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Stochastic final-state dynamics of widening entanglement — a possible description of quantum measurement

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Abstract. The measurement process of quantum mechanics is analysed in the scattering theory of quantum field theory. A matrix of bilinear forms of the scattering amplitudes (the R-matrix) is used as the basic descriptive tool. The measurement process is viewed as a final-state interaction described through a series of linear stochastic mappings of the R-matrix, not changing the observable to be measured. The unknown details of the measurement apparatus enter through the stochasticity of the mappings. Although linear in terms of the R-matrix, the mappings are non-linear in the density matrix, which is obtainable from the R-matrix through normalization. The eigenstates of the observable are the attractors of the mapping process. This result, known from previous generalizations of quantum mechanics, is here obtained within linear quantum mechanics. The conclusion is that the measurement process can be understood within relativistic quantum field theory itself without any generalization or metatheory.

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1. Introduction

A non-destructive measurement on a quantum system involves a bifurcation process. In terms of the squared moduli of the components of the wave function (the diagonal elements of the density matrix), i.e., in the relevant probability simplex, it involves a movement from an initial position to the corners of the simplex. In the ensemble of such movements, the centre of mass remains at the point of origin.

For an ensemble of measurement processes, this bifurcation can be seen as a diffusion process. A *single* measurement process, can then be viewed as a random walk determined by stochastic variables and ending up in one corner of the probability simplex. The stochastic variables of the random walk in a single process then come from intrinsic properties in the measurement apparatus which is a macroscopic system impossible to know in detail.

The theoretical framework for our discussion is scattering theory within quantum field theory, where the stochastic variables determine factors in the final-state interaction of the quantum system and the measurement apparatus. The discussion is carried out in terms of a set of bilinear forms of scattering amplitudes, here called the R-matrix. This matrix plays the rôle of a non-normalized density matrix for the final state. Stochastic linear mappings of this matrix leading to random walk (single case) or diffusion (ensemble case) of the density matrix, have been used in previous discussions of the measuring process [1-3]. The point of view taken here

is that such mappings can be interpreted as a final-state interaction in a scattering process, through which the quantum system becomes increasingly entangled with the measurement apparatus [4].

The R-matrix is introduced in the next section together with the density matrix obtained from it through normalization. There we also describe the diffusion process as a series of stochastic linear mappings of the R-matrix. The individual paths of a series of such mappings as well as the ensemble of paths are discussed in Section 3. In Section 4 these paths are identified as describing a series of final-state interactions leading to a widening entanglement of the quantum system with the measurement apparatus.

A linear theory of the interaction between the quantum system μ and the measurement apparatus (described by stochastic variables) leads to a non-linear (and stochastic) dependence on the density matrix resulting from the internal interaction within μ itself (prior to measurement). An explanation within quantum field theory for this is sketched in Section 5. A brief summary is given in the concluding section together with a sketch of an experimental test..

2. The R-matrix — a non-normalized density matrix of final states

Consider a scattering process in a quantum system μ from a certain initial state $\rho^{(0)} = |0\rangle\langle 0|$ into a set of outgoing states in a Hilbert space spanned by an orthonormal basis formed by the eigenstates of a certain operator F with non-degenerate eigenvalues, i.e.,

$$\begin{aligned} |j\rangle, \quad j = 1, \dots, n; \\ \langle j|0\rangle = 0, \quad \langle j|k\rangle = \delta_{jk}, \\ F|j\rangle = f_j|j\rangle, \quad \text{with } f_j \neq f_k, \quad \text{for } j \neq k. \end{aligned} \tag{1}$$

We shall consider a hermitean matrix, the *R-matrix*, defined to be bilinear in the scattering amplitudes M_{j0} and their complex conjugates,

$$R = (R_{jk}) = M\rho^{(0)}M^\dagger, \quad R_{jk} = M_{j0}M_{k0}^* = R_{kj}^*. \tag{2}$$

Then R also satisfies the projector property,

$$R^2 = wR, \quad w = \text{Tr } R = \sum_{j=1}^n |M_{j0}|^2. \tag{3}$$

The trace w is proportional to the total transition rate. (As such it will have the rôle of a *weight* in a competition between several paths determined by the stochastic parameters.) The density matrix/projection matrix for the final state of the scattering process is

$$\rho = w^{-1}R = R / \text{Tr } R. \tag{4}$$

We shall use the basis (1), where F is diagonal and consider linear stochastic transformations of the R-matrix,

$$\begin{aligned} R &\rightarrow R + \Delta R, \\ \Delta R &= \Delta \Lambda R + R \Delta \Lambda^\dagger + \text{higher order terms in } \Delta \Lambda, \Delta \Lambda^\dagger; \\ \langle \langle \Delta \Lambda \rangle \rangle &= 0. \end{aligned} \tag{5}$$

