Dimensioning and control for heat pump systems using a combination of vertical and horizontal ground-coupled heat exchangers

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Abstract

A model has been developed which simulates a system consisting of a horizontal and vertical ground-coupled heat exchanger connected in parallel to the same heat pump. The model was used in computer simulations to investigate how the annual minimum and mean fluid temperatures at the heat pump varied as several parameters of the combined system were changed. A comparison was also made between different control settings for fluid flow rate distribution between the two exchangers. For the case when the flow rate distribution was not controlled, the effect of viscosity differences between a colder and warmer exchanger was investigated. The short term effects of letting the vertical heat source rest during the warm summer months was then tested. Lastly, the results of the model was compared to a simple ’rule of thumb’ that have been used in the industry for this kind of combined system. The results show that using a combined system might not always result in increased performance, if the previously existing exchanger is a vertical ground-coupled heat exchanger. The effects of viscosity differences on the flow distribution seems to be negligible, especially for high net flows. Controlling the fluid flow rates seems to only be worth the effort if the the pipe lengths of the two combined exchangers differ heavily. Letting the vertical ground-coupled heat exchanger rest during summer was shown to in some cases yield an increased short-term performance in addition to the already known positive long term effects. The rule of thumb was shown to recommend smaller dimensions for combination systems than the more realistic analytical model.
Acknowledgements

I would like to express my deepest gratitude to my supervisor Krister Svensson, for both his interest and his efforts to understand and assist with problems encountered along the way. I would also sincerely like to thank my supervisor at Danfoss Värnepumpar AB, Erik Olsson, who kept drowning me in useful literature. Many thanks goes out as well to Ulf Olsson at Danfoss Värnepumpar AB, who provided me with insights that helped me greatly in my work. Last of all I’d like to thank my two cats, who have been my steadfast companions, telling me loudly when they wanted food and providing distraction from the hard work. Maybe my dearest life companion Therese deserves to be mentioned here as well, but she’s been away most of the time, writing her own thesis, having her own problems.
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Nomenclature

BHE Borehole heat exchanger
GSHE Ground surface heat exchanger
GSHP Ground source heat pump
\(COP_{HP, rev}\) Coefficient of performance for a reversible heat pump, see (1.2)
\(COP_{HP}\) Heat pump coefficient of performance, see (1.1)
\(Q_C\) (J) Amount of heat lifted from the cold reservoir
\(Q_H\) (J) Amount of heat ejected to the hot reservoir
\(T_C\) (K) Temperature at cold reservoir
\(T_H\) (K) Temperature at hot reservoir
\(W\) (J) Input work
\(\alpha\) \((^\circ\text{C}/\text{m})\) Geothermal gradient
\(\dot{V}_f\) \((\text{m}^3/\text{s})\) Fluid flow rate through the pipe
\(\gamma\) Euler’s constant \(\approx 0.5772\),
\(\lambda\) \((\text{W}/(\text{m}\cdot\text{K}))\) Thermal conductivity of medium (bedrock or soil)
\(\mu_f\) \((\text{Pa}\cdot\text{s})\) Dynamic viscosity of the fluid evaluated at the bulk temperature along the pipe.
\(A_p\) \((\text{m}^2)\) Cross-sectional area of the pipe through which fluid can flow
\(C_p\) \((\text{J}/(\text{m}^3\cdot\text{K}))\) Volumetric heat capacity of medium (bedrock or soil)
\(D_i\) (m) Insulated depth
\(H\) (m) Active borehole depth
\(R_0\) (m) Borehole radius, radial distance to borehole wall
\(R_{p,o}\) (m) Pipe/hose inner radius
\( R_{p,o} \) (m) Pipe/hose outer radius
\( T_0 \) (°C) Annual average air temperature
\( T_R \) (°C) Average temperature at the borehole wall or soil nearest the hose
\( T_f \) (°C) Average heat carrier fluid temperature
\( T_{0m} \) (°C) Undisturbed bedrock temperature at depth \( D_i + H/2 \), see (2.2)
\( T_{0s}(t, z) \) (°C) Soil temperature at depth \( z \) at time \( t \)
\( \lambda' \) (W/(m-K)) Thermal conductivity of the pipe wall
\( \lambda_f \) (W/(m-K)) Thermal conductivity of the fluid
\( \lambda_w \) (W/(m-K)) Thermal conductivity of ground water/grout
\( \rho_f \) (kg/m\(^3\)) Density of the fluid
\( \bar{q} \) (W/m) Average heat load per unit length
\( a \) (m\(^2\)/s) Thermal diffusivity of medium (bedrock or soil)
\( b \) Relative distance of the two pipes in terms of \( R_0 \) from the center of the borehole
\( c_{p,f} \) (J/(kg-K)) Specific heat of the fluid
\( d_0 \) Penetration depth (m)
\( m'_p \) (m-K/W) Thermal resistance between the outer and inner radius of the pipe
\( m_{f,c} \) (m-K/W) Thermal resistance between the fluid and and the inner pipe wall
\( q(t) \) (W/m) Heat load per unit length at time \( t \)
\( t_0 \) Period of the temperature variation at the surface (typically a year) (s)
\( t_s \) (s) Steady-state break-time
\( \mu_{bh} \) (Pa-s) Dynamic viscosity of the fluid evaluated at the bulk temperature of the BHE
\( \mu_{gs} \) (Pa-s) Dynamic viscosity of the fluid evaluated at the bulk temperature of the GSHE
\( A_{bh} \) (m\(^2\)) Cross-sectional area of the BHE pipe through which fluid can flow
\( A_{gs} \) (m\(^2\)) Cross-sectional area of the GSHE pipe through which fluid can flow
\( C_f \) (J/(m\(^3\)K)) Volumetric heat capacity of the heat carrier fluid
\( D_{bh} \) (m) Inner diameter of the BHE pipe
\( D_{gs} \) (m) Inner diameter of the GSHE pipe
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<thead>
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<td>$K_{gs}$</td>
<td>Defined in (3.14)</td>
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<td>$L_{bh}$</td>
<td>(m) Active borehole depth</td>
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<td>(m) Length of hose used for the GSHE</td>
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<td>$Q(t)$</td>
<td>(W) Total heat load at time $t$</td>
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<tr>
<td>$Q_{bh}$</td>
<td>(W) Heat load on the BHE</td>
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<td>$Q_{gs}$</td>
<td>(W) Heat load on the GSHE</td>
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<td>$R_{b}$</td>
<td>(m-K/W) Borehole thermal resistance</td>
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<tr>
<td>$R_{h}$</td>
<td>(m-K/W) Thermal resistance between fluid and soil</td>
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<td>$T_{HP, out}$</td>
<td>($^\circ$C) Heat pump outlet fluid temperature</td>
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<tr>
<td>$T_{R,bh}$</td>
<td>($^\circ$C) Average temperature at the borehole wall</td>
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<td>$T_{f,gs}$</td>
<td>($^\circ$C) Average heat carrier fluid temperature in the GSHE</td>
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<td>($^\circ$C) GSHE inlet fluid temperature</td>
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<td>$\Delta T_{bh}$</td>
<td>($^\circ$C) Fluid temperature difference across the BHE (Outgoing - Ingoing)</td>
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<td>($^\circ$C) Fluid temperature difference across the GSHE (Outgoing - Ingoing)</td>
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<tr>
<td>$\dot{V}_{bh}$</td>
<td>(m$^3$/s) Fluid flow rate through the BHE</td>
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<td>$\dot{V}_{gs}$</td>
<td>(m$^3$/s) Fluid flow rate through the GSHE</td>
</tr>
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<td>$f_{bh}$</td>
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<tr>
<td>$f_{gs}$</td>
<td>Darcy friction factor of the GSHE</td>
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<td>$l_{bh}$</td>
<td>(m) Length of the BHE pipe ($2L_{bh} + 2D_t$ – equivalent to a single U-pipe)</td>
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<td>$q_{bh}(t)$</td>
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<td>$q_{gs}(t)$</td>
<td>(W/m) GSHE average heat load per unit length at time $t$</td>
</tr>
<tr>
<td>$t$</td>
<td>(s) Time</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>(s) Time step size</td>
</tr>
</tbody>
</table>
\( \lambda_{bh} \) (W/(m·K)) BHE bedrock thermal conductivity

\( \lambda_{gs} \) (W/(m·K)) GSHE soil thermal conductivity

\( D \) (m) GSHE pipe/hose burial depth

\( R_{c,bh} \) Defined in (A.6)

\( R_{c,gs} \) Defined in (A.7)

\( R_{past,bh} \) Defined in (A.8)

\( R_{past,gs} \) Defined in (A.9)

\( R_{s,gs} \) Defined in (A.12)

\( \tilde{q}_{gs} \) (W/m) GSHE average heat load per unit length, defined as in (2.10)

\( a_{bh} \) (m\(^2\)/s) BHE bedrock thermal diffusivity

\( a_{gs} \) (m\(^2\)/s) GSHE soil thermal diffusivity

\( f_{den} \) Defined in (A.5)

\( r_b \) (m) Borehole radius

\( r_h \) (m) GSHE pipe/hose outer radius
Chapter 1

Introduction

The work presented here was done for and in cooperation with Danfoss Värmepumpar AB, as well as for fulfilling the last requirements for the authors M.Sc degree. It was completed during a period of 20 weeks between January and June 2015.

1.1 Background

1.1.1 Danfoss Värmepumpar AB

Danfoss Värmepumpar AB, based in Arvika, Sweden, is a developer and manufacturer of heat pump systems used in both heating and cooling applications, most known for heat pumps sold under the brand name "Thermia". Danfoss Värmepumpar AB is one of the leading companies on the European market for heat pump systems, with a revenue of about 600 Mkr, and about 200 employees.

1.1.2 Ground source heat pump systems

Ground source heat pump (GSHP) systems are common in Sweden and are mainly used for the purpose of heating single-family houses. They use thermal energy stored in the ground during warmer days to heat the inside of a building during colder days and also to generate domestic hot water. For a simple sketch of GSHP system operation, see Figure 1.1.

The heat is extracted from the ground by circulating a heat carrier fluid (brine) through a closed loop of plastic pipes installed in a deep borehole in the case of a vertical ground-coupled heat exchanger (borehole heat exchanger, BHE) or buried horizontally in the top few meters of soil (ground surface heat exchanger, GSHE). The former type of ground-coupled heat exchanger are more expensive to install, but generally performs better in colder climates, while the latter might be much more economical in warm enough climates. The GSHP systems may also operate in reverse, moving heat from inside a building to the ground or bedrock during hot days.
1.1.3 Basic heat pump operation

The basic main functionality of a GSHP is to use supplied work (produced through the use of electrical energy) to transport several times that energy amount in the form of heat from a colder reservoir at temperature $T_C$ (the ground surface soil or bedrock) to a hotter reservoir at temperature $T_H$ (the indoor radiator system), see Figure 1.2 for a simple schematic.

\[
\text{Figure 1.2: A simple diagram of a heat pump operation. Input work } W \text{ is used to lift a heat amount } Q_C \text{ from a lower temperature reservoir to a higher temperature reservoir, which will receive a heat amount } Q_H = Q_C + W. \]

The ratio between the output heat and the input work (electrical energy energy spent) is known as the heat pump coefficient of performance, $COP_{HP}$,

\[
COP_{HP} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C} = \frac{1}{1 - \frac{Q_C}{Q_H}}. \tag{1.1}
\]

The higher the COP is, the more efficient the GSHP is. For a more efficient heat pump, a larger portion of the supplied heat comes from the cold reservoir, and a lower portion from the supplied work. The theoretical maximum $COP_{HP}$ is obtained for a reversible heat pump cycle (Carnot cycle)\(^1\), and may be calculated as follows:

\[
COP_{HP, rev} = \frac{1}{1 - \frac{T_C}{T_H}}. \tag{1.2}
\]
This means that for a reversible heat pump (a fictional, most efficient heat pump), the $COP_{HP}$ is maximized by using the least possible temperature difference between the hot and the cold reservoir, and this is also true for real heat pump.

1.1.4 Using the ground as a source

In a heat pump system, the hot reservoir temperature will be fixed for a certain outdoor temperature, matching the supply line temperature required by the radiator system to keep a comfortable indoor air temperature. The cold reservoir temperature may vary greatly depending on what is used as a source. Using an outdoor air convector as a source, for example, means that the cold reservoir temperature will be at its lowest when the heating requirement will be at its highest – during the coldest winter days – which can be very detrimental. It means that the COP will low as heating is most needed, resulting in a poor annual efficiency.

Because of thermal inertia, the ground surface soil at a few meters depth (1.5-2 m) maintain a more steady temperature, without the cold spikes of the outdoor air, and the bedrock at a larger depth (>10 m) maintain a virtually constant temperature during the year equal to the annual average temperature. This is the great advantage of using the ground as a heat source, since the higher cold reservoir temperatures during winter lead to a better $COP_{HP}$ during the time when the heating requirements is at its highest.

Why is the outdoor air ever used as a source when the efficiency during cold days is worse? The answer is that whether an outdoor air convector is a good alternative or not will depend on the climate. For colder climates it will become relatively worse, compared to a ground-coupled heat exchanger. There are also limitations on where ground-coupled heat exchangers may be used, due requirements on the thermal conductivity of the top soil or bedrock. Another issue might be lack of space, especially in the case of horizontal ground-coupled heat exchangers, which require a large and preferably flat area. The biggest deterrent for ground-coupled heat exchangers systems, however, may be the large one time installation costs for drilling or digging.

A deeper borehole or longer ground surface loop generally yields better performance, but the installation costs becomes steeper, and it might not be economically justifiable above a certain limit. A BHE is especially expensive to install, because of the drilling involved, making a GSHE more economically preferable if it is an alternative, except for in very cold climates.

The thermal energy extracted from the soil or bedrock is recovered through heat transfer from the air, solar irradiation and heat transfer from the hot core of the earth. It may also be recovered to some extent by running the heat pump in reverse mode during summer, depending on the ratio between the heating and cooling loads.

For a constant heat extraction rate, a typical GSHE will reach it’s steady-state – the state when the rate of supplied heat from the surroundings equals the extraction rate – after about a month. The steady-state for a borehole will not be reached until after about 25 years, meaning that the temperature of the bedrock will be lowered for each year up to the 25th, where the final temperature will depend on the size on the constant heat load and the depth of the borehole. A smaller borehole and larger heat load will cause a lower bedrock temperature.

