraj-Blatchford, and Taggart 2011).

Taggart 2011, 36): 'A wide variety of shapes are accessible and adults draw children's attention to shape names'; 'Staff draw children's attention to shape in their own work [drawings and models]'.

The lowest scores and mean values

The lowest scores and mean values are related to the item on Reading and Representing Simple Numbers, and to the item on Mathematical Activities: Sorting, Matching and Comparing. As regards these two items, the preschool unit did not attain 'good' quality during the three observations of item 8, and in the observations related to item 9b, quality was between 'minimal' and 'good'. Indicators for 'good' quality under item 8 (Reading and Representing Simple Numbers) are the following (Sylva, Siraj-Blatchford, and Taggart 2011, 34): 'Children are regularly encouraged to read and/or represent simple numbers'; 'Children have materials available that support them in representing numbers'. Indicators of 'good' quality under item 9b (Mathematical Activities: Sorting, Matching and Comparing) are the following (Sylva, Siraj-Blatchford, and Taggart 2011, 37): 'Activities occur regularly that develop and extend sorting, comparing and mathematical skills'; 'Characteristics that form the basis for sorting, matching and comparing are made explicit by the adults'; 'Staff encourage children to use comparative language when sorting, matching, comparing or measuring'.

The mean value of early childhood mathematics in the preschool unit is approaching 'good' (m = 4.5), and there are both strengths and needs for improvement in the preschool unit. The mean value is just about the same (m = 4.6) when item 9b is not included.

Early childhood mathematics and related proximal processes in the preschool unit – characterising themes

Six themes emerged in the analyses of data obtained from the semi-structured observations of early childhood mathematics and related proximal processes in the preschool unit at a microsystem level and from the focus group interview.

One to ten mathematics

The first theme is *one to ten mathematics*. This theme reflects the attention given to the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 in the preschool unit. In the preschool unit the children took part in activities (e.g. circle times and mealtime preparation) where counting, most often no higher than to ten, was used. These 10 numbers were also displayed on posters (indoors on the wall as well as outdoors on the school yard fence) in the preschool unit, showing, for example, the number 1 next to one animal, the number 2 next to two animals and the number 3 next to three animals, and so forth. In the preschool unit, the children were also encouraged to sort objects (e.g. plastic fruits by colour) and to count these objects (the objects were no more than ten). During the observations there were children who easily sorted objects, read numbers and applied counting to 10, and children who did not. The preschool teacher and the other preschool staff members supported the children who had difficulties so that they could complete a task and answer a question given to them; for example, they counted together with these children.

This theme also reflected that there were few mentions and uses of the number 11 and above in the preschool unit.

A short question and a short answer

The second theme is called *A short question and a short answer*. This reflects short mathematical dialogues that occurred in the preschool unit between staff members and children. These short dialogues encompassed a question from teacher to child (e.g. How many plates do we need? Do you want a whole or a half sandwich? Do you want four or five pearls?), and an answer from the child to the teacher (e.g. Three plates. A half sandwich. Five.). There were also short dialogues and questions which were initiated by a child. For example, one child said, 'My father is big'. One of the preschool staff members answered, 'Yes, he is tall'. This theme reflects the relative rarity of long mathematical dialogues between teacher and children in which both a teacher and a child constructed a shared meaning and had a sustained shared thinking on such matters as counting, written numbers, comparing, matching and measuring. One example of a longer dialogue was identified during the observation of an interaction between a teacher and a child working together when programming a robot. The dialogue between the two reached a deeper level of longer, more complex questions demanding reflections, exploration and discussions resulting in new questions and answers.

Embedded mathematical concepts in everyday routines and play activities

The third theme concerns mathematical concepts such as side, under, as many as, lesser, whole, half, over, tall, both, under, high, low, circle, square, rectangle, triangle, heavy, light, long, big, small, few, many, 1st, 2nd, 3rd and so forth. In the preschool unit the preschool teacher and other preschool staff members used mathematical concepts when talking to children in daily routines (e.g. breakfast) and play situations (e.g. art activities and free play). One example was when a staff member said to a child, 'You are so tall'! The child answered that she had been growing the whole day. These concepts were neither foregrounded (i.e. drawn attention to) nor backgrounded; these were, as many other terms, interwoven into everyday preschool conversations during daily routines and play activities. This theme is termed *Embedded mathematical concepts in everyday routines and play activities.*

Whole-group mathematical circle times incorporating support for younger children and those who are struggling