Here we have used the notation $\langle \langle \rangle \rangle$ for a mean over the stochastic variables. We shall assume $\Delta \Lambda$ to be real and commuting with F , i.e., diagonal in the basis (1). We shall also assume the different channels to be independent. Then

$$\begin{aligned} \Delta \Lambda &= \frac{1}{2} \sum_{j=1}^n \Delta \chi_j |j\rangle \langle j|, \quad \Delta \chi_j^* = \Delta \chi_j; \\ \langle \langle \Delta \chi_j \rangle \rangle &= 0, \\ \langle \langle \Delta \chi_j \Delta \chi_k \rangle \rangle &= \eta_j^2 \delta_{jk}; \quad \eta_j \ll 1. \end{aligned} \tag{6}$$

Further, we assume that the transformations, in the mean, do not favour any channel at the expense of the others, i.e., that

$$\langle \langle \Delta R_{jj} \rangle \rangle = \frac{\langle \langle \Delta w \rangle \rangle}{w} R_{jj}. \tag{7}$$

Also $R + \Delta R$ has to be an R-matrix, satisfying (3),

$$\begin{aligned} (R + \Delta R)^2 &= (w + \Delta w)(R + \Delta R), \\ \Delta w &= \text{Tr } \Delta R. \end{aligned} \tag{8}$$

These assumptions together determine the second order terms in (5) and we get

$$\Delta R = \Delta \Lambda R + R \Delta \Lambda - \frac{1}{2} [\Delta \Lambda, [\Delta \Lambda, R]]; \tag{9}$$

(Making $\Delta \chi_j$ complex would not affect w , nor the diagonal elements of R or ρ .) For simplicity we also assume all η_j in (6) to have the same value $\eta \ll 1$. Furthermore, for simplicity we choose $\Delta \chi_j$ to take on only the following two values with equal a priori probability,

$$\Delta \chi_j = \pm \eta. \tag{10}$$

We express (9) in terms of the matrix elements and the trace,

$$\begin{aligned}\Delta R_{jk} &= \frac{1}{2} R_{jk} [\Delta \chi_j + \Delta \chi_k - \frac{1}{2} (\eta^2 - \Delta \chi_j \Delta \chi_k)], \\ \Delta w &= \sum_{j=1}^n R_{jj} \Delta \chi_j,\end{aligned}\tag{11}$$

which have the means

$$\begin{aligned}\langle\langle \Delta R_{jk} \rangle\rangle &= -\frac{1}{4} \eta^2 (1 - \delta_{jk}) R_{jk}, \\ \langle\langle \Delta w \rangle\rangle &= 0,\end{aligned}\tag{12}$$

and variances/correlations

$$\begin{aligned}\langle\langle \Delta R_{jk} \Delta R_{lm} \rangle\rangle &= \frac{1}{4} \eta^2 R_{jk} R_{lm} (\delta_{jl} + \delta_{jm} + \delta_{kl} + \delta_{km}), \\ \langle\langle \Delta w \Delta R_{jk} \rangle\rangle &= \frac{1}{2} \eta^2 R_{jk} (R_{jj} + R_{kk}), \\ \langle\langle \Delta w^2 \rangle\rangle &= \eta^2 \sum_{j=1}^n R_{jj}^2.\end{aligned}\tag{13}$$

We shall consider (11) to (13) to constitute a step in a random walk, and we shall study an ensemble of such random walks.

For the density matrix/projection matrix ρ defined in (4), the transformation induces the change

$$\Delta \rho_{jk} = \rho_{jk} \left(1 + \frac{\Delta w}{w}\right)^{-1} \left(\frac{1}{2} (\Delta \chi_j + \Delta \chi_k) - \sum_{l=1}^n \rho_{ll} \Delta \chi_l - \frac{1}{4} (\eta^2 - \Delta \chi_j \Delta \chi_k)\right),\tag{14}$$

which is non-linear in ρ . The last bracket in (14) makes non-diagonal elements of ρ drift to zero. To obtain the mean of the step (14), we have to use the relative weight $1 + \frac{\Delta w}{w}$, which balances the the corresponding inverse factor. We thus get the drift

$$\langle\langle \Delta \rho_{jk} \rangle\rangle = -\frac{\eta^2}{4} \rho_{jk} (1 - \delta_{jk}),\tag{15}$$

which leaves the means of the diagonal elements unchanged. Because of this and since ρ remains a one-dimensional projector with its non-diagonal elements going to zero, the attractors of a random walk of steps (11) have to be $|j\rangle\langle j|$ with frequency ρ_{jj} .