1.1.5 Dimensioning the heat exchangers

A proper model which can predict the behaviour of a GSHP system using a GSHE or BHE may be used to find the minimum borehole depth/loop length for which satisfactory performance will still be
retained, a procedure often referred to as "dimensioning". Here "satisfactory performance" means for example that the temperature of the fluid circulating in the ground-coupled heat exchanger does not go below the lowest operating temperature for the heat pump and that the annual efficiency of the system will be enough to justify its existence. Other factors may also weigh in, such as effects on the local biosphere. Examples of existing models that may be used to dimension individual vertical and horizontal ground-coupled heat exchangers are presented in Chapter 2.

1.1.6 Using a complementary heat source

An emerging problem today is that ground source heat pumps installed 15-20 years ago are beginning to reach the end of their lifetimes, whereas the ground-coupled heat exchangers used as a source have a lifetime of about 50 or even 100 years. During the last two decades, the efficiency of heat pumps have been significantly improved. Two important contributors to this was the introduction of the very efficient scroll compressor and the use of multi-stage compression cycles.

Because of the increased efficiency, more of the heat ejected by the heat pump into the radiator system will come from the ground sources, and less from the heat pump itself, which means that the heat load on the existing ground-coupled heat exchanger will be increased. The old ground source, be it horizontal or vertical, may be under-dimensioned to cope with an increased heat extraction rate, and might not be able to supply the heat needed to fully utilize the new heat pump without the help of a complementary source.

Another effect of the increased efficiency of newer heat pumps is that it has become economically preferable to let the GHSP system cover a larger portion of the annual household heating requirements. Old less efficient GSHP systems were designed to provide about 70% of the net required heat, whereas with the newer more efficient heat pumps it may be advantageous to let the GSHP system cover up to about 95%. The exact break point will depend on the price of other sources such as electrical heating. The benefits of allowing the GSHP system to carry larger portion of the heating requirement is yet another reason to use a complementary source when replacing an old heat pump.

For an already installed BHE, a complementary source would also help to alleviate the annual average heat load on the BHE, and should thereby reduce the long term temperature drop in the bedrock. This effect have already been studied for the case of adding a complementary outdoor air convector to an existing BHE. The set-up was shown to improve the long term annual COP of the heat pump system when the borehole was allowed to recharge passively during the warm part of the year. It also reduced the long term temperature drop in the ground surrounding the exchanger, as expected, with increased effect for multiple-borehole BHEs.

Instead of using an outdoor air convector as a complement, another ground-coupled heat exchanger of a different type could be used to increase the heat load capacity. It might be preferable to add a second source of a different kind, either because there is no space to add another GSHE to an existing GSHE, or because it is cheaper to add a GSHE to an existing BHE than to drill another borehole. There is an issue, however – an issue that the work presented here is attempting to address: Plenty of models have been developed to describe individual horizontal and vertical ground-coupled heat exchanger systems – even multiple systems of the same kind in the case of boreholes – but there is a lack of models describing a system of two ground-coupled heat exchangers of different types.
1.2 Scope of the thesis

An analytical model has been developed, which have then been used in computer simulations in order to study the behaviour of a system consisting of a BHE and a GSHE connected in parallel to the same heat pump. The model is based on previously existing models for individual BHEs and GSHEs. Simulations were also performed for the purpose investigating how different parameters affect the performance of the combined system. The effects of viscosity and geometry on fluid flow rate distribution between the two exchangers was also studied, as well as the effects of controlling the fluid flow rates through each of the exchangers. The short-term effects of letting the BHE rest during summer for different combination cases was also investigated. Lastly, a rule of thumb that, in lack of more realistic models, have been used in the industry to dimension combined systems of GSHEs and BHEs is evaluated by comparing it with the more realistic analytical model.

1.3 Principal Findings

Surprisingly, the results show that in some cases a combination might perform worse than the previously existing heat exchanger alone, if the previous exchanger is a BHE. The effects of the viscosity on the natural fluid flow distribution between the two exchangers is small, and is increasingly negligible for an increased net flow rate. There is no tangible gain in controlling the fluid flow rate distribution unless the pipe lengths of the two exchangers differ by a lot. Letting the BHE rest during summer might in some cases provide both long-term and short-term benefits. The rule of thumb was shown to always recommend smaller dimensions than the analytical model for a given minimum fluid temperature at the evaporator of the heat pump, indicating that it is overestimating the performance of a combined system.
Chapter 2

Modelling the individual heat exchangers

There are two major lines of approach for modelling ground-coupled heat exchangers; analytical and numerical. Analytical solutions generally yield simple formulas that may be used in computer programs for very fast calculations. Numerical solutions typically involve the use of a finite element or finite difference method, which has better accuracy, but far greater computational times. Another advantage of analytical solutions is that it’s easier to change parameters such as the geometries of the modelled system, while such a change would mean a lot of work in the case of a numerical finite element or finite difference method.

Examples of important analytical models are the infinite line source model presented by Ingersoll et al in 19545 and the cylindrical source model presented by Carslaw and Jaeger6 in 1959. Both methods involve integral functions which may be further simplified through the use of approximate algebraic expressions or tabular values.

An analytical model for horizontal ground-coupled heat exchangers based on the infinite line source model, was presented by Claesson and Dunand in 19837.

In 1987, Eskilson8 presented a semi-analytical approach for modelling systems of multiple boreholes by using numerically calculated so-called dimensionless g-functions for an array of different borehole field geometries. These were obtained through the superposition of numerical solutions for individual boreholes, and once known, could be stored in a database and reused at will.

An example of a purely numerical model is the one presented by Muraya in his PhD Thesis in 19949, where made use of non-linear finite element methods. Other numerical models was presented by Al-Khoury in 2005/200610 for steady-state and transient state, respectively.

One of the biggest drawbacks of most existing analytical models is that simplifications make them so crude that they are only well suited for calculating long term responses to heat extraction or injection. An attempt to improve the short-term precision was presented by Javed12 in 2012, but for applications such as dimensioning heat exchangers for use by single family houses a bad short-term precision is not necessarily a great hindrance. In fact, considering the superior calculation speeds and adaptability, such analytical models are commonly used in modern energy design software.

In Section 2.2 a simple analytical model will be presented, that may be used for determining suitable dimensions for a single borehole heat exchanger. In Section 2.3 a similar model that may be used for dimensioning a ground surface heat exchanger will be introduced. Both are based on the work of Claesson7,13 and Eskilson8 and these two models will provide the basis for a combination model.
2.1 General simplifications

The following simplifications have been made for all of the models presented in this thesis:

- Effects of soil or ground water freezing have been ignored.
- The effect of a potential snow cover on the undisturbed ground temperature have been ignored.
- Potential ground water movements have been ignored.

Freezing of the ground water would reduce the borehole resistance discussed in Section 2.2.5, and thereby lead to an increased performance of a BHE (higher fluid temperatures). It might also in rare circumstances cause difficulties by squeezing off the fluid flow through the pipes inserted into the borehole. This is discussed in more detail in a master thesis by Ahlström\textsuperscript{14}. For a numerical investigation on the effect of freezing and snow cover on ground temperature, see the work of Ling and Zhang\textsuperscript{15}. In an article from 1982 by Goodrich\textsuperscript{16} it's concluded that snow coverage may increase the average annual ground temperature by several degrees Celsius. Eskilson\textsuperscript{8} concludes in his PhD Thesis that in most cases the effect of ground water movement on performance should be negligible.

2.2 The single BHE model

A typical borehole heat exchanger consists of one or several U-shaped plastic tubes with a weight at the bottom which have been inserted into a drilled borehole which is 50-250 meters deep and with a radius $R_0$ of about 0.05 m. The top few meters are normally enveloped by an insulated casing, penetrating at least 2 meters into the bedrock to prevent ground water contamination. To extract heat from the bedrock, a heat carrier fluid is pumped down through one leg of the U-pipe and up the other. Commonly used heat carrier fluids – also known as 'brine' – are water with a high concentration of salt, or a mix of water and alcohol. Both mixture types gives a below zero freezing point which is necessary for operation in cold climates. The alcohols used may be ethanol, propanol or ethylene glycol, but ethylene glycol is not allowed in the ground in many countries. Thermal contact between the U-shaped plastic tubes and the borehole wall is established either by naturally occurring ground water (common in Sweden) or a by a filling of grout with high thermal conductivity such as bentonite. Grout can also be used for the purpose of reducing ground water movement between different depths. A picture of a BHE being installed may be seen in Figure 2.1

Now a model will be presented that may be used to obtain the variation of heat carrier fluid temperature $T_f$ for a single borehole heat exchanger, given a specified heat extraction schedule. This may in turn be used to obtain an active borehole depth $H$ for which $T_f$ is above a required minimum for the whole of its expected lifetime. The model is based on the work of Claesson \textit{et al.}\textsuperscript{13} and handles the problem in two steps: First the temperature at the borehole wall, $T_R$, is calculated, and then the temperature drop between the borehole wall and heat carrier fluid is treated as a separate problem.
\subsection*{2.2.1 The heat equation}

The main mechanism of heat transfer involved is conduction. The governing equation for this in a homogeneous isotropic medium is

$$\nabla^2 T = \frac{1}{a} \frac{dT}{dt}, \quad a = \frac{C_p}{\lambda},$$

(2.1)

where

\(\lambda\) is the thermal conductivity of the medium (W/(m·K))
\(C_p\) is the volumetric heat capacity (J/(m³K))
\(a\) is the thermal diffusivity (m²/s)

\subsection*{2.2.2 Assumptions and simplifications}

In reality a borehole heat exchanger is a rather complex system. Thus, in order to obtain a concise analytical solution, some assumptions and simplifications are required:

\(i\) The full depth of the borehole is \(H + D_i\) where \(H\) is the active depth of the borehole (the part which lies under the ground water/grout level) and \(D_i\) is the distance from the ground water/grout level to the surface (see Figure 2.2). The whole of \(D_i\) is assumed to be insulated and does not contribute to any heat extraction or injection.

\(ii\) The ground will be treated as a homogeneous medium, with a constant heat conductivity and volumetric heat capacity equal to the average values between \(D_i\) and \(D_i + H\). In reality there may be inhomogeneities, for example a depth varying conductivity due to stratification of quartz levels in the bedrock.

\(iii\) The temperature at the borehole wall \(T_R\) is assumed to be constant and equal to the mean temperature along the active borehole depth. In reality the temperature will differ with depth due to varying fluid temperature along the U-pipe and the geothermal gradient (an increase in temperature with depth caused by the hot core of the earth).

\(iv\) The thermal properties of the top soil is different from that of the bedrock, but this is ignored.
v) The undisturbed ground temperature $T_{0m}$ is assumed to be constant along the depth of the borehole, its value fixed as

$$T_{0m} = T_0 + \alpha \cdot (D_i + H/2), \quad (2.2)$$

where

- $\alpha$ is the local geothermal gradient ($^\circ$C/m)
- $T_0$ is the annual average air temperature ($^\circ$C)

At only a few meters beneath the surface, any short term temperature variations is greatly reduced due to the thermal inertia of soil and bedrock. Therefore this assumption mainly simplifies the effect of the geothermal gradient.

vi) The net heat extracted from the borehole is

$$Q(t) = q(t) \cdot H, \quad (2.3)$$

where

- $q(t)$ is the heat load per unit length at time $t$ (W/m)

It will be assumed that $q(t)$ is constant along the active borehole depth and equal to the true average value. In reality it will differ with depth.

Figure 2.2: Schematic of a borehole with important variables\textsuperscript{13}. 

Figure 2.2: Schematic of a borehole with important variables\textsuperscript{13}. 

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2.2.3 Solution for a single heat extraction step

The solution for a step function with constant heat extraction beginning at \( t = 0 \),

\[
q(t) = \begin{cases} 
q_1 & \text{for } t \geq 0 \\
0 & \text{for } t < 0
\end{cases}, \tag{2.4}
\]

is according to Claesson et al.\textsuperscript{13} approximately

\[
T_R(t) = \begin{cases} 
T_0 \left( m - \frac{q_1}{4\pi\lambda} \left( \ln \left( \frac{4at}{R_0^2} \right) - \gamma \right) \right) & \text{for } \frac{5R_0^2}{a} < t < \frac{H^2}{9a} \\
T_0 - \frac{q_1}{2\pi\lambda} \ln \left( \frac{H}{2R_0} \right) & \text{for } t > \frac{H^2}{9a}
\end{cases}, \tag{2.5}
\]

where

\[
\gamma = 0.5772 \text{ is Euler's constant.}
\]

The first expression is the transient, purely radial solution, based on Ingersoll’s\textsuperscript{5} infinite line source model, while the second yields the temperature once an equilibrium state has been reached (normally after about 25 years). The factors in \((2.5)\) by which \( q_1/2\pi\lambda \) is multiplied are referred to as the thermal response function \( g_\ast \), by Claesson et al.\textsuperscript{13} and Eskilson\textsuperscript{8}. In a comparison made by Claesson et al.\textsuperscript{13} and Eskilson\textsuperscript{8} between more exact, numerically calculated values for the \( g \)-function and the approximative expressions in \((2.5)\), the error was greatest at the breakpoint time (see Figure 2.3). In an example it was determined to be about 7%.

![Figure 2.3: A comparison\textsuperscript{13} between the approximative expressions in (2.5) (dashed lines) and numerically calculated values for the \( g \)-function (solid line). The maximum difference of about 7% occurs at the break point time, \( t = t_1 \)](image)

It should be mentioned that Ingersoll recommends the use of the line source model only for \( t > \frac{20R_0^2}{a} \), but Claesson et al. uses the model from the lower limit of \( t > \frac{5R_0^2}{a} \). Claesson et al.’s lower limit is based on when the exponential integral in Ingersoll’s solution may be approximated
with a simple algebraic expression with a maximum error of about 2%, while the limit given by Ingersoll is the lower limit for when the line source method gives reasonable results for a source that in reality is not a line. One study\textsuperscript{18} claims that the error is about 10% for the lower limit and about 2.5% for Ingersoll's limit.