Theme number four reflects the whole-group and staff-initiated early childhood mathematics. In the preschool unit, these took the form of circle times: the children sat in a circle on the floor, they listened to the teacher's instructions, answered her questions, performed tasks provided in front of peers and listened to one another. Therefore, the first part of this theme's name is *Whole group mathematical circle times*. The observations showed that all of the children in the preschool unit took part in the same mathematical circle time, listened to the same instructions, practised the same skills and performed the same kinds of task. One example of such a task was counting plastic objects of various kinds placed in boxes. Another example was counting plastic fruits and pointing at a number displayed on a wall poster. A further example of such a task was following directions from teachers and peers, such as 'Go and sit in front of Anna [fictitious name]' or 'Go and sit next to Fredrik' or 'Stand up behind the chair'. During these whole-group mathematical circle times there were children who were given extra support to be able to

take part, for example gentle prompts and extra feedback. Therefore, the second part of this theme's name is *incorporating support for younger children and those who are struggling*.

This theme also reflects what seemed to be an absence of stimulation, challenges and enrichments to highly able children in the preschool unit (during whole-group mathematical circle times). During the interview, one of the staff members said that the older children in the preschool unit group often inspired the younger ones; the younger children look at the older children and learn from them during circle times as well as other activities. This theme overall is therefore entitled *Whole group mathematical circle times incorporating support for younger children and those who are struggling*.

Accessible Robots, Materials and Tools

The fifth theme is termed *Accessible robots, materials and tools*. This reflects all the physical materials in the preschool unit that the children could choose to play, create and construct with during free-time activities. Some examples were robots, pebbles, pearls, jigsaw puzzles, board games, dice, blocks, money toys, a cash register toy, shapes and so forth.

This theme also reflected relatively little use of digital tools in this preschool unit. During the interview, one of the staff members said that educational apps could facilitate learning and development of children who learn easily and know a lot, but these apps were not used any more due to a regulation on data protection and privacy.

Small group mathematics

Theme number six reflects small group, staff-initiated early childhood mathematics and is termed *Small group mathematics*. In the preschool unit, there were children who took part in mathematical song time, dancing, waste management and board games which incorporated counting and the use of a colour dice, as well as art activities that incorporated short conversations about shapes. The staff initiated and led these activities. One staff member also said that small group mathematics enables them to facilitate and enhance the mathematical learning and development of each child. She said, 'It is easier [than whole group circle times], since you have more time for each child'.

This theme also reflects that not all children took part in these activities but instead chose to play house, play with dough or clay, dress up as ghosts, do drawings, do block building, run and jump in a space for gross motor activities, etc. During the interview, one staff member said that they did not require these children to take part in small group mathematics.

Discussion

The aim of this study was to investigate early childhood mathematics in a Swedish preschool unit (the case). In keeping with the frame, influential microsystem factors in children's development, as well as opportunities for learning in proximal processes in the unit, were given attention.

Efforts needed to reach a good quality

The results of this study shows that the quality of early childhood mathematics in the preschool unit was rated as approaching 'good'. This was also the case when 9b was not included. This part of the results suggests that only small efforts should be needed in the unit to reach a 'good' quality of early childhood mathematics. In order to reach a 'good' quality of early childhood mathematics, as suggested by Sylva, Siraj-Blatchford, and Taggart (2011), the participating preschool teacher and other preschool staff members, for example, need to increase their attention to activities such as reading and representing simple numbers, as well as to activities such as sorting, matching, and comparing, on a daily basis. However, the quality of other activities (e.g. counting and the application of counting, and mathematical activities related to shapes) needs to be maintained and to remain of 'good' quality in the preschool unit, on a daily basis. The results of this study also show that a 'good' quality of early childhood mathematics in a preschool unit is related to several aspects (referred to as items in the ECERS-E).

One implication for those working in preschools at a microsystem level is to ensure that all days in a preschool unit include at least some proximal processes directed towards: counting and the application of counting; reading and representing simple numbers; mathematical activities concerning shape, and mathematical activities concerning sorting, matching and comparing. This means that staff members' attention to one or two aspects will not be enough.

Scope for improvement as regards differentiated instruction for the highly able

Moreover, the results of this study show that early childhood mathematics in the preschool unit was characterised by: *one to ten mathematics; A short question and a short answer; embedded mathematical concepts in everyday routines and play activities; whole group mathematical circle times incorporating support for younger children and those who are struggling; accessible robots, materials and tools*, and *small group mathematics*. These include child-initiated and staff-initiated activities, spontaneous and planned activities, indoor and outdoor activities, using materials as well as mind and body.

