How the word ‘mathematical’ influences students’ responses to explanation tasks in a dynamic mathematics software environment

Maria Fahlgren and Mats Brunström

Karlstad University, Department of Mathematics and Computer Science, Sweden, maria.fahlgren@kau.se

Task design is a central issue in mathematics education, not least in relation to digital technology. This paper reports how a small but significant change in wording affects students’ explanatory responses. The study is comparative and involves 229 10th grade students working on tasks designed for a dynamic mathematics software environment. The findings indicate that inclusion of the word ‘mathematical’ prompted students to use algebraic symbols and algebraic arguments, to a higher degree.

Keywords: task design, dynamic mathematics software, explanation task.

INTRODUCTION

Task design within mathematics education has been an important issue for decades, and the increased availability of different kinds of technology in mathematics classrooms has made this issue even more important. Designing tasks that utilize the affordances provided by digital technologies is recognized in the literature as being a complex and subtle process (Joubert, 2017).

In relation to Dynamic Mathematics Software (DMS), the literature emphasizes the possibility of visualizing and linking various representations of mathematical objects, particularly in relation to functions and graphs (Hegedus et al., 2017). With DMS it is possible to make direct manipulation of dynamically linked representations of functions, e.g. algebraic and graphic representations (Drijvers, 2003). However, there is an identified risk that students, while working on a task in a DMS environment, will only relate to the empirical/visual objects obtained on the screen without reflecting on the mathematics involved (Drijvers, 2003; Joubert, 2017). Hence, one important issue to consider in the design of tasks for DMS environments is how to formulate tasks that encourage students to move from the empirical/visual to the mathematical/systematic field (Joubert, 2017).

Several studies, focusing on task design in DMS environments, emphasize the importance of asking students to explain their empirical findings (Leung, 2011). According to Leung, “A meaningful mathematical explorative task should be one that involves conjecturing and explanation activities.” (p. 328). The issue of student explanations in the teaching and learning of mathematics has been a focus of research literature for several decades (e.g. Dreyfus, 1999; Silver, 1994). Silver (1994) acknowledge the challenge for students to provide explanations in writing, and suggests “…the need for explanations, especially written explanations, to become a more prevalent feature of school mathematics instruction.” (p. 315).
Moreover, Sierpinska (2004) discusses the importance of ‘task problematization’, and pinpoints that small differences in the formulation of tasks might have a significant impact on students’ responses. In line with this, we found, in a previous study, that the wording is crucial in the formulation of questions where students are asked for explanations (Brunström & Fahlgren, 2015). Particularly, we found that students’ responses tend to be superficial and more descriptive than explanatory. The result from the study prompted us to further investigate how small but potentially significant changes in wording might influence students’ explanatory responses in a DMS environment.

The change in task wording investigated in the study reported in this paper involves moving from asking students simply for an explanation to asking them for a ‘mathematical’ explanation. The explanation tasks in question are embedded in a task sequence developed for a DMS environment with the aim of developing students’ awareness of some of the connections between the standard form of quadratic function $f(x) = ax^2 + bx + c$ and the corresponding graphical representation and quadratic equation. The research question will be presented in detail later.

THE PRACTICE OF EXPLANATION

In (digitalized) mathematics classrooms, technology provides feedback in response to students’ action with the environment (Joubert, 2017). Using the terminology introduced by Noss and Hoyles, Joubert argues that for students to move from the ‘pragmatic/empirical’ to the ‘mathematical/systematic’ field, the students must go beyond just reporting what they have seen (Joubert, 2017). The literature suggests asking students for explanations as a way to encourage students’ movement between these fields (e.g. Dreyfus, 1999).

However, the literature recognizes the challenge for students to provide mathematical explanations. According to Dreyfus (1999), students have rarely learned what counts as a satisfactory explanation. Moreover, Levenson (2013) argues that the properties of a task as well as the mathematical concepts under consideration affect the features of the explanations used. In a study investigating students’ conceptions of the qualities of mathematical explanations, Healy and Hoyles (2000) found that many students preferred explanations described in everyday narratives. For these students “empirical data convince whereas words and pictures, but not algebra, explain.” (p. 415). However, the study showed that although students predominantly used narrative explanations they were aware of their limitations, and they thought that to receive a good mark, explanations should include algebraic arguments (Healy & Hoyles, 2000).

Researchers refer to ‘expository writing’ as “writing which is intended to describe and explain mathematical ideas” (Shield & Galbraith, 1998, p. 29). The main idea is that by communicating their mathematical thinking in writing, students improve their mathematical understanding (Santos & Semana, 2015; Shield & Galbraith, 1998). In the study reported by Santos and Semana (2015), students used several types of
representations in their written expositions, e.g. verbal language, iconic representations, numerical and/or algebraic symbols.

These ideas lead us to formulate more precise research questions: What impact, if any, has inclusion of the word ‘mathematical’ on students’ responses when asked to explain observations made in a DMS environment on: (a) the forms of representation employed, and (b) the characteristics of explanations used?