### 2.2.4 Solution for a multi-step heat extraction function

For an arbitrary multi-step heat extraction function with \( n \) steps, each step being active only during a time period \( t_i < t < t_{i+1} \), superposition of the transient solution gives:

\[
T_R(t) = T_{0m} - \sum_{i=0}^{n} \frac{q_i - q_{i-1}}{4\pi\lambda} \left( \ln \left( \frac{4a(t - t_i)}{R_0^2} \right) - \gamma \right), \quad t_{n+1} \geq t > t_n + \frac{5R_0^2}{a}, \quad q_{-1} = 0. \tag{2.6}
\]

To understand this superposition formula, imagine that for each time step \( i \), the next heat load step \( q_i \) begins, as well as the negative of the previous heat load step, \( -q_{i-1} \). Equivalently one may imagine that for each heat load step, \( q_i \), beginning at \( t = t_i \), a heat load of \( -q_i \) starts at \( t = t_{i+1} \), leading to the following formula:

\[
T_R(t) = T_{0m} - \frac{q_n}{4\pi\lambda} \left( \ln \left( \frac{4a(t - t_n)}{R_0^2} \right) - \gamma \right) - \sum_{i=0}^{n-1} \frac{q_i}{4\pi\lambda} \ln \left( \frac{t - t_i}{t - t_{i+1}} \right), \quad t_{n+1} \geq t > t_n + \frac{5R_0^2}{a}. \tag{2.7}
\]

In order to take long-term effects into account when having a multi-step heat load function, the following method is suggested:

1. Let the break-time at which steady-state is reached for a single-heat extraction step be denoted \( t_s \).

2. When calculating \( T_R \) at time \( t \), only take into account heat load steps active after the time \( t - t_s \). Any heat load steps before \( t - t_s \) should cause a near zero temperature change at time \( t \).

3. Divide the the multi-step heat load function in the time interval \([t - t_s, t]\) into two parts:
   - i) One constant single heat load step equal to the average in the interval.
   - ii) One multi-step function equal to the deviations from the average value in the interval.

4. Use the steady-state thermal response function in row two of (2.5) for Part i) and the summation formula in (2.7) for Part ii).

The described method should result in an accurate formula for the case of a borehole, which have a typical steady-state time of about 25 years. If a typical annual heat load function is used for each year, the annual mean heat load should be the same for each of the 25 years before the steady state is reached, making the use of a formula for a constant heat load appropriate. The seasonal variations on the other hand would only last a very small fraction of the steady state time, and would only cause a local temporary disturbance, making it suitable for a the transient superposition formula.

If only heat load steps beginning after \( t - t_s \) is included, the formula then becomes

\[
T_R(t) = T_{0m} - \frac{\bar{q}}{2\pi\lambda} \ln \left( \frac{H}{2R_0} \right) - \frac{q_n(t)}{4\pi\lambda} \left( \ln \left( \frac{4a(t - t_n)}{R_0^2} \right) - \gamma \right) - \sum_{i=m}^{n-1} \frac{q'_i(t)}{4\pi\lambda} \ln \left( \frac{t - t_i}{t - t_{i+1}} \right), \tag{2.8}
\]

\[
t_{n+1} \geq t > t_n + \frac{5R_0^2}{a}, \quad t_m > t_{n+1} - t_s > t_{m-1}
\]
where

\[ q_j' = q_j - \tilde{q}, \quad (2.9) \]

\[ \tilde{q} = \sum_{i=m-1}^{n-1} \frac{t_{i+1} - t_i}{t_{n-1} - t_{m-1}} q_i. \quad (2.10) \]

The average heat load calculation includes the last step before \( t - t_s \), \( q_{m-1} \), but not \( q_n \). This is because it is desirable to have the average heat load independent of where the interval \([t_n + \frac{5R_0^2}{a}, t_{n+1}]\) we choose \( t \).

### 2.2.5 Heat carrier fluid temperature, \( T_f \)

Once an expression for \( T_R \) has been obtained, it remains to find the average heat carrier fluid temperature, \( T_f \). For a given borehole wall temperature, the fluid temperature depends both on the flow velocity and thermal properties of the fluid, as well as the geometry and thermal properties of the borehole itself (including both the ground water/grout and the pipe/pipes). This relation may be simply expressed as

\[ T_R - T_f = R_b q, \quad (2.11) \]

where

- \( R_b \) is the borehole thermal resistance (m·K/W).

The value of the borehole resistance includes all of the above mentioned factors and in general depends on time. The simple formulation of (2.11) is only possible if it is assumed that the fluid temperature is constant along both the legs of the U-pipes, and equal to the true average value of the fluid temperature. In reality the fluid temperature will vary between the inlet and outlet which should cause heat to be transferred from the hotter leg to the colder leg, but the approximation used have been compared\(^{13}\) to numerical calculations and will suffice for the purposes of this thesis.

### 2.2.6 Calculating the borehole resistance

The problem may be treated as 2-dimensional with a uniform cross section along the length of the borehole. In the work of Claesson et al.\(^ {13}\), the variable corresponding to \( R_b \) is denoted \( m_R \). A formula for calculating the borehole resistance is presented in (2.12). It’s the result of using Claessons et al.’s formula for \( m_R \) with the special case of \( b_1 = b_2 = b \) and \( b_{12} = 2b \). This corresponds to a single U-tube, with the down-going and up-going pipe being placed with their cross-sectional center at distance \( bR_0 \) from the borehole center, on opposite sides (see Figure 2.4). Other arrangements would result in a different formula, but for the described case we get

\[ R_b = \frac{1}{4\pi \lambda_w} \left[ \left( \ln \frac{R_0}{R_{p,0}} \right) - \frac{\lambda_w - \lambda}{\lambda_w + \lambda} \ln(1 - b^2) \right] - \left( \ln 2b + \frac{\lambda_w - \lambda}{\lambda_w + \lambda} \ln(1 + b^2) \right) + m_p' + m_f c \quad (2.12) \]

where

- \( \lambda_w \) is the thermal conductivity of the ground water/grout (W/(m·K))
- \( \lambda \) is the thermal conductivity of the bedrock (W/(m·K))
Figure 2.4: Cross section of a borehole.

\( R_0 \) is the borehole radius (m)

\( R_{p,o} \) is the outer radius of the pipe/hose (m)

\( b \) is the relative distance of the two pipes in terms of \( R_0 \) from the center of the borehole

\( m'_p \) is the thermal resistance between the outer and inner radius of the pipe (m·K/W)

\( m_{fc} \) is the thermal resistance between the fluid and and the inner pipe wall (m·K/W)

The thermal resistance across the pipe wall may be calculated as

\[
m'_p = \frac{1}{2\pi \lambda'} \ln \frac{R_{p,o}}{R_{p,i}}
\]  \hspace{1cm} (2.13)

where

\( \lambda' \) is the thermal conductivity of the pipe wall (W/(m·K))

\( R_{p,i} \) is the inner radius of the pipe/hose (m)

and thermal resistance between the fluid and inner pipe wall may be calculated as

\[
m_{fc} = \frac{1}{\pi \lambda_w \text{Nu}}
\]  \hspace{1cm} (2.14)

where

\( \text{Nu} \) is the average Nusselt number along the pipe

In reality it would be difficult to maintain a certain fixed distance between pipes for the whole depth of the borehole, or to know exactly the distance at each depth. However, for calculations purposes an exact geometry needs to be specified. For more advanced analysis concerning the borehole resistance, see the work done by Claesson and Hellsöm.
2.2.7 Calculating the Nusselt number

The Nusselt number is in essence a ratio between the convection and conduction part of heat transfer in cases when both are involved. It depends on the Reynold’s number:

\[
\text{Re} = \frac{\rho_f D_p \dot{V}_f}{A_p \mu_f},
\]  

(2.15)

where

- \(\mu_f\) is the dynamic viscosity of the fluid evaluated at the bulk temperature along the pipe (Pa·s)
- \(\rho_f\) is the density of the fluid (kg/m\(^3\))
- \(A_p\) is the cross-sectional area of the pipe through which fluid can flow (m\(^2\))
- \(\dot{V}_f\) is the fluid flow rate through the pipe (m\(^3\)/s)

which is a ratio of inertial forces to viscous forces, determining the type of flow (laminar or turbulent).

For turbulent flow the Nusselt number also depends on the Prandtl number:

\[
\text{Pr} = \frac{\mu_f c_{p,f}}{\lambda_f}
\]  

(2.16)

where

- \(\lambda_f\) is the thermal conductivity of the fluid (W/(m·K))
- \(c_{p,f}\) is the specific heat of the fluid (J/(kg·K))

which is a ratio of momentum diffusivity to thermal diffusivity. The Nusselt number for turbulent flow also depends on the Darcy friction factor, according to the formulas presented by Gnielinski\(^{20}\) in 1976. The Darcy friction factor may be calculated using the following formula:

\[
f = \begin{cases} 
  f_A = \frac{64}{\text{Re}}, & 0 < \text{Re} < 2300 \\
  \frac{(3000 - \text{Re})f_A + (2300 - \text{Re})f_B}{3000 - 2300}, & 2300 < \text{Re} < 3000 \\
  f_B = (0.79 \ln \text{Re} - 1.64)^{-2}, & 3000 < \text{Re} < 5 \cdot 10^6, \text{ Gnielinski}^{20}
\end{cases}
\]  

(2.17)

The expression on the second row is a weighted average of the expression on the first row and the third row. This will be used as an approximation for 2300 <\text{Re}< 3000, since no valid formula could be found for this interval when performing a search. A plot of the Darcy friction factor calculated according to (2.17) can be found in Figure 2.5.

Now, using the recently presented expressions, the Nusselt number may be calculated as follows:

\[
\text{Nu} = \begin{cases} 
  \text{Nu}_A = 4, & 0 < \text{Re} < 2300 \\
  \frac{(3000 - \text{Re})\text{Nu}_A + (2300 - \text{Re})\text{Nu}_B}{3000 - 2300}, & 2300 < \text{Re} < 3000 \\
  \text{Nu}_B = \frac{(f/8)(\text{Re} - 1000)\text{Pr}}{1 + 12.7*(f/8)^{0.5}(\text{Pr}^{2/3} - 1)}, & 3000 < \text{Re} < 5 \cdot 10^6, \text{ Gnielinski}^{20}
\end{cases}
\]  

(2.18)
A weighted average of the first and third row expressions are used here as well, due to the same reasons mentioned in the case of the Darcy friction factor. The Nusselt number can not be plotted as easily as were the friction factor; it depends on multiple variables \((f, \Re, \Pr)\). The value 4 in the first row of (2.18) is taken from an example where Claesson et al.\textsuperscript{13} uses it for the case of laminar flow.

### 2.3 The single GSHE model

Horizontal ground-coupled heat exchangers, here called ground surface heat exchangers, usually consists of a series of parallel hoses/pipes buried in the top soil 0.6-1.4 m below the surface. The pipes are generally buried deeper in cold climates, and shallower in warmer climates. Heat is extracted from the soil by circulating a heat carrier fluid through the pipes, similarly to the borehole heat exchanger, and the resulting lowered soil temperature will in turn cause the soil to act as a solar or air heat collector. It is assumed that the net heat extraction throughout a year will be completely compensated by solar irradiation and heat from the surrounding air. No long term temperature changes to the soil will occur. One important aim when dimensioning a GSHE is to not have too large short term temperature changes, since this can cause various problems such as disturbance of the local biosphere and visible height variations on the surface. A picture of a GSHE being installed may be seen in Figure 2.1

Now a model will be presented that can be used to obtain the fluid temperature, \(T_f(t)\), as a function of a given heat load, \(Q(t)\) for a GSHE system consisting of a number of parallel hoses. The model is based on the work of Claesson et al.\textsuperscript{13,7}.
2.3.1 Assumptions and simplifications

Similarly to the case of the borehole heat exchangers, an array of assumptions and simplifications will be made:

i) The soil will be treated as a homogeneous medium, with a constant heat conductivity and volumetric heat capacity, equal to the average value.

ii) The temperature of the ground nearest the hose/pipe $T_R$ is assumed to be constant along the pipe and equal to the average value. In reality it will differ along the pipe/pipes.

iii) The effects of raining and ground water movement will be ignored. Both these effects improves the performance of the heat exchanger, which means that ignoring them will give an error on the pessimistic side.

2.3.2 Dividing the problem into parts

The problem may be divided into parts which are solved separately. The separate solutions are then superposed to obtain the solution of the complete system. The procedure is as follows:

a) Let the heat extraction at all of the parallel hoses be zero, and find the undisturbed ground temperature field.

b) Let the undisturbed ground temperature be 0 °C everywhere at $t = 0$ and always be 0 °C at the surface ($z = 0$). Find the static temperature field due to the average heat extraction rate $\tilde{q}_i$ at hose $i$. This is equivalent to obtaining the steady-state temperature field for a constant heat extraction rate $q_i = \tilde{q}_i$, while setting $q_j \neq i = 0$ for hose $j \neq i$. See Figure 2.7 for the case of one hose and a constant heat extraction $q_1$. This procedure will yield the shift from undisturbed conditions which may then be added to the undisturbed temperature field. The second condition also states that there will never be any temperature shift due to extraction at the surface, $z=0$. For multiple hoses, the shifts due to the individual hoses are added together to obtain the total static temperature field.

c) Find the dynamic temperature field around hose $i$ due to the time-dependent deviation from average heat extraction rate, $q'_i(t) = q_i(t) - \tilde{q}_i$. The dynamic field around a hose can be seen
Figure 2.7: A schematic\textsuperscript{13} of the single hose case with a step extraction $q_1$.

independent of other hoses if the heat extraction deviation is sufficiently short term (dependent on hose separation distance).

d) Superpose the solutions obtained from a), b) and c). Since the undisturbed ground temperature is set to 0 °C in b), the static temperature field will be equal to the shift from undisturbed conditions. Similarly the dynamic temperature field obtained in c) will be equal to the shift relative to the static field.

It should be stressed that superposition of solutions is only allowed if (2.1) is linear, which means that it’s not applicable for scenarios where the soil is partially and temporarily frozen, yielding local and time-dependent variations in the diffusivity, $a$. Ignoring freezing will underestimate the performance of the GSHE, according to numerical study by Gan, 2013\textsuperscript{21}.