For the correlations and variances of the differences (14), we get from (4) and (13)

$$\begin{aligned} \langle\langle \Delta\rho_{jk}\Delta\rho_{lm} \rangle\rangle &= \eta^2 \rho_{jk}\rho_{lm} \left(\frac{1}{4}(\delta_{jl} + \delta_{jm} + \right. \\ &\left. \delta_{kl} + \delta_{km}) - \frac{1}{2}(\rho_{jj} + \rho_{kk} + \rho_{ll} + \rho_{mm}) + \sum_{r=1}^n \rho_{rr}^2 \right). \end{aligned} \quad (16)$$

In particular, for the diagonal elements [1,2],

$$\langle\langle \Delta\rho_{jj}\Delta\rho_{kk} \rangle\rangle = \eta^2 \rho_{jj} \rho_{kk} (\delta_{jk} - \rho_{jj} - \rho_{kk} + \sum_{r=1}^n \rho_{rr}^2). \quad (17)$$

Thus, whereas there is no systematic drift for the diagonal elements of ρ , there is diffusion with the corners of the probability simplex as attractors.

The entropy $-\text{Tr}[\rho \ln \rho]$ stays equal to zero and the entropy based on the diagonal elements of ρ ,

$$S = - \sum_{j=1}^n \rho_{jj} \ln \rho_{jj}, \quad (18)$$

changes through the mapping according to

$$\langle\langle \Delta S \rangle\rangle = -\frac{1}{2} \left\langle \left\langle \sum_{j=1}^n \frac{1}{\rho_{jj}} \Delta\rho_{jj}^2 \right\rangle \right\rangle = -\frac{1}{2} \eta^2 \sum_{j=1}^n \rho_{jj} (1 - \rho_{jj}) \leq 0. \quad (19)$$

Thus, the entropy (18) decreases until all diagonal elements of ρ are zero or one [2], in accordance with the previous discussion.

3. Random walk for the R-matrix

Let $R^{(x+1)} = R^{(x)} + \Delta R^{(x)}$ be the new R-matrix after one step. Then to second order, we have from (11),

$$\begin{aligned} R_{jk}^{(x+1)} &= R_{jk}^{(x)} \sqrt{(1 + \Delta\chi_j^{(x)})(1 + \Delta\chi_k^{(x)})}, \\ w^{(x+1)} &= \sum_{j=1}^n R_{jj}^{(x)} (1 + \Delta\chi_j^{(x)}). \end{aligned} \quad (20)$$

Following a whole random walk, consisting of $2X$ steps from $R = R^{(0)}$ to $R = R^{(2X)}$ maintaining, in each step, terms to second order in η ,

$$\begin{aligned}
R_{jk}^{(2X)} &= R_{jk}^{(2X)}(\underline{X}) = R_{jk}^{(0)}(1+\eta)^{X+\frac{1}{2}(X_j+X_k)}(1-\eta)^{X-\frac{1}{2}(X_j+X_k)} = \\
&\exp(-\eta^2 X) R_{jk}^{(0)} e^{\eta(X_j+X_k)} = \exp(-\eta^2 X) w^{(0)} \rho_{jk}^{(0)} e^{\eta(X_j+X_k)}, \\
w^{(2X)} &= w^{(2X)}(\underline{X}) = \exp(-\eta^2 X) w^{(0)} \sum_{j=1}^n \rho_{jj}^{(0)} e^{2\eta X_j},
\end{aligned} \tag{21}$$

where we have introduced

$$\underline{X} = (X_1, X_2, \dots, X_n), \quad X_j = \frac{1}{2} \sum_{x=1}^{2X} \Delta \chi_j^{(x)} / \eta, \quad -X \leq X_j \leq X. \tag{22}$$

Here X_j is an integer.

From (4) and (21), we immediately get the density matrix/ projection matrix,

$$\rho_{jk}^{(2X)}(\underline{X}) = \frac{\rho_{jk}^{(0)} e^{\eta(X_j+X_k)}}{\sum_{l=1}^n \rho_{ll}^{(0)} e^{2\eta X_l}}, \tag{23}$$

i.e., the final density matrix after mapping is highly non-linear in the original one.

There are 2^{2nX} random walks in the ensemble, and $\prod_{k=1}^n \binom{2X}{X+X_k}$ out of them are characterized by $\underline{X}$. We have to complement this weight with the weight factor $w^{(2X)}/w^{(0)}$, coming from dynamics, to get the following probability distribution for $\underline{X}$,

$$\begin{aligned}
Q(\underline{X}) &= 2^{-2nX} \left(\prod_{j=1}^n \binom{2X}{X+X_j} \right) \frac{w^{(2X)}(\underline{X})}{w^{(0)}} = \sum_{l=1}^n \rho_{ll}^{(0)} Q_l(\underline{X}), \\
Q_l(\underline{X}) &= \left(\prod_{j \neq l} P_0(X_j) \right) P_\eta(X_l),
\end{aligned} \tag{24}$$