Several characteristics (i.e. *one to ten mathematics; A short question and a short answer* and *whole group mathematical circle times incorporating support for younger children and those who are struggling*) seem to be more beneficial for children who are young and/or who have difficulties in learning mathematics than for children who are older and/or highly able. It is important to pay attention to children who are young and who have difficulties in learning in preschool, but it is also important to acknowledge the needs of children who learn easily and know a lot (Swedish National Agency for Education, SNAE 2018; Swedish Code of Statues 2010:800).

One further implication for those working in a preschool unit at a microsystem level is to ensure that mathematical teaching is differentiated for the highly able children too. As in the preschool unit investigated, other preschools might be focused on children who are young and who have difficulties in learning in preschool. This, however, necessitates that these children have been identified in preschool. Differentiation for highly able children can take the form of content that goes beyond one to ten mathematics and mathematical interactions that are longer and problem-oriented. A preschool staff member could

introduce ten to twenty mathematics or count to a hundred together with children when appropriate. Extended and problem-oriented mathematical dialogues between teachers and children – in which both a staff member and a child construct a shared meaning and have sustained shared thinking about such matters as counting, written numbers, comparing, matching and measuring – could also be beneficial for children's learning and constitute a necessary complement to shorter dialogues. Other possible enrichments could be to embed mathematical concepts such as circumference in playful situations or in small groups and to play more complex board games, for example chess.

A comparison with previous studies and the contribution of this study

This study confirmed several results from previous studies. Both this study and previous studies shed light on, for example, the role of a child-centred approach and play (Björklund 2014; Fisher, Hirsh-Pasek, and Golinkoff 2012; Vogt et al. 2018), a variety of content (Bäckman 2015), to confirm direction of interest, to provide strategies, to situate known concepts, to challenge concept meaning (Björklund et al. 2018), children's use of mathematics in communication with other children and staff (Lundström 2015), child-initiated and preschool-teacher-initiated mathematics (Delacour 2013), spontaneous and planned situations (Björklund 2017) and to involve the body (Delacour 2013; Franzén 2014, 2015a, 2015b).

Concerning the quality of early childhood mathematics, the mean value of the rating ($m = 4.5$) in the participating preschool unit is above the mean values ($m = 2.38$, 3.08 and 2.03 as well as 3.58) of ratings found in preschools enrolled in previous studies (Doverborg and Pramling Samuelsson 2009; Larsson et al. 2021; Sheridan, Pramling Samuelsson, and Johansson 2009). Even though there is scope for improvement, the participating preschool unit seems to have come far in comparison with several other preschools/preschool units in Sweden. One explanation could be their positive attitudes; they wanted to take part in this study. Positive attitudes can bring about sensitivity towards mathematical content in preschool activities (Anders and Roβbach 2015).

The present study contributes new knowledge of such matters as early childhood mathematics and differentiation. For example, it provides examples of what early childhood mathematics can be in a preschool and the need for improvement in relation to quality and differentiation. The results should be generalised with much caution, since this is one case study.

Identify strengths and needs for improvement

Yet another implication for those working in a preschool unit at a microsystem level is to evaluate continuously the quality of early childhood mathematics, and in so doing identify the strengths and needs for improvement. As in the preschool unit investigated, other preschools may be in need of some improvement. Evaluations and self-reflections of early childhood mathematics can be made by ECERS-E or using other structured observation rating scales (Harms, Clifford, and Cryer 2016); such tools are not only useful for researchers.

A basis for early childhood mathematics discussion and a suggestion for further research

At a time when the value of early childhood mathematics is receiving recognition (Gasteiger and Benz 2018), and considered important for children's educational success (Cross, Woods, and Schweingruber 2009), the Swedish preschool should provide mathematical content and adapt teaching for each child (Björklund et al. 2018; Swedish National Agency for Education, SNAE 2018). Since studies demonstrate the importance of early mathematics in school (Bailey, Siegler, and Geary 2014; Claessen and Engel 2013) and pre-kindergarten (Watts et al. 2014) for later achievement and grade retention from kindergarten, the study has relevance for several researchers. These are researchers in the fields of inclusive education, special education and education as well as heads of preschools, preschool teachers and other preschool staff members. It also has relevance for policy makers, responsible authorities, preschool teacher students and others interested in early childhood mathematics and preschool education and care. The study can form a basis for their early childhood mathematics discussions.

More research is needed on the topic of early childhood mathematics and differentiation. For example, further research could investigate whether the themes identified in this study are present in other preschool units.

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Author contributions

JL and KF planned the study. JL, KF and ACM collected the data. JL conducted the analysis, and KF and ACM contributed to the analysis. JL and KF wrote the paper. ACM contributed to the revision of the paper.

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