\subsection*{2.3.3 Calculating the undisturbed ground temperature, $T_{0s}(t, z)$}

Since the burial depths of the pipes/hoses of a GSHE is only between 0.6-1.4 m, it cannot simply be assumed that the ground temperature will always be equal to the annual average air temperature and that the oscillations at the surface are completely damped out as was the case with the BHE. Instead there will be a damped temperature oscillation in the ground, with increased dampening as the depth increases. There will also be a phase delay because of the thermal inertia of the soil which increases with depth.

The problem that needs to be solved is that of one-dimensional conduction in a semi-infinite homogeneous medium with a periodic temperature boundary condition at the surface. In the case of a simple harmonic,

\[ T_{0s}(t, 0) = T_{\text{air}}(t) = T_0 - A \cdot \cos \left( \frac{2\pi t}{t_0} \right) \]

where

$t_0$ is the period of the temperature variation at the surface (typically a year)(s)
$A_1$ is a constant determining the oscillation amplitude ($\degree C$)

$T_0$ is the annual average air temperature ($\degree C$)

the resulting analytical solution, given in various sources$^{13, 7, 22}$ is

$$T_{0s}(t, z) = T_0 - A \cdot e^{-z/d_0} \cdot \cos\left(\frac{2\pi t}{t_0} - \frac{z}{d_0}\right)$$  \hspace{1cm} (2.20)

where

$d_0$ is the penetration depth (m)

The penetration depth is

$$d_0 = \sqrt{\frac{at_0}{\pi}}. \hspace{1cm} (2.21)$$

It’s common to use data for a typical year repetitively for multi-year simulations. Given exact air temperature data for a sample year, one may fit a Fourier series to the data, using the least square method. This Fourier series is just a superposition of the boundary condition in (2.19) with terms having different periods $\frac{t_0}{n}$, $(n = 1, 2, 3, \ldots)$. The solution therefore becomes

$$T_{0s}(t, z) = T_0 - \sum_{n=1}^{k} e^{-z/d_{0,n}} \left[ A_n \cdot \cos\left(\frac{2\pi t}{t_0} - \frac{z}{d_{0,n}}\right) + B_n \sin\left(\frac{2\pi t}{t_0} - \frac{z}{d_{0,n}}\right) \right]$$  \hspace{1cm} (2.22)

where

$$d_{0,n} = \sqrt{\frac{at_0}{n\pi}} \hspace{1cm} (2.23)$$

and

$k$ is the order of the Fourier series

$A_n$ is the coefficient in front of the $n$th cosine term, obtained from the least square fitting of the Fourier series to the temperature data ($\degree C$)

$B_n$ is the coefficient in front of the $n$th sine term, obtained from the least square fitting of the Fourier series to the temperature data ($\degree C$)

This procedure with $k = 30$ was used on a data set containing the measured average daily temperatures in Uppsala, Sweden (2013)$^{23}$, shifted upwards with $+3\degree C$. The shift was added in order to achieve a climate where the effects of snow and freezing would be minimal, since the models presented will not take those factors into consideration. The resulting undisturbed ground temperature at a depth $z = 1$ m is presented in Figure 2.8. This semi-fictional temperature data was used to obtain the results in this report.

As the air temperatures rises and drops, there is a visible delay on the accompanying ground temperature rises and drops. There is also a very obvious smoothing effect for temperature spikes, since the soil has a high thermal inertia. Mathematically this can be explained by looking at the dampening factor in (2.22), which will cause an increased dampening effect for higher frequency oscillations $- d_{0,n}$ will get smaller as $n$ increases.
It should be mentioned here that, freezing would affect the undisturbed ground temperature solution, just as it would the heat extraction solutions. An increased conductivity here would reduce the thermal inertia of the ground, and thereby reduce the dampening and time delay of temperature variations occurring at the surface.

For another approach on how to calculate the undisturbed ground temperature, see the work by Cleall et al., where derivatives are also included in the surface boundary condition.

### 2.3.4 Solution for a single heat extraction step, single hose

For the sake of simplicity it will be assumed that there is no thermal interference between parallel hoses for neither static nor dynamic heat loads. This is equivalent to having a system consisting of only one long hose. Claesson et al. provides suggestions on how to handle the case of thermal interference between hoses, but the expressions are rather cumbersome and will not help the goal of this thesis. Their inclusion would not complicate much, only lead to larger expressions, which is not helpful. Their exclusion should not affect the general behaviour of a combined system.

The solution suggested by Claesson for a single heat extraction step for a single hose (see (2.4)) is very similar to that of the solution for the BHE:

\[
T_R(t) = \begin{cases} 
T_0s(t, D) - \frac{q_1}{4\pi\lambda} \left( \ln \left( \frac{4at}{r_h^2} \right) - \gamma \right) & \text{for } \frac{5r_h^2}{a} < t < \frac{1.78D^2}{a} \\
T_0s(t, D) - \frac{q_1}{2\pi\lambda} \ln \left( \frac{2D}{r_h} \right) & \text{for } t > \frac{1.78D^2}{a}
\end{cases}
\]

(2.24)

where
\( r_h \) is the outer pipe/hose radius, elsewhere denoted \( R_{p,o} \) or simply \( R_p \) (m)

\( D \) the burial depth of the pipes/hoses of the GSHE (m)
2.3.5 Solution for an arbitrary multiple-step heat extraction function, single hose

We may here use the same approach as in the case of the BHE, dividing the multiple-step heat extraction function into two parts, and only looking back to $t - t_s$ in order to calculate the temperature at time $t$.

A big difference here compared to the BHE is that the steady-state break time is about a month, which is shorter than the seasonal variations. Here one may argue that if we in the beginning of winter have a strictly increasing heat load during a time period of length $2t_s$, is it reasonable to use a formula that includes a steady-state solution obtained by assuming a constant heat extraction rate? The average heat extraction rate is obviously not the same across two adjacent intervals of length $t_s$. Yet somehow the heat flux from the surface and surrounding ground needs to be taken into account. It is reasonable to assume that the true value of the temperature drop obtained at a time $t_s$ after the beginning of a strictly increasing heat load should be bounded by the temperatures that would be obtained when using constant heat loads equal to the left or right hand values for the duration of the interval. It is also reasonable that the average heat load during the interval should affect the location of the true value between the two limits. Because of this, the formula in (2.8) will also be used for GSHE:s, replacing $T_{0m}$ with $T_{0s}(t, D)$, $\frac{H^2}{\pi a}$ with $\frac{1.78D^2}{a}$ and $\frac{H}{2r_0}$ with $\frac{2D}{r_h}$.

2.3.6 Heat carrier fluid temperature, $T_f$

Once $T_R$ is known, the average fluid temperature along the GSHE may be found through

$$T_f = T_R - qR_h$$  \hspace{1cm} (2.25)

where

$R_h$ is the thermal resistance between the ground outside the hose and the fluid (m·K/W)

This can be divided into three sub-parts: the ground-hose contact resistance $m_{cs}$, the hose resistance $m'_p$ and the hose-fluid resistance, $m_{fc}$:

$$R_h = m'_p + m_{fc} + m_c.$$  \hspace{1cm} (2.26)

It will be assumed that the thermal contact between the hose and the ground is perfect, meaning that $m_c = 0$. In reality it is normally not zero, and is a result of cracks in the soil, or imperfect filling when burying the hoses during installation. The other two components, $m'_p$ and $m_{fc}$ is calculated according to (2.13) and (2.14) respectively.
Chapter 3

Dual heat exchanger model

We do not know of any previous work where ground-coupled collectors of different kinds connected to a single heat pump have been studied. Therefore the ideas presented here are basically developed from scratch, the ultimate goal being a model that can describe how a combination of two ground source heat collector of different types will behave. The way of approach will be to attempt to utilize the models described in the previous chapter to obtain the heat carrier fluid temperature at the heat pump. These models give the fluid temperature as a function of the heat load and the problem is therefore mainly to find the heat load distribution, given a certain total required heat load function, $Q(t)$.

It will here be assumed that there is no thermal influence between the two heat exchangers except through the heat carrier fluid. The validity of this assumption should be studied in more detail. Any influence between the two exchangers should depend on the average distance between them. In an example calculation by Claesson et al.\textsuperscript{13} the maximum temperature drop at a depth of 1 m is around -0.1°C for a single BHE. This is reduced to less than -0.03°C at a radial distance of 20 m from the borehole, which suggests that the external thermal influence between should be minimal. It should be taken into account, however, that one of the recharging mechanisms of the borehole is heat transfer from the surface. This mechanism should be at least partially disrupted by a GSHE, which in turn may affect the long term steady-state temperature drop of the borehole.
3.1 Dual heat exchangers - Rule of thumb

Even though there is a lack of previous models, a rule of thumb that have been used by people in the industry will be presented in this section. This rule of thumb may be 'tested' within the scope of the here developed model.

The rule will be described for an example case of a GSHE being added to an already existing BHE. The scenario is that an increased net heat load $Q$ will cause the minimum fluid temperature $T_{f,min}$ to drop below the desired limit which we will denote $T_{lim}$, and this is why a GSHE will be added. The procedure then is as follows:

1) Calculate the active depth required for a pure BHE to meet the requirement of $T_{f,min} > T_{lim}$. This will be denoted $L_{bh,req}$.

2) Calculate the pipe length required for a pure GSHE to meet the same requirement of $T_{f,min} > T_{lim}$. This will be denoted $L_{gs,req}$.

3) Calculate the ratio $k = L_{gs,req}/L_{bh,req}$. This ratio will be used as an indicator of the relative value of GSHE pipe length to borehole depth.

4) The pipe length of the added GSHE may now be calculated according to

$$L_{gs,add} = k \cdot (L_{bh,req} - L_{bh,cur}) \quad (3.1)$$

where $L_{bh,cur}$ is the currently existing borehole depth.

The method thus simply tries to match a missing borehole depth to its corresponding GSHE pipe length, by using the relative value calculated in step 3). It does not take into account any form of interaction between the two systems, external or internal, which is why would be interesting to compare it to a more realistic model.
3.2 Dual heat exchangers connected in parallel

The system that is to be modelled here is that of a GSHE and a BHE connected in parallel by two T-junctions at the inlets and the outlets (see Figure 3.1). The fluid flow is driven by a circulation pump placed somewhere along the shared pipe section before and after the heat pump. The fluid flow rate through the heat pump, \( \dot{V}_{\text{tot}} \), is regulated to achieve a pre-set value for \( \Delta T_{HP} \), for example 3°C. As a result, conservation of energy gives that \( \dot{V}_{\text{tot}} \) becomes a function of the net heat load extracted from the fluid by the heat pump, \( Q \):

\[
\dot{V}_{\text{tot}} = \dot{V}_{gs} + \dot{V}_{bh} = \frac{Q}{\Delta T_{HP} C_f} \tag{3.2}
\]

where

\( \dot{V}_{bh} \) is the fluid flow rate through the BHE (m\(^3\)/s)

\( \dot{V}_{gs} \) is the fluid flow rate through the GSHE (m\(^3\)/s)

\( Q \) is the net required heat load, sum of \( Q_{bh} \) and \( Q_{gs} \) (W)

\( C_f \) is the volumetric heat capacity of the heat carrier fluid (J/(m\(^3\)K))

\[ T_{f,bh} = T_{bh,in} + \frac{1}{2} \Delta T_{bh} \]

\[ T_{f,gs} = T_{gs,in} + \frac{1}{2} \Delta T_{gs} \]

Figure 3.1: A simple diagram of a BHE and GSHE connected in parallel.

The net required heat load as a function of time, \( Q(t) \), is assumed to be a known input parameter and will in general depend on the building that needs to be heated as well as the outdoor air temperature. Through conservation of energy, we also have that
\[ Q(t) = Q_{gs}(t) + Q_{bh}(t) = L_{gs}q_{gs}(t) + L_{bh}q_{bh}(t), \] (3.3)

where

- \( L_{gs} \) is the length of the hose in the GSHE (m)
- \( L_{bh} \) is the active depth of the BHE (previously referred to as \( H \)) (m)
- \( q_{gs}(t) \) is the heat load per unit length on the GSHE (W/m)
- \( q_{bh}(t) \) is the heat load per unit length on the BHE (W/m)
- \( Q_{bh} \) is the heat load on the BHE (W)
- \( Q_{gs} \) is the heat load on the GSHE (W)

If the models of the previous chapter are to be useful, we must find \( q_{bh}(t) \) and \( q_{gs}(t) \). If one is found, then according (3.3) the other can be found as follows

\[ q_{gs}(t) = \frac{Q(t) - L_{bh}q_{bh}(t)}{L_{gs}}, \quad q_{bh}(t) = \frac{Q(t) - L_{gs}q_{gs}(t)}{L_{bh}}. \] (3.4)

Equation (3.4) links the heat loads on the two exchangers through conservation of energy. Next step is to find an equation which describes the relation between the average fluid temperatures of the two exchangers, because then equations (2.11) and (2.25) may be used to obtain a second equation with the current heat loads as unknowns.