**METHOD**

**Research Setting**

In total, 229 tenth grade students at a secondary school in Sweden participated in the study, conducted during a year-long school development project with the overarching aim to develop and test sequences of tasks designed for a DMS environment. The aim of the particular task sequence was to introduce graphical representations of quadratic functions written in the standard form \( f(x) = ax^2 + bx + c \), and the corresponding quadratic equation. The students had previously worked with linear functions, linear equations and with solving quadratic equations algebraically. In Sweden, the predominant method for solving a quadratic equation of the form \( ax^2 + bx + c = 0 \), is first to reduce it to the equivalent quadratic equation \( x^2 + px + q = 0 \), and then to apply the so called \( pq \)-formula.

**Material**

In total, the task sequence includes three embedded explanation tasks formulated in the following two versions: “Explain why…” and “Give a mathematical explanation why…”. Besides the word ‘mathematical’, these phrases also differ grammatically (i.e. ‘explain’ vs. ‘explanation’). However, we suggest that this has an insignificant impact on student responses. The two versions are labeled U and M for Unspecified and Mathematical explanation respectively. Due to limitation of space, this paper only reports on two of the tasks; Task 1c and Task 3c (see Figure 1).

**Task 1**

(a) Investigate, by dragging the slider \( c \), in what way the value of \( c \) alters the graph. Describe in your own words.

(b) The value of the constant \( c \) can be found in the coordinate system. How?

(c) Explain why/Give a mathematical explanation why the value of \( c \) can be found in this way.

**Task 3**

(a) Solve the quadratic equation \( x^2 - 4x + 3 = 0 \) algebraically (using pen and paper).

(b) Set the sliders so that the graph of the function \( f(x) = x^2 - 4x + 3 \) is shown. The solutions to the corresponding quadratic equation, \( x^2 - 4x + 3 = 0 \), can be found in the coordinate system. How?

(c) Explain why/Give a mathematical explanation why the solutions to the equation can be found in this way.

**Figure 1:** Two tasks including a request for explanation (subtask c)

**Data Collection**

The empirical data for each task consist of the written responses from students. Only the teachers were told that there were two versions of the task sequence. Each student received one or other version, distributed at random in each class. However, not all
students provided answers to all of the tasks. The number of student responses on Task 1c are 109 (version U) and 100 (version M). The corresponding numbers for Task 3c are 102 (version U) and 99 (version M).

Data Analysis

The analysis process was conducted in several phases. Initially, only Task 1c was analysed, which resulted in preliminary results that were presented as a poster at the 13th International Conference on Technology in Mathematics Teaching (Fahlgren & Brunström, 2017). This analysis provided insights into what kind of results we could get from the empirical material, and thus, how to continue the analysis process.

The initial analysis was followed by a more structured content analysis. Student responses were inspected and compared to identify a basic set of elements of explanation which could be used to summarise the content of any response. This made it possible to create a manual used to code all responses in terms of the presence or absence of each of the explanation elements and representation types.

The Coding Manual

While the categories of representation type are the same for all tasks, based on predefined general patterns of use of verbal and algebraic representation (Santos & Semana, 2015), most of the categories of explanation elements are task specific (as suggested in Levenson, 2013). The later ones were developed inductively through analysis of the substantive content of students’ responses to the specific tasks. To illustrate and clarify the categorization, exemplars of student responses in each category, including representation type, are presented for one of the tasks (Task 3c).

Representation type

In this study, the types of representation were divided into four categories. Students’ responses were classified as “Verbal only” (V) even if they included single letter coefficients or variables. In student responses classified as “Verbal with elements of Algebraic symbols” (VeA) formulas or other algebraic symbols are just included without being evaluated or manipulated in some way. Hence, the categories “Algebraic symbols only” (A) and “Verbal and Algebraic symbols” (VA) are the only categories where students really use algebraic symbols (even if not always in an appropriate way). No student responses included a graph, although some made reference to graphs (See elements B and H in Table 1 below).

Explanation elements

Due to limitation of space, we choose one task (Task 3c) to present as exemplary of the analysis process in terms of identified elements of explanation (see Table 1) making up a particular response.

<table>
<thead>
<tr>
<th>Code</th>
<th>Explanation element</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Express that ‘y = 0’ (where the solutions occur)</td>
</tr>
<tr>
<td>B</td>
<td>Relates ‘y = 0’ to intersection with the x-axis</td>
</tr>
</tbody>
</table>
Table 1: The categories of explanation element in Task 3c

Below are exemplars of six student responses and the suggested categorization for each of them is shown in Table 2:

S1: “Because that is where y is 0”

S2: “f(x) = x^2 – 4x + 3 = 0, hence y = 0, where x is the correct answer. That is, where the line intersects the x-axis (where y = 0)”

S3: “I use the pq-formula. That is the easiest way.”

S4: (Handwritten solution)

S5: “We inserted the formula $x^2 – 4x + 3$, and received the solution”

S6: “Because we have got 2 x values.”