Let’s begin with looking at Figure 3.1 and draw the conclusions that

\[ T_{gs, in} = T_{bh, in} = T_{HP, out} \] (3.5)
\[ T_{gs, in} = T_{f, gs} - \frac{1}{2}\Delta T_{gs}, \] (3.6)
\[ T_{bh, in} = T_{f, bh} - \frac{1}{2}\Delta T_{bh}, \] (3.7)

where

- \( T_{gs, in} \) is the GSHE inlet fluid temperature (°C)
- \( T_{bh, in} \) is the BHE inlet fluid temperature (°C)
- \( T_{HP, out} \) is the heat pump outlet fluid temperature (°C)
- \( T_{f, bh} \) is the average heat carrier fluid temperature in the BHE (°C)
- \( T_{f, gs} \) is the average heat carrier fluid temperature in the GSHE (°C)
- \( \Delta T_{bh} \) is the fluid temperature difference across the BHE (Outgoing - Ingoing) (°C)
- \( \Delta T_{gs} \) is the fluid temperature difference across the GSHE (Outgoing - Ingoing) (°C)
Using (3.5) to connect (3.6) and (3.7) we obtain the equality
\[ T_{f,bh} - \frac{1}{2} \Delta T_{bh} = T_{f,gs} - \frac{1}{2} \Delta T_{gs}. \] (3.8)

From (2.11) and (2.25) we have that
\[ T_{f,bh} = T_{R,bh} - q_{bh} R_b \] (3.9)
\[ T_{f,gs} = T_{R,gs} - q_{gs} R_h \] (3.10)

where
\( T_{R,bh} \) is the temperature at the borehole wall (°C)
\( T_{R,gs} \) is the temperature just outside the hose (°C)
\( R_b \) is the thermal resistance of the borehole (K·m/W)
\( R_h \) is the thermal resistance between the ground outside the hose and the fluid in the hose (K·m/W)

and using conservation of energy across each of the two exchangers:
\[ \Delta T_{bh} = \frac{Q_{bh}}{C_f V_{bh}} = \frac{L_{bh} q_{bh}}{C_f V_{bh}}, \] (3.11)
\[ \Delta T_{gs} = \frac{Q_{gs}}{C_f V_{gs}} = \frac{Q - L_{bh} q_{bh}}{C_f V_{gs}}, \] (3.12)

Expressing \( q_{gs} \) in (3.10) in terms of \( q_{bh} \) by using (3.4) and inserting (3.11), (3.12), (3.9) and (3.10) into (3.8) yields
\[ T_{R,bh} - q_{bh} R_b - \frac{L_{bh} q_{bh}}{2 C_f V_{bh}} = T_{R,gs} - \frac{Q - L_{bh} q_{bh}}{L_{gs}} R_h - \frac{Q - L_{bh} q_{bh}}{2 C_f V_{gs}}. \] (3.13)

Henceforth the following abbreviations will be used in order to reduce the size of expressions:
\[ K_{bh} = \frac{1}{2 C_f V_{bh}}, \quad K_{gs} = \frac{1}{2 C_f V_{gs}} \] (3.14)

Using these abbreviations in (3.13), results in:
\[ T_{R,bh} - q_{bh} R_b - K_{bh} L_{bh} q_{bh} = T_{R,gs} - \frac{Q - L_{bh} q_{bh}}{L_{gs}} R_h - K_{gs} (Q - L_{bh} q_{bh}). \] (3.15)

For times \( t < t_s \), the formula in (2.7) is used to calculate \( T_{R,bh} \) and \( T_{R,gs} \), using the bedrock and soil conductivities and thermal diffusivities respectively. For times \( t > t_s \) the formula in (2.8) is used. In both formulas, \( T_{0m} \) is replaced with \( T_{0b}(t, D) \) for the GSHE.

For each of the cases just mentioned, (3.15) may be algebraically manipulated to solve for \( q_{bh} \) in terms of previous heat loads on the GSHE and BHE, \( R_b \) and \( R_h \), as well as the current fluid flows via \( K_{bh} \) and \( K_{gs} \). Exact calculation formulas may be found in Appendix A.
Once $q_{bh}$ is known for each time step, so is $q_{gs}$. Through (3.9) and (3.10), $T_{f,bh}$ and $T_{f,gs}$ is also known, and subsequently the average temperature at the heat pump may be calculated as

$$T_f = \frac{\dot{V}_{bh} T_{f,bh} + \dot{V}_{gs} T_{f,gs}}{\dot{V}_{tot}}$$  \hspace{1cm} (3.16)$$

$R_b$ and $R_h$, depends on both the fluid flow rate and the dynamic viscosity via the Nusselt number. The dynamic viscosity of the fluid depends on the bulk temperatures of the two exchangers, and these temperatures is what the model is supposed to predict. This means that in order to obtain a correct solution, iterations will have to be made, until the input temperatures are equal to the output temperatures for each time step. A simple flow chart of the iteration procedure can be found in Figure 3.2.

In practice the iterations currently stops once the last four calculated values for the heat load, flow rate and viscosity on each exchanger have not differed more than 1 permille (0.001) from the one before.
3.3 Controlling the heat load distribution

The distribution of the net required heat load $Q$ on the two different collectors may be controlled by controlling the individual fluid flow rates through each of the collectors. Rearranging (3.11) and (3.12) yields

$$Q_{bh} = C_f \dot{V}_{bh} \Delta T_{bh}, \quad (3.17)$$

$$Q_{gs} = C_f \dot{V}_{gs} \Delta T_{gs}. \quad (3.18)$$

If the net heat load is positive (extraction), the largest possible flow rate and heat load for any one exchanger is equal to $\dot{V}_{tot}$ and $Q$, while the smallest is obtained for zero flow rate and thereby zero heat load.

There are several reasons why one might want to exert some kind of control over how the heat load is distributed between the two exchangers. Generally it should be better if the heat load and flow rate is always shifted in favour of the currently best performing source, the source which would alone in it’s current state yield the highest fluid temperature for a certain heat load. Such shifting should lead to a higher COP and would be desirable, but it would also be hard to achieve in practice, since many different factors affects which ground-coupled heat exchanger is currently the "best". What would be more feasible however, is to force the relative flow rates between the exchangers to at all times remain at certain ratio, like 0.5/0.5, or 0.6/0.4, if it will result in a higher net performance.

Another factor to take into account is that the BHE, as opposed to the GSHE, will be subject to a long term temperature drop due to the annual average heat load on the borehole. This temperature drop is directly proportional to the value of the annual average heat load, and will therefore be reduced by shifting as much of the total heat load over to the GSHE as possible. Since the BHE is generally the best performing heat exchanger during winter, when the heat extraction rates is high, it should not be favourable to redirect the heat load towards the GSHE at this time of year. Instead it should be done during summer months, when the GSHE may be the best individually performing exchanger due to the high undisturbed ground temperature at the surface. The easiest way to achieve such a temporary redirection of heat load and flow rate would be to simply cut-off the flow through the BHE completely during summer.

3.3.1 Fluid flow scenarios

The following scenarios will be investigated using the dual heat source model described in this chapter:

A) No fluid flow distribution control, the flow distribution will be determined by the geometries and the bulk viscosities of two exchangers (see Section 3.4).

B) Fixed ratios between the flow rates through the two exchangers at all times, presumably achieved through some type of dynamic throttling.

C) Time-limited shut-off of the BHE during summer.
3.4 No flow distribution control

The natural flow distribution will in general depend on pipe geometries, the roughness of the pipes (if not smooth), as well as fluid viscosity differences in the case of a bulk temperature difference between the GSHE and BHE. The general formula for pressure drops along pipes is given by the Darcy-Weisbach equation

\[
\Delta p = f \cdot \frac{l}{D_{\text{pipe}}} \cdot \frac{\rho u^2}{2}
\]  

(3.19)

where

- \(f\) is the Darcy friction factor
- \(l\) is the length of the pipe (m)
- \(D_{\text{pipe}}\) is the inner diameter of the pipe (m)
- \(\rho_f\) is the density of the fluid (kg/m\(^3\))
- \(u\) is the mean velocity of the fluid (m/s)

Since the pressure drop across each exchanger is equal (they are connected in parallel) equation (3.19) can be used to obtain a relation between the volume flows of the GSHE and BHE. This is under the assumption that the majority of the pressure drop is caused by the long straight section of pipes in each of the two exchangers, ignoring the effects of bends and ignoring the short common section. The resulting equation is

\[
\dot{V}_{bh} = \frac{\dot{V}_{tot}}{1 + A_{gs} \sqrt{\frac{D_{gs} f_{bh} l_{bh}}{D_{bh} f_{gs} l_{gs}}}}
\]  

(3.20)

where

- \(f_{bh}\) is the Darcy friction factor of the BHE
- \(f_{gs}\) is the Darcy friction factor of the GSHE
- \(l_{bh}\) is the length of the BHE pipe \((2L_{bh} + 2D_i - \text{equivalent to a single U-pipe})\) (m)
- \(l_{gs}\) is the length of the GSHE pipe/hose (m)
- \(D_{bh}\) is the inner diameter of the BHE pipe (m)
- \(D_{gs}\) is the inner diameter of the GSHE pipe (m)
- \(A_{bh}\) is the cross-sectional area of the BHE pipe through which fluid can flow (m\(^2\))
- \(A_{gs}\) is the cross-sectional area of the GSHE pipe through which fluid can flow (m\(^2\))
All of the variables in (3.20) is independent of time, excepting the Darcy friction factors and $\dot{V}_{tot}$. The latter is already known for all times, if the heat load $Q(t)$ is known, whilst the former will depend on the Reynold's numbers of the two exchangers:

$$Re_{bh} = \frac{\rho D_{bh} \dot{V}_{bh}}{A_{bh} \mu_{bh}}, \quad Re_{gs} = \frac{\rho D_{gs} \dot{V}_{gs}}{A_{gs} \mu_{gs}}$$ (3.21)

where

$\mu_{bh}$ is the dynamic viscosity of the fluid evaluated at the bulk temperature of the BHE (Pa·s)

$\mu_{gs}$ is the dynamic viscosity of the fluid evaluated at the bulk temperature of the GSHE (Pa·s)
Chapter 4

Simulations and input parameters

In this chapter, an array of simulation scenarios is presented which have been run through the model developed in Chapter 3. The results is presented and discussed in Chapter 5. Before that, in the first section, input parameters common for all of the simulations will be presented. All of the simulations were performed using MATLAB.

4.1 Common input parameters for all simulations

All of the simulations were performed with many of the input parameters untouched. These parameters will be presented here.

4.1.1 Circulation pump settings

The circulation pump which drives the heat carrier fluid through the exchanger/exchangers was set to keep a flow rate which yields $\Delta T_{HP} = 3^\circ C$. Any changes from this default value will be clearly stated.

4.1.2 Air temperature data, $T_{air}(t)$

As a base, the daily average temperature in Uppsala, Sweden (2013)\textsuperscript{23} was chosen. Since the model does not take freezing into account and will be less accurate for below zero temperature, a temperature shift of $+3^\circ C$ to each day was introduced. This yields an annual average temperature above $10^\circ C$, which will be similar to performing the simulations for a warmer country for example in central/southern Europe. A plot of the resulting data may be seen in Figure 4.1. The reason why not real, measured data for a warmer climate have been used is simply availability.

4.1.3 Net required heat load, $Q(t)$

A plot of the net required heat load as a function of time, used for all of the simulations, is shown in Figure 4.2. The net heat load extracted from the source by the pump in order to heat a building will generally depend on many parameters, such as heat lost by the building due to outdoor conditions and instantaneous COP of the heat pump. The COP of the heat pump will depend on the source fluid temperature, as well as the supply line temperature on the hot end of the heat pump. For a realistic net required heat load, the heat supply needed by the radiator system should first be
calculated, using a proper simulation software. Then the COP of the heat pump should be iterated by using the resulting fluid temperatures in the source for each iteration, until a converging net required heat load from the source is obtained. However, for the purposes of this thesis, it was sufficient to use an artificially constructed heat load, based on the outdoor air temperature data in Figure 4.1. It was constructed by scaling the difference between the outdoor air temperature and a comfortable inside air temperature of 20°C to the size of a few kilowatts. During a period where the heat load would otherwise be below 1 kW (day 135 to 263), it was set to a static value of 1 kW in order to manage domestic hot water. The reason why this approximative heat load is sufficient, is that it will take into account that the heat load will be at its highest when the temperature difference between the outside and inside is at its highest, and at its lowest when the temperature difference is at its lowest, while still staying within realistic power limits for the heat pump. The specific numbers should not matter since the model should be usable for any real heat load, which may differ a lot between different building types.

4.1.4 BHE input parameters
Input parameters for the BHE used in all simulations is shown in Table 4.1.

4.1.5 GSHE input parameters
Input parameters for the GSHE used in all simulations is shown in Table 4.2.

4.1.6 Fluid input parameters
The fluid properties used for all simulations is that for 29.7 wt-% ethyl alcohol, as published by Melinder. The thermal properties less sensitive to temperature changes was set to be independent of temperature, and equal to the rounded value between 0-10°C (see Table 4.1.6).
Figure 4.2: A plot of the net required heat load used in every simulation. Day 135-263 the load is set to a fixed value of 1 kW in order to manage domestic hot water.

Table 4.1: Parameters used for simulating the BHE. All of the parameters come from examples presented by Claesson et al.\textsuperscript{13}, except one. The geothermal gradient is calculated by assuming an average heat flux near the surface of 0.06 W/m\textsuperscript{2}, and dividing this by the bedrock thermal conductivity.

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<th>Parameter</th>
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Table 4.2: Parameters used for simulating the GSHE. All of the parameters come from examples presented by Claesson et al.13, except the soil parameters. These are the unfrozen values for soil type 4 presented in a report from 1986 written by Rhen25.

<table>
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<th>Parameter</th>
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</tr>
<tr>
<td>Soil volumetric heat capacity</td>
<td>( C_{gs} )</td>
<td>3.1\cdot10^6</td>
<td>J/(m^3·K)</td>
</tr>
<tr>
<td>Soil thermal diffusivity</td>
<td>( a_{gs} )</td>
<td>0.40\cdot10^{-6}</td>
<td>m^2/s</td>
</tr>
<tr>
<td>GSHE U-pipe outer radius</td>
<td>( r_h; R_{p,o} )</td>
<td>0.0200</td>
<td>m</td>
</tr>
<tr>
<td>GSHE U-pipe inner radius</td>
<td>( R_{p,i} )</td>
<td>0.0176</td>
<td>m</td>
</tr>
<tr>
<td>GSHE pipe thermal conductivity</td>
<td>( \lambda' )</td>
<td>0.43</td>
<td>W/(m·K)</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters used for simulating the heat carrier fluid, based on data from Melinder26 for 29.7 wt-% ethyl alcohol in the temperature interval 0-10°C

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid thermal conductivity</td>
<td>( \lambda_f )</td>
<td>0.4</td>
<td>W/(m·K)</td>
</tr>
<tr>
<td>Fluid specific heat</td>
<td>( c_{p,f} )</td>
<td>4.2\cdot10^3</td>
<td>J/(kg·K)</td>
</tr>
<tr>
<td>Fluid density</td>
<td>( \rho_f )</td>
<td>0.96\cdot10^3</td>
<td>kg/m^3</td>
</tr>
</tbody>
</table>
The dynamic viscosity of 29.7 wt-% ethyl alcohol varies greatly with temperature in the operation interval of a BHE or GSHE. In the simulations it also needs to be evaluated continuously at any temperature, not just those listed in Melinder’s table. Therefore, an exponential function was least square-fitted to the table data, and the resulting function together with the data points it was fitted to is plotted in Figure 4.3.

![Dynamic Viscosity fit](image)

Figure 4.3: Melinder’s data for the dynamic viscosity of 29.7 wt-% ethyl alcohol, together with a least-square fitted exponential function.

### 4.2 Simulations

A series of simulations were performed, using the model developed in Chapter 3. These simulations were aimed at testing the model, as well as identifying trends when certain parameters are changed. All of the simulations were performed for one year of runtime, starting from a "newly installed" system (i.e. zero heat load before day 1). The results are not expected to change notably for multi-year simulations, since the long-term temperature change in the borehole should lessen with the presence of an added GSHE to help carry the heat loads.

Two of the most important parameters that were tracked was

- The annual mean average fluid temperature at the heat pump $T_{f,\text{mean}}$ – related to the time average COP of the heat pump.
- The minimum average fluid temperature at the heat pump $T_{f,\text{min}}$ – important dimensioning factor.
A brief description of each of the simulations performed for this thesis will now be given. The results is presented and discussed in Chapter 5.

1) Single heat exchangers

1.1) Single BHE: The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on the active borehole depth $L_{bh}$ for a single BHE was studied.

1.2) Single GSHE: The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on the pipe length $L_{gs}$ as well as burial depth $D$ for a single GSHE was studied.

2) Dual heat exchangers – No flow distribution control

2.1 Fixed GSHE/Varying BHE: A BHE of varying active borehole depth $L_{bh}$ is added to a GSHE with pipe length $L_{gs}=300$ m and burial depth $D=1$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{bh}$ was studied. The annual mean values of $\dot{V}_{bh}$, $V_{gs}$, $Q_{bh}$ and $Q_{gs}$ as a function of the increased $L_{bh}$ was also tracked.

2.2 Fixed BHE/Varying GSHE: A GSHE of varying pipe length $L_{gs}$ and burial depth $D$ is added to a BHE with active borehole depth $L_{bh}=80$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{gs}$ and $D$ was studied.

2.3 Flow distribution, viscosity dependence: A difference in average fluid temperature between the GSHE and BHE should give a difference in bulk viscosity. The effect of this bulk viscosity difference on the fluid flow distribution was studied for the case of a BHE with $L_{bh}=80$ m and a GSHE with $L_{gs}=500$ m and $D=1$ m.

3) Comparison between different fixed flow ratios and the case of no flow distribution control

3.1 Fixed GSHE/Varying BHE: A BHE of varying active borehole depth $L_{bh}$ is added to a GSHE with pipe length $L_{gs}=300$ m and burial depth $D=1$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{bh}$ was studied for different fluid flow distributions.

3.2 Fixed BHE/Varying GSHE: A GSHE of varying pipe length $L_{gs}$ and burial depth $D$ is added to a BHE with active borehole depth $L_{bh}=80$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{gs}$ and $D$ was studied for different fluid flow distributions.

4) BHE shut-off during summer – No other fluid distribution control

4.1 Fixed GSHE/Varying BHE: A BHE of varying active borehole depth $L_{bh}$ is added to a GSHE with pipe length $L_{gs}=300$ m and burial depth $D=1$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{bh}$ was studied when the BHE was shut-off in the summer (day 150-250), redirecting the whole flow to the GSHE.

4.2 Fixed BHE/Varying GSHE: A GSHE of varying pipe length $L_{gs}$ and burial depth $D$ is added to a BHE with active borehole depth $L_{bh}=80$ m. The dependence of $T_{f,\text{mean}}$ and $T_{f,\text{min}}$ on $L_{gs}$ and $D$ was studied when the BHE was shut-off in the summer (day 150-250), redirecting the whole flow to the GSHE.

5) Net flow rate dependence (varying $\Delta T_{HP}$) – No flow distribution control
5.1 Fixed BHE/Varying GSHE A GSHE of varying pipe length $L_{gs}$ and burial depth $D$ is added to a BHE with active borehole depth $L_{bh}=80$ m. The dependence of $T_{f,mean}$ and $T_{f,min}$ on $L_{gs}$ and $D$ was studied as the net flow rate through the system was varied by varying $\Delta T_{HP}$.

6) Comparison: Rule of thumb vs Model

6.1 Required length comparison: As a GSHE is added to a BHE with $L_{bh}=80$ m, the required length $L_{gs}$ recommended for a certain $T_{f,min}$ by the rule of thumb in Section 3.1 is compared with the required length recommended by the simulation model presented in this thesis. The data used for the rule of thumb was the data obtained from simulating a single GSHE and a single BHE.
Chapter 5

Results and discussion

The results from the simulations described in Section 4.2 will be presented and discussed within this chapter. It should be kept in mind that the model developed in this thesis has not been validated and it doesn’t take into account any freezing of the ground water in the borehole or soil. It will, however, give the general idea of how changing different parameters would affect a real system. Any combination of BHE and GSHE mentioned in this chapter (and in this thesis as a whole) implies a parallel connection such as described in Figure 3.1.

5.1 Single heat exchangers

5.1.1 Single BHE

The resulting change in minimum and mean fluid temperatures as the active borehole depth of a single BHE is increased can be seen in Figure 5.1. As the active depth nears 250 meters, the minimum and mean temperatures both approaches a value of about 10 °C. This is because for deep boreholes, the heat extraction will be distributed on a large volume of bedrock, and therefore the temperature drop on average will be very small.

5.1.2 Single GSHE

The resulting change in minimum fluid temperature as the pipe length of a single GSHE is increased is shown in Figure 5.2 for different burial depths. It can be clearly seen that the minimum fluid temperature increases with increased burial depth for any fixed length, as well as with increasing length for the same burial depth. Comparing with Figure 5.1, it can also be seen that the maximum obtainable $T_{f,min}$ for a GSHE is much lower than for a BHE (2 °C vs 10 °C). This is natural, since the undisturbed ground surface soil temperature varies with the air temperature during the colder parts of the year, whilst the bedrock does not.

The change of annual mean fluid temperature with increased pipe length for a single GSHE may be seen in Figure 5.3. It shows that the annual mean temperature also is increased by increasing the pipe length, but here the depth relation is reversed. At a shallower burial depth, the mean temperature is slightly increased, but the effect of depth is notably lower than for the minimum temperature. The mean temperature changes less than 1 °C with depth, while the minimum changes about 3 °C.
Figure 5.1: A plot of the minimum fluid temperature $T_{f,\text{min}}$ and the annual mean fluid temperature $T_{f,\text{mean}}$ for different active borehole depths $L_{bh}$ of a single BHE.

During winter, because of thermal inertia, an increased depth will cause an increased dampening effect on the largest cold spikes, which is why the minimum temperature will increase significantly with increased depth. Conversely, an increased depth will also dampen out the largest hot spikes during summer, which is the period with the lowest heat load. During periods of low heat loads, the undisturbed ground temperature will be a larger portion of the determining factor for the fluid temperature than during periods of high heat loads. A depth reduction would therefore increase the fluid temperatures more during summer than it would decrease the fluid temperatures during winter. As $L_{gs}$ increases, the temperature dependence on the heat load will lessen also during winter, causing the annual mean fluid temperatures for different depths to converge and approach the annual mean ground temperature.

Comparing 5.2 with 5.1, it can be noted that the same annual mean temperature is obtained for a BHE with $L_{bh}=110$ m as for a GSHE with $L_{gs}=300$ m; about $5^\circ$C. But if we then also compare the minimum temperatures for $L_{bh}=110$ m and $L_{gs}=300$ m, the BHE has a minimum temperature of $-4^\circ$C, while the GSHE has a minimum temperature of less than $-8^\circ$C, for all depths. The matching annual mean temperatures and differing minimum temperatures, with the GSHE having a lower minimum temperature, can be explained by the higher undisturbed ground temperature around the GSHE during summer. The annual mean fluid temperature is related to the time average COP, but it does not say when the COP is high. A high COP will be most important during times of large heat loads, which usually makes the minimum fluid temperature more important than the mean.
Figure 5.2: A plot of the minimum fluid temperature $T_{f,min}$ for different lengths $L_{gs}$ and burial depths $D$ of a single GSHE.

Figure 5.3: A plot of the annual mean fluid temperature $T_{f,mean}$ for different lengths $L_{gs}$ and burial depths $D$ of a single GSHE.
5.2 Dual heat exchangers – No flow distribution control

5.2.1 Fixed GSHE/Varying BHE

The effects on mean and minimum fluid temperatures by adding a BHE to an already existing GSHE with \( L_{gs} = 300 \) m and \( D = 1 \) m can be seen in Figure 5.4. Comparing this with the case of a single BHE in Figure 5.1, it’s interesting to see that the mean and minimum temperatures no longer seem to approach the same value. The already existing GSHE is strongly influencing and limiting the maximum obtainable \( T_{f,min} \) for the combined system, making it seem as if a deep single borehole might be preferable to a combination of a deep borehole and a GSHE. Yet if one compares the minimum temperature values at a borehole depth of 100 m, it is increased from -5 °C to 0 °C through the combination. The conclusion that may be drawn from this is that for certain values of the borehole depth, the combination is preferable to a pure borehole, but above a certain limit – about 150 meters – it is not.

Since there is no flow distribution control, an increased borehole depth will cause an increased fraction of the total fluid flow to go through the GSHE. This behaviour may be seen in Figure 5.5 and is a result of (3.20). Note that the curves intersect at about \( L_{bh} = 160 \) m. The pipe length traversed by the fluid across each exchanger is equal when \( 2L_{bh} + 2D_i = L_{gs}, \) with \( D_i \) being 6 m for the simulations. This means that the the pipe lengths are equal at \( L_{bh} = 144 \) m, and that the average flow rates does not intersect until the BHE pipe length becomes larger than the GSHE pipe length. The reason for this is most likely because the average fluid temperature of the GSHE is lower for most of the year, resulting in a higher viscosity. An increased viscosity will cause an increased resistance to flow through the GSHE, just as an increased pipe length would.

One may think that an increased fluid flow rate fraction through the GSHE should also shift a larger part of the heat load over to the GSHE. This is not the case, as may be seen in Figure 5.6. Increasing the borehole depth will not only reduce the fluid flow rate, but also increase the total heat extracted for a certain average temperature difference between the fluid and bedrock. We have from (3.9) and (2.3) that

\[
Q_{bh} = \frac{1}{R_b} (T_{R,bh} - T_f) L_{bh}. \tag{5.1}
\]

The effect of an increased borehole depth on \( Q_{bh} \) is much stronger than the effect of increased borehole resistance \( R_b \) and average fluid temperature \( T_f \) because of reduced flow rate.

The borehole depth for which the annual average heat loads intersect is at about 75 m. The exact depth for which the intersection occurs should depend heavily on how the heat load is distributed between the two exchangers during winter for different borehole depths. Winter is the time of highest heat loads, which means the average heat load distribution is largely determined by the winter distribution. For given system geometries and a given net heat load function, the fractional heat load distribution should then depend on the difference in undisturbed ground temperatures around the two exchangers. Increasing the undisturbed ground temperature around one exchanger should shift a larger portion of the heat load fraction over to that exchanger, if everything else is held constant. Therefore the intersection depth, if the borehole depth is varied, should also depend the undisturbed ground temperature difference, as well as the length of the GSHE.
Figure 5.4: A plot of the minimum fluid temperature $T_{f,min}$ and the annual mean fluid temperature $T_{f,mean}$ for different active borehole depths $L_{bh}$ of a BHE added to a GSHE with $L_{gs}=300$ m and $D=1$ m and no flow distribution control.

Figure 5.5: A plot of the the annual mean flow rates $\dot{V}_{bh,mean}$ and $\dot{V}_{gs,mean}$ for different active borehole depths $L_{bh}$ added to a GSHE with $L_{gs}=300$ m and $D=1$ m and no flow distribution control.
Figure 5.6: A plot of the annual mean heat loads $Q_{bh,mean}$ and $Q_{gs,mean}$ for different active borehole depths $L_{bh}$ added to a GSHE with $L_{gs}=300$ m and $D=1$ m and no flow distribution control.

5.2.2 Fixed BHE/Varying GSHE

The effects on the minimum fluid temperature by adding a GSHE to an already existing BHE with $L_{bh}=80$ m can be seen in Figure 5.7. Comparing Figure 5.7 with Figure 5.2 leads us to draw the conclusion that a combined system will always perform better than a single GSHE with an equal GSHE length, at least if the borehole depth is 80 m. Remember from Section 5.2.1, that for a BHE above 150 meters, adding a GSHE with pipe length 300 m and burial depth 1 m will lower the minimum temperature, instead of increasing it. It seems that a BHE will always be helpful addition to an existing GSHE, but it might not always be the other way around.

The dependence of the mean fluid temperature on $L_{gs}$ and $D$ is shown in Figure 5.8. An interesting thing to note here is that even though the mean fluid temperature for short GSHE lengths in a combined system is much higher than that of a single GSHE, at above $L_{gs}=400$ m it actually starts to drop below that for a single GSHE (compare with Figure 5.3). It is not unreasonable to believe that this has to do with the bedrock being colder than the top soil during summer, and increasing the length of the GSHE will push more of the total flow over to the BHE which is fixed in length and colder during the summer months.
Figure 5.7: A plot of the minimum fluid temperature $T_{f,\text{min}}$ for different lengths $L_{gs}$ and burial depths $D$ of a GSHE added to a BHE with $L_{bh}=80$ m and no flow distribution control.

Figure 5.8: A plot of the annual mean fluid temperature $T_{f,\text{mean}}$ for different lengths $L_{gs}$ and burial depths $D$ of a GSHE added to a BHE with $L_{bh}=80$ m and no flow distribution control.
5.2.3 Flow distribution, viscosity dependence

The average BHE and GSHE fluid temperatures in a combined system with \( L_{gs} = 500 \) m, \( D = 1 \) m and \( L_{bh} = 80 \) m may be seen in Figure 5.9, where it is plotted as a function of time. The temperature difference is at its maximum about 5 °C at about day 270, and looking at Figure 5.10, this coincides with a maximum bulk dynamic viscosity difference of about 1 mPa·s.