Table 2: The categorisation of student responses S1 to S6 on Task 3c

RESULTS AND ANALYSIS

This section provides the results from the comparison between two groups of students; Group U and Group M, the ones answering Versions U and M respectively. First, the results concerning representation type for the two tasks are introduced. Then, the results related to explanation elements are presented for each task separately.

Representation type

The results indicate that the task formulation including “mathematical” (i.e. Version M) prompts more students to use algebraic symbols in their explanations (see Table 3).
In particular, merging the two categories where students really use algebraic symbols (A and VA) the tendency becomes clear for both tasks, with all differences statistically significant (Task 1c: \( p<0.001 \) and Task 3c: \( p<0.01 \)).

<table>
<thead>
<tr>
<th>Task</th>
<th>Version</th>
<th>A</th>
<th>VA</th>
<th>A or VA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Task 1c</td>
<td>U</td>
<td>0.0%</td>
<td>6.4%</td>
<td>6.4%</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>10.0%</td>
<td>20.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>Task 3c</td>
<td>U</td>
<td>1.0%</td>
<td>7.8%</td>
<td>8.8%</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>14.1%</td>
<td>8.1%</td>
<td>22.2%</td>
</tr>
</tbody>
</table>

Table 3: The proportion of student responses using algebraic symbols (A), verbal and algebraic symbols (VA), and one or the other of these

**Explanation elements**

**Task 1c**

The results indicate some differences between the groups. Compared to Group U, Group M were, as an element of their explanation:

- less inclined to repeat their answer to the previous subtask (29.0% vs 45.9%)
- less inclined to refer to the feedback from the DMS environment (11.0% vs 26.6%)
- more inclined to use the fact that \( x = 0 \) when the graph intersects the \( y \)-axis (14.0% vs 6.4%)
- more inclined to use linear analogy (64.0% vs 44.0%)

Concerning the categories ‘Repeating the answers from the previous subtask’ and ‘Referring to the DMS feedback’, in several cases these elements of explanation were combined with other more relevant elements. Therefore, it is interesting to investigate the proportion of student responses using one or both of these explanation elements only. In this analysis, a significant difference (\( p<0.001 \)) between the groups emerged (32.1% for group U vs 6.0% for group M).

The explanation element using the fact that \( x = 0 \) when the graph intersects the \( y \)-axis focuses on an algebraic expression, and in this aspect aligns with several other explanation elements; ‘\( c \) is the constant term’, ‘\( c \) is independent of \( x \)’, ‘\( c \) is independent of \( a \) and/or \( b \)’, and ‘solves for \( c \)’. A further analysis, looking at responses including one or several of these categories, revealed a significant difference (\( p<0.001 \)) between the groups (50.0% in group M vs 28.4% in group U).

**Task 3c**

There are some differences between the groups in Task 3c. Compared to Group U, Group M were, as an element of explanation:

- more inclined to refer to the \( pq \)-formula (Category F; 19.2% vs 2.9%; \( p<0.001 \))
- more inclined to verify the solution by inserting the values 1 and 3 into the equation (Category G; 12.1% vs 2.0%; \( p<0.01 \))
- less inclined to refer to DMS feedback (Category H; 7.1% vs 11.8%)
The first two categories involve, although in different ways, references to the equation elaborated on in subtask a). That is, students using these explanation elements are referring back to the algebraic subtask a) rather than the graphical subtask b). Concerning the category ‘Referring to the DMS feedback’, the tendency is the same as in Task 1c. The difference between the groups in Task 3c appears somewhat clearer when looking at responses with this element only (10.8% in group U vs 5.1% in group M).

**DISCUSSION**

As stated in the Introduction, one overarching goal of letting students work in DMS environments is to encourage their movement from the empirical/visual to the mathematical/systematic field (Joubert, 2017). One way of doing this, is to prompt students to explain, in writing, what they notice when interacting with the technology (e.g. Leung, 2011). By comparing student responses from two versions of explanation tasks, this study sought to investigate whether a small but significant change in task wording influences students’ explanatory responses in a DMS environment. The results show that there were significant differences between the groups, both in relation to type of representation and explanation elements used.

Students asked for a ‘mathematical’ explanation, used algebraic symbols and algebraic arguments to a higher degree. Moreover, they were more likely to use linear analogy, and hence to utilize their prior knowledge in mathematics. In contrast, the other group of students were more inclined to use the feedback from the DMS environment as an element of explanation and/or to repeat the answer to the previous subtask. In both these cases, student responses were descriptive based on reporting visual information rather than explanatory based on accounting for that information, which indicate that the students still are in the empirical/visual field.

Taken together, the findings in this study indicate that the word ‘mathematical’ signals a request for an algebraic explanation. Consequently, this small change in wording might enhance students’ movement from the empirical/visual to the mathematical/systematic field.

**REFERENCES**


Technology in Mathematics Teaching ICTMT 13 (pp. 443-446). Université Claude Bernard Lyon 1: École Normale Supérieure de Lyon.