The fractional flow rates as a function of time, belonging to the BHE and GSHE, is shown in Figure 5.11. The flow rate fractions appear to be as good as constant, varying slightly around the flow fractions obtained when the bulk temperatures equal. At the time of the biggest temperature difference, the BHE flow fraction is only shifted down from 0.75 to 0.70. We may therefore conclude that the viscosity differences in the temperature interval between 0 °C and 15 °C has a small impact on the flow rate distributions. This might change slightly for colder temperatures, however, since the dynamic viscosity changes more rapidly with temperature below zero (see Figure 4.3).

In the example presented in this section, the flow through both exchangers are rather low – in the laminar region. For very high net flow rates, the temperature difference in Figure 5.9 should almost vanish, since the out-temperatures of both exchangers should approach their shared in-temperature, causing the average temperatures to align. Due to this, the viscosity difference effect should be even more negligible for higher flow rates.

![Graph showing fluid temperatures for BHE and GSHE](image)

Figure 5.9: The fluid temperatures of a GSHE and BHE connected in parallel as a function of time. \( L_{bh} = 80 \) m, \( L_{gs} = 500 \) m, \( D = 1 \) m. No flow distribution control.
Figure 5.10: A plot of $\mu_{bh}$ and $\mu_{gs}$ as a function of time for a combination of GSHE and BHE. $L_{bh}=80$ m, $L_{gs}=500$ m, $D=1$ m. No flow distribution control.

Figure 5.11: Fractional flow rates for a combined BHE and GSHE system, as well as the corresponding pipe length fractions (no flow rate control) $L_{bh}=80$ m, $L_{gs}=500$ m, $D=1$ m. No flow distribution control.
5.3 Comparison between different fixed flow ratios and the case of no flow distribution control

5.3.1 Fixed GSHE/Varying BHE

The effect of $L_{bh}$ on mean and minimum fluid temperatures for different flow distribution scenarios as a BHE is added to an existing GSHE with $L_{gs}=300$ m and $D=1$ m is shown in Figure 5.12. An obvious trend that can be seen there is that $T_{f,\text{min}}$ is increased by redirecting the larger flow to the "best" individual exchanger. At $L_{bh}=250$ m, the clear winner is 0.8 BHE/0.2 GSHE, and at $L_{bh}=30$ m, 0.2 BHE/0.8 GSHE gives the highest minimum temperature.

For medium range borehole depths, between 80-150 meters, the 'Auto' or 'no flow distribution control' solution is almost identical with the 0.5/0.5 solution, and both are better than the 0.8 BHE/0.2 GSHE solution up to about $L_{bh}=125$ m. Ultimately, it seems that regulating the flow rate when adding a BHE to a GSHE would only give a significant gain in $T_{f,\text{min}}$ for large differences in pipe lengths, $l_{bh} = 2(L_{bh} + D)$ and $l_{gs} = L_{gs}$, and is otherwise not worth it.

The "wobbly" part of the 0.8 BHE/0.2 GSHE line is most likely due to shifts in the flow regime caused by a reduced dynamic viscosity as the fluid temperature increases. A transient or turbulent flow will reduce the thermal resistance between the inner pipe wall and the fluid, thereby increasing the minimum fluid temperature. It may also be due to a shift of the day when the coldest fluid temperature is obtained at the heat pump.

Now, looking at Figure 5.13, we can see the effect on the annual mean fluid temperature. It seems to be following the same trend as for the minimum temperature. This leads to the same verdict as before: it should not be worth it to control the flow distribution, when adding a BHE to a GSHE, if not the pipe length differences is very large.

![Figure 5.12: A plot over how the minimum fluid temperature depends on $L_{bh}$ as a BHE is added to a GSHE with $L_{gs}=300$ m and $D=1$ m for different flow distribution scenarios.](image-url)
Figure 5.13: A plot over how the annual mean fluid temperature depends on $L_{bh}$ as a BHE is added to a GSHE with $L_{gs}=300$ m and $D=1$ m for different flow distribution scenarios.

### 5.3.2 Fixed BHE/Varying GSHE

The effect of flow distribution settings on $T_{f,min}$ when adding a GSHE to a BHE with $L_{bh}=80$ m, may be seen in Figure 5.14. Here the curve positions are inverted when compared to Figure 5.12, which is natural, since forcing 0.8 of the flow rate into a GSHE that is 50 meter long should be much worse than letting 0.8 of flow ratio go through the BHE which has $L_{bh}=80$ m. The verdict still remains from the previous section, that control should only be worth it for large difference in pipe lengths.

An interesting phenomena here is that the ‘Auto’ or ‘no flow distribution control’ option starts off close to the 0.2 BHE/0.8 GSHE option, and then approaches the 0.8 BHE/0.2 GSHE option as the length increases, touching the 0.5/0.5 option along the way. This is a result of the ‘natural’ distribution for parallel pipe flow, where a higher flow rate is obtained through a shorter pipe and a lower flow rate is obtained through a longer pipe. The same phenomena would most likely occur for very large borehole depths in Figure 5.12 and 5.13.

It seems that there is a breakpoint for when it is more beneficial to shift the flow towards the GSHE, rather than the BHE, at about $L_{gs}=280$ m, but interestingly enough it is better or equal to have 0.5/0.5 a bit before, during and a bit after the break point.

In Figure 5.15, the effects on the annual mean fluid temperature is shown. For $L_{gs}>300$ m there is a distinct trend of increased mean temperature with increased flow ratio directed to the GSHE. We may also see that the only setting which leads to a decreased $T_{f,mean}$ for larger $L_{gs}$ above a certain limit is the ‘Auto’ setting. This is because of the tendency for a flow shift towards the BHE for an increased GSHE pipe length.
Figure 5.14: A plot over how the minimum fluid temperature depends on $L_{gs}$ as a GSHE with $D=1$ m is added to a BHE with $L_{bh}=80$ m for different flow distribution scenarios.

Figure 5.15: A plot over how the annual mean fluid temperature depends on $L_{gs}$ as a GSHE with $D=1$ m is added to a BHE with $L_{bh}=80$ m for different flow distribution scenarios.
5.4 BHE shut-off during summer – No other fluid distribution control

5.4.1 Fixed GSHE/Varying BHE

The results of shutting off the BHE completely during summer (day 150-250), and thereby redirecting the whole flow to the GSHE, may be seen in Figure 5.16 and Figure 5.17. No other fluid distribution control was exerted.

As $L_{bh}$ is varied, it seems as if there is no discernible loss in minimum fluid temperature by letting the borehole rest during summer, but a slight increase in annual mean temperature, up to an active borehole depth of about 150 m.

The lack of change of minimum temperature is most certainly because the minimum temperature occurs before summer. If it would occur after the summer, allowing the BHE to rest should prove an increase in minimum temperature, since there is no long term effects of temporary heat loads on a GSHE.

As for the mean temperature increase up to a certain break point, it is not unreasonable to believe that an additional increase of $L_{bh}$ after 150 m will push the BHE summer fluid temperature up over a threshold, where it is slightly higher than that of a GSHE with $L_{gs}$=300 and $D$ =1 m operating single-handedly. In this case, no short term benefits would come from letting the BHE rest. A very important reason to let the BHE rest during summer, however, is to reduce the long term effect on the bedrock temperature. Having a zero heat load on the borehole during summer, will always reduce the annual average heat load on the BHE and thereby also the long term temperature drop described in the second row of (2.5). This in turn means that the individual performance of the BHE will be improved for each winter following the first, compared to when not letting it rest.

Based on this discussion we may conclude that under some conditions, letting the BHE rest during summer will improve the short-term performance of the combined system in addition to an increase in long-term performance.

It should be mentioned that the average heat load on the BHE is already alleviated simply by adding a GSHE, even if it is not allowed to rest. Since the heat load is split up between the two exchangers, the average heat load on the BHE during a year will be lower than if it had to carry the load alone. In either case, rest or no rest, it is expected that the long term benefits of adding a GSHE to a BHE with multiple boreholes should be even larger, since the long term temperature drop in a multiple borehole set-up for a certain annual average heat load is generally larger.

It should also be mentioned that the break point and effect of resting should depend on the choice of time interval for the rest.
Figure 5.16: A plot of the minimum fluid temperature $T_{f,min}$ for different active borehole depths $L_{bh}$ of a BHE added to a GSHE with $L_{gs}=300$ m and $D=1$ m when the BHE is shut off during summer months (day 150-250), no other flow distribution control.

Figure 5.17: A plot of the annual mean fluid temperature $T_{f,mean}$ for different active borehole depths $L_{bh}$ of a BHE added to a GSHE with $L_{gs}=300$ m and $D=1$ m when the BHE is shut off during summer months (day 150-250), no other flow distribution control.
5.4.2 Fixed BHE/Varying GSHE

The results of shutting off the BHE completely during summer (day 150-250), and thereby redirecting the whole flow to the GSHE, may be seen in Figure 5.18 and Figure 5.19. No other fluid distribution control was exerted.

As $L_{gs}$ is varied, there is no discernible change in minimum temperature when letting the BHE rest during summer, just as in the case with varying $L_{bh}$. The inversion break point in annual average fluid temperature, when the BHE fluid temperatures during summer most likely starts to become lower than the GSHE temperatures seems to here be around $L_{gs}=200$ m. After this break point, there will be an added short term benefit of shutting down the BHE during summer, in addition to the long term benefits.

We also see a decline in annual mean temperature for larger GSHE pipe lengths, when the BHE is left active during summer. This is due to the flow favouring the increasingly relatively shorter BHE as $L_{gs}$ increases, which in turn lowers the utilization of the warmer top soil in summer time. It should be commented that for higher heat loads during summer, the break point for a gain should be shifted towards higher $L_{gs}$. The break points should also be heavily dependent on the choice of shut-off interval.

![Figure 5.18: A plot of the minimum fluid temperature $T_{f,min}$ for different lengths $L_{gs}$ of a GSHE with $D=1$ m added to a BHE with $L_{bh}=80$ m when the BHE is shut off during summer months (day 150-250), no other flow distribution control.](image)
Figure 5.19: A plot of the annual mean fluid temperature \( T_{f,\text{mean}} \) for different lengths \( L_{gs} \) of a GSHE with \( D=1 \, \text{m} \) added to a BHE with \( L_{bh}=80 \, \text{m} \) when the BHE is shut off during summer months (day 150-250), no other flow distribution control.

5.5 Net flow rate dependence (varying \( \Delta T_{HP} \)) – No flow distribution control

5.5.1 Fixed BHE/Varying GSHE

The flow regime and the exact flow rate should affect the fluid temperatures, with an increased fluid temperature for a higher flow rate, since it lowers the thermal resistance between the inner pipe wall and the fluid. In Figure 5.20 and 5.21 the effect of varying net flow rate through changing \( \Delta T_{HP} \) is shown for a varying GSHE with \( D=1 \, \text{m} \) added to a BHE with \( L_{bh}=80 \, \text{m} \). As expected, both the minimum and annual mean fluid temperatures are increased by increasing the flow rate. A decreased \( \Delta T_{HP} \) will lead to a higher \( \dot{V}_{\text{tot}} \) for the same \( Q \), see equation (3.2).

The case of double flow rate (\( \Delta T_{HP}=1.5 \, ^\circ \text{C} \)) yields a very uneven dependence on \( L_{gs} \) after about 250 m, most likely because of shifts in the flow regime in the BHE due to lower viscosity at higher temperatures, and also because more of the flow will be distributed towards the the BHE.

The minimum temperatures for the case of \( \Delta T_{HP}=1.5 \, ^\circ \text{C} \) does not differ much from the regular \( \Delta T_{HP}=3 \, ^\circ \text{C} \) for \( L_{gs}<250 \, \text{m} \), and this is most likely due to the Nusselt number for laminar flow being independent on flow rate (see equation (2.18)). If the Nusselt number is equal, then the borehole and soil-fluid thermal resistance is equal, which means that the only improvement in average fluid temperatures in the two exchangers will come from the inlet temperature being higher due to the lower heat pump delta.

The same reasoning may be applied to explain the very closely matching annual average temperatures for \( L_{gs}<300 \, \text{m} \) in Figure 5.21. The flow regimes is most likely laminar for all \( L_{gs} \) up to 800 m for both \( \Delta T_{HP}=1.5 \, ^\circ \text{C} \) and \( \Delta T_{HP}=3 \, ^\circ \text{C} \), since for most of the year, the net heat load will not be large enough to cause turbulent flow in any of the exchangers. The reason for the dip and separation of the \( \Delta T_{HP}=3 \, ^\circ \text{C} \) setting from the \( \Delta T_{HP}=1.5 \, ^\circ \text{C} \) setting for \( L_{gs}>300 \, \text{m} \) is most
likely because an increased portion of the flow rate and heat load will be delegated to the BHE during summer, which will lower the annual mean temperature. A larger portion of the flow rate will go through the BHE during summer also for the case $\Delta T_{HP}=1.5 \, ^\circ C$, lowering the annual mean temperature in the same way, but for this case the heat load redistribution is likely not as large as for the $\Delta T_{HP}=3 \, ^\circ C$. As long as a "large enough" flow rate goes through the GSHE, it should still carry most of the heat load during summer. There will most likely be a similar dip at higher $L_{gs}$ perhaps above 1000 m for the $\Delta T_{HP}=1.5 \, ^\circ C$ case, when the flow rate through the GSHE is low enough to allow the BHE characteristics to manifest.

For the case of tenfold flow rate increase ($\Delta T_{HP}=0.3 \, ^\circ C$) it is safe to assume that the flow rate is turbulent for both the GSHE and BHE during all times of the year, which clearly yields a significant increase in minimum temperature for low $L_{gs}$, a smaller increase for higher $L_{gs}$, but still significant, and a about a 1-2 $^\circ C$ boost to the mean temperature at all different GSHE pipe lengths.

Figure 5.20: Plot of how $T_{f,min}$ depends on the net flow rate ($\Delta T_{HP}$) and $L_{gs}$ as a GSHE with $D=1 \, m$ is added to a BHE with $L_{bh}=80 \, m$, no flow distribution control
5.6 Comparison: Rule of thumb vs Model

5.6.1 Required length comparison

A comparison between using the rule of thumb presented in Section 3.1 and the simulation model to dimension a system with a GSHE \((D=1 \text{ m})\) added to an existing BHE with \(L_{bh}=80 \text{ m}\) is shown in Figure 5.22 and 5.23. The corresponding fractional errors, in terms of the simulation model result, is found in Figure 5.24 and 5.25. There was no fluid distribution control for the dual system simulation results presented in these two Figures. The basis for the rule of thumb was the data obtained for a single BHE and a single GSHE via the same simulation model used for dual systems.

The comparison shows that the recommended \(L_{gs}\) for a certain \(T_{f,min}\) is always lower when using the rule of thumb. The fractional difference varies between -50% and -5% on the short side. The same comparison for other borehole depths, have shown that the error varies greatly with \(L_{bh}\), and to a less extent with \(L_{gs}\), ranging between 5-50% on the low side.

As for the lengths required for certain mean temperatures, the rule of thumb always gives a severe underestimation, always at least -10% and may go down to more than -70%.

The increasingly large difference above -1 °C may be explained by flow being redistributed towards the BHE as the GSHE length increases, partly negating the benefits of an increased length. In fact, if the flow is forced to remain at a 50/50 ratio between the two exchangers, as it is in Figure 5.26, the phenomena disappears.

In general, the lengths recommended by the rule of thumb seem to match the 50/50 flow scenario much more closely than a scenario with no fluid distribution control. This is probably because it is more similar to a case where the systems are completely independent of each other.
Figure 5.22: A comparison between using the rule of thumb (see Section 3.1) and the simulation model to calculate the required $L_{gs}$ for a certain $T_{f,min}$, as a GSHE with $D=1$ m is added to a BHE with $L_{bh}=80$ m, no fluid distribution control.

Figure 5.23: A comparison between using the rule of thumb (see Section 3.1) and the simulation model to calculate the required $L_{gs}$ for a certain $T_{f,mean}$, as a GSHE with $D=1$ m is added to a BHE with $L_{bh}=80$ m, no fluid distribution control.
Figure 5.24: Fractional difference between the simulation model and rule of thumb in Figure 5.22

Figure 5.25: Fractional difference between the simulation model and rule of thumb in Figure 5.23
Figure 5.26: A comparison between using the rule of thumb (see Section 3.1) and the simulation model to calculate the required $L_{gs}$ for a certain $T_{f,min}$, as a GSHE with $D=1$ m is added to a BHE with $L_{bh}=80$ m, 0.5/0.5 flow ratio.
Chapter 6

Conclusions

Even though the results presented in this thesis were obtained for the simplest possible form of BHE/GSHE, for a specific climate and heat load, as well as for specific soil/bedrock/fluid parameters, they may be used as indicators of general phenomena. Much of the general behaviour of the combined system may most likely be expanded to more complicated systems, such as multiple boreholes used together with a GSHE. The results provide insights into which parameters are most important for optimizing performance, and more importantly, in which direction the parameters should be changed in order to improve performance. The main conclusions that may be drawn based on the results are these:

- The annual mean and minimum temperatures of a GSHE have an opposite dependence on pipe burial depth – the minimum temperature is increased by an increased depth, while the annual mean temperature is reduced. Keep in mind, however, that a high fluid temperature is a larger improvement to heat pump efficiency during a period of high heat load than during a period of low heat load, meaning that the minimum temperature is more important.

- A single BHE may be better than a combination of a BHE and GSHE, if the BHE is deep enough. Adding a GSHE may limit the highest possible minimum fluid temperature.

- Adding a BHE to an existing GSHE is most likely always a gain in minimum fluid temperature.

- The viscosity differences between a warmer and a colder exchanger normally causes a negligible effect on the flow rate distribution. This is because the inlet fluid temperatures of the two exchangers are the same, which means that the average temperatures will not differ by much, especially not for high flow rates.

- The natural flow rate distribution is heavily dependent on pipe length differences between the two exchangers.

- Controlling the flow rates to remain at a certain ratio between the two exchangers only yields a tangible benefit for very large pipe length differences, otherwise the natural flow rate distribution might as well be used for a similar effect but less hassle.

- Shutting off the BHE completely during the warm summer months will increase the mean temperature above a certain break point for the GSHE pipe length, depending on the summer
heat loads. This means that under some conditions the summer rest will improve the short-
term performance of the combined system, in addition to improving the long term performance
of the BHE. If the BHE has multiple boreholes, the long-term benefits should be even more
pronounced.

- Increasing the net flow rate through both exchangers is always an improvement, with the shift
to turbulent flow giving an additional bump.

- In all the simulations that has been performed, the rule of thumb have ended up recommending
a shorter GSHE length than the simulation model for a certain minimum temperature, as a
GSHE is added to a BHE. The error varies greatly with $L_{bh}$, and to a less extent with $L_{gs}$,
ranging between 5-50% on the low side when compared to the case of no distribution flow
control. The rule of thumb more closely matches the case of 50/50 fluid distribution.
Chapter 7

Future work

While the model presented in this thesis have led to some interesting results, it is far from complete. There are an array of things that haven’t been taken into account that could be, and there is also many optimizations that could be done in order to speed up calculations. There are also conditions that need to be fulfilled in order for all of the equations involved to be valid. A few suggestions on how to improve the model will now be presented.

- In real life applications of ground coupled heat exchanger systems, there will almost always be freezing involved. It is either the ground water in the borehole freezing during periods of high heat loads, or the water absorbed in the top soil freezing during winter. Freezing will change the thermal conductivity, and thereby affect performance, and it should somehow be taken into account.

- Effect of snow on the annual average ground temperature should be taken into account.

- Support for multiple hoses in the GSHE should be added, where the influence between parallel pipes is taken into account. In reality there is usually more than one long single pipe in a GSHE.

- Functionality should be added for handling the fact that an old BHE to which a GSHE is added, have already been run for years, maybe even into it’s steady state. This may be handled by assuming steady state based on the old average heat load, and start from there, using the formula (2.8).

- Ideally there should be more air temperature data points per year, preferably spaced in time by an hour. The temperature of the air may vary greatly between night and day, and even though high frequency temperature variations are quickly damped out underground, it will affect the heat load required to keep a building at an even temperature.

- In the simulations, the heat pump is assumed to always be running, with stepwise constant heat loads. It should be investigated how this assumption holds up against a real system, where the heat pump will typically be operating in intervals. The instantaneous heat loads and thereby fluid temperatures in the heat exchangers should be higher and lower respectively.

- Support for a system of multiple boreholes connected to a GSHE should be added, since this is common for larger buildings.
• The same methodology used here could be used for modelling a GSHE and BHE connected in series.

• It is in this model assumed that the average fluid temperature is equal to the average of the inlet and outlet temperature. At low flow rates and/or for very long pipes, this assumption may be false. Corrections for low flow rates should be investigated and added.

• Aggregations of heat loads on the BHE should be added in order to speed up calculations, letting heat loads far in the past be averaged and spanned over an increased amount of time steps. A suggestion on how this can be done is given by Eskilson$^8$.

• And more... There is always room for more improvements!
Bibliography


Appendices
Appendix A

Calculation formulas

A.1 Calculating $T_{R,bh}$, $t < t_{s,bh}$

Expressions used for calculating the borehole wall temperature $T_{R,bh}(t)$ at the end of time step $n$, each time step being of the same length $\Delta t$ is found in (A.1). The formula is based on (2.7).

$$T_{R,bh}(n) = T_{0m} - \frac{q_{bh}(n)}{4\pi \lambda_{bh}} \left( \ln \left( \frac{4a_{bh} \Delta t}{r_{b}^2} \right) - \gamma \right) - \sum_{i=1}^{n-1} \frac{q_{bh}(i)}{4\pi \lambda_{bh}} \ln \left( \frac{n + 1 - i}{n - i} \right)$$

(A.1)

where

$T_{0m}$ is the undisturbed temperature of the borehole bedrock ($^\circ$C)

$r_b$ is the radius of the borehole, previously referred to as $R_0$ (m)

$a_{bh}$ is the thermal diffusivity of the bedrock around the BHE ($m^2/s$)

$\lambda_{bh}$ is the thermal conductivity of the bedrock around the BHE ($W/(m\cdot K)$)

The formula (A.1) describes the the temperature change at $t = t_{n+1}$ as a result of the latest and ongoing extraction step $q_{bh}(n)$, beginning at time $t = n\Delta t$ and ending at time $t = (n + 1)\Delta t$, as well as all the previous heat loads times $t < t_s$. 

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A.2 Calculating $T_{R,gs}$, $t < t_{s,gs}$

For $T_{R,gs}(t)$ an expression equivalent to (A.1) is obtained:

$$T_{R,gs}(n) = T_{0s}(n, D) - \frac{q_{gs}(n)}{4\pi\lambda_{gs}} \left( \ln \left( \frac{4a_{gs}\Delta t}{r_h^2} \right) - \gamma \right) - \sum_{j=1}^{n-1} \frac{q_{gs}(j)}{4\pi\lambda_{gs}} \ln \left( \frac{n+1-j}{n-j} \right) \quad (A.2)$$

$$\gamma = 0.5772, \quad \frac{5r_h^2}{a_{gs}} < \Delta t, \quad (n+1)\Delta t < \frac{1.78D^2}{a_{gs}}$$

where

- $T_{0s}(n, D)$ is the temperature at the hose depth $z = D$ at time $t = (n+1)\Delta t$ (°C)
- $r_h$ is the outer radius of the hose
- $a_{gs}$ is the thermal diffusivity of the soil (m)
- $a_{gs}$ is the thermal diffusivity of the soil around the GSHE (m$^2$/s)
- $\lambda_{gs}$ is the thermal conductivity of the soil (W/(m·K))
- $D$ is the GSHE pipe/hose depth (m)

A.3 Calculating $T_{R,gs}$, $t > t_{s,gs}$

The expression in (A.2) is under normal circumstances valid for about a month. For times larger than $t_{s,gs}$ the following expression is used, based on (2.8, but with GSHE parameters.

$$T_{R,gs}(n) = T_{0s}(n, D) - \frac{q_{gs}(n) - \bar{q}_{gs}}{4\pi\lambda_{gs}} \left( \ln \left( \frac{4a_{gs}\Delta t}{r_h^2} \right) - \gamma \right) - \sum_{j=m}^{n-1} \frac{q_{gs}(j) - \bar{q}_{gs}}{4\pi\lambda_{gs}} \ln \left( \frac{n+1-j}{n-j} \right) \quad (A.3)$$

$$\gamma = 0.5772, \quad \frac{5r_h^2}{a_{gs}} < \Delta t, \quad \frac{1.78D^2}{a_{gs}} < (n+1)\Delta t,$$

$$m\Delta t > (n+1)\Delta t - \frac{1.78D^2}{a_{gs}} > (m-1)\Delta t$$

where

- $\bar{q}_{gs}$ is the average heat load per unit length on the GSHE, defined by (2.10) but with GSHE parameters (W/m)
A.4 Calculating the heat load, \( t < t_{s,gs}, t_{s,bh} \)

Using the expressions (A.1) to (A.3) and (3.4) in (3.15) and solving for \( q_{bh} \) one obtains the following for short times:

\[
q_{bh}(n) = \frac{R_h + R_{c,gs} + L_{gs}K_{gs}}{f_{den}} Q(n)
\]

\[
+ \frac{L_{gs}}{f_{den}} (T_{0m} - T_{0s}(n, D))
\]

\[
- \frac{L_{gs}}{f_{den}} \sum_{i=1}^{n-1} q_{bh}(i) R_{past,bh}(i, n))
\]

\[
+ \frac{L_{gs}}{f_{den}} \sum_{j=1}^{n-1} q_{gs}(j) R_{past,gs}(j, n),
\]

\[
\gamma = 0.5772, \quad \frac{5r_h^2}{a_{gs}} < \Delta t, \quad \frac{1.78D^2}{a_{gs}} > (n + 1)\Delta t,
\]

where the following variables have been introduced to reduce the size of the expressions

\[
f_{den} = L_{gs}(R_{c,bh} + R_b) + L_{bh}(R_{c,gs} + R_h) + L_{gs}L_{bh}(K_{gs} + K_{bh})
\]

\[
R_{c,bh} = \frac{1}{4\pi \lambda_{bh}} \left( \ln \left( \frac{4a_{bh} \Delta t}{r_b^2} \right) - \gamma \right),
\]

\[
R_{c,gs} = \frac{1}{4\pi \lambda_{gs}} \left( \ln \left( \frac{4a_{gs} \Delta t}{r_h^2} \right) - \gamma \right),
\]

\[
R_{past,bh}(i, n) = \frac{1}{4\pi \lambda_{bh}} \ln \left( \frac{n + 1 - i}{n - i} \right),
\]

\[
R_{past,gs}(j, n) = \frac{1}{4\pi \lambda_{gs}} \ln \left( \frac{n + 1 - j}{n - j} \right).
\]
A.5 Calculating the heat load, $t > t_{s,gs}, t < t_{s,bh}$

For longer times we obtain

$$q_{bh}(n) = \frac{R_h + R_{c,gs} + L_{gs}K_{gs}}{f_{den}}Q(n)$$

$$+ \frac{L_{gs}}{f_{den}}(T_{0m} - T_{0s}(n, D))$$

$$- \frac{L_{gs}}{f_{den}} \sum_{i=1}^{n-1} q_{bh}(i)R_{past,bh}(i, n))$$

$$+ \frac{L_{gs}}{f_{den}} \sum_{j=m}^{n-1} (q_{gs}(j) - \bar{q}_{gs})R_{past,gs}(j, n)$$

$$+ \frac{L_{gs}}{f_{den}} \bar{q}_{gs}(R_{s,gs} - R_{c,gs})$$

(A.11)

where

$$\gamma = 0.5772, \quad \frac{5r_h^2}{a_{gs}} < \Delta t, \quad \frac{1.78D^2}{a_{gs}} < (n + 1)\Delta t,$$

$$m\Delta t > (n + 1)\Delta t - \frac{1.78D^2}{a_{gs}} > (m - 1)\Delta t$$

$$R_{s,gs}$$ is the steady state thermal resistance (m·K/W), defined as

$$R_{s,gs} = \frac{1}{2\pi \lambda_{gs}} \ln \left( \frac{2D}{r_h} \right).$$

(A.12)

Equation (A.12) is valid if the parallel hoses are placed far enough apart for any influence to be negligible, otherwise another expression is needed.