where the binomial distribution

$$P_\beta(Y) = \binom{2X}{X+Y} \left(\frac{1+\beta}{2} \right)^{X+Y} \left(\frac{1-\beta}{2} \right)^{X-Y} \tag{25}$$

has the means $\langle\langle Y \rangle\rangle = X\beta$ and $\langle\langle Y^2 \rangle\rangle = X^2\beta^2 + \frac{1}{2}X$. From this we get the following means and variances of (24),

$$\begin{aligned} \sum_{\underline{X}} Q_l(\underline{X}) \underline{X} &= \underline{X}^{(l)}, \quad X_j^{(l)} = X\eta \delta_{jl}; \quad \langle\langle X_j \rangle\rangle = \sum_{\underline{X}} Q(\underline{X}) X_j = X\eta \rho_{jj}^{(0)}; \\ \sum_{\underline{X}} Q_l(\underline{X}) (\underline{X} - \underline{X}^{(l)})^2 &= \frac{1}{2}nX. \end{aligned} \quad (26)$$

The distribution $Q_l(\underline{X})$ has a convenient approximation in terms of continuous vector variables $\underline{z}$ with components in the interval $(-\eta^{-1}, \eta^{-1})$:

$$\begin{aligned} Q_l(\underline{X}) &= d^n \underline{z} \, q_l(\underline{z}); \quad q_l(\underline{z}) = \left(\frac{X\eta^2}{\pi} \right)^{\frac{1}{2}n} \exp\left(-X\eta^2 (\underline{z} - \underline{z}^{(l)})^2\right); \\ \underline{X} &= X\eta \underline{z}; \quad \underline{X}^{(l)} = X\eta \underline{z}^{(l)}; \quad z_j^{(l)} = \delta_{jl}. \end{aligned} \quad (27)$$

With increasing X , $Q(\underline{X})$ splits from a unimodal distribution around $X_j = X\eta \rho_{jj}^{(0)}$ to an n -modal distribution with the centres $\underline{X} = \underline{X}^{(l)}$. The splitting occurs around $X\eta^2 = 1$. In the thermodynamic limit, i.e., for $X\eta^2 \rightarrow \infty$, we have for the total distribution and the partial distributions in $\underline{z}$,

$$q(\underline{z}) = \sum_{l=1}^n \rho_{ll}^{(0)} q_l(\underline{z}); \quad q_l(\underline{z}) = \delta(\underline{z} - \underline{z}^{(l)}). \quad (28)$$

In the limit, the state projector (23) corresponding to $\underline{z} = \underline{z}^{(l)}$ is

$$\rho_{jk}^{(2X)} = \delta_{jl} \delta_{kl}. \quad (29)$$

and the mean of the density matrix is

$$\langle\langle \rho_{jk}^{(2X)} \rangle\rangle = \sum_{l=1}^n \rho_{ll}^{(0)} \delta_{jl} \delta_{kl}, \quad (30)$$

in agreement with the previous discussion. This movement of ρ from $\rho^{(0)}$ to (29) for one l (with the mean (30)) describes the bifurcation that takes place in an ensemble of non-destructive measurements of F on μ .

We shall see that the random walk presented here, can describe a microsystem starting with the R-matrix $R^{(0)}$ (density matrix $\rho^{(0)}$), which, through soft final-state interaction becomes

successively entangled with the macroscopic system A (the measurement apparatus), through an interaction characterized by the stochastic variables.

4. Entanglement through soft final-state interaction

When a small quantum system μ , with states in the Hilbert space spanned by (1), interacts non-destructively with a large system A , triggered to be influenced by μ , what happens is a series of consecutive entanglements of μ with a rapidly growing part of A . The system A is assumed to consist of various parts,

$$A = (A_1, A_2, \dots, A_n), \quad (31)$$

where A_j is prepared to interact non-destructively and get entangled with μ in the state $|j\rangle$, here denoted by $|j\rangle_\mu$. The entanglement is assumed to start small and to propagate through A_j in a way that depends on the stochastic variables through the aggregated quantities $\underline{X}$ defined in (22).

With μ entangled with A , there is no longer any pure state describing μ itself. Therefore we have to extend the description to include the total system of μ and A *together*.

To do this, we introduce the (orthonormal) basis states,

$$\begin{aligned} (a) \quad & |0, \bar{0}, \underline{X}, \Delta \underline{\chi}\rangle = |0\rangle_\mu \otimes \prod_{k=1}^n |0, 0, X_k, \Delta \underline{\chi}_k\rangle_{A_k}; \\ (b) \quad & |j, \bar{0}, \underline{X}, \Delta \underline{\chi}\rangle = |j\rangle_\mu \otimes \prod_{k=1}^n |0, 0, X_k, \Delta \underline{\chi}_k\rangle_{A_k}; \\ (c) \quad & |j, x, \underline{X}, \Delta \underline{\chi}\rangle = |j\rangle_\mu \otimes |1, x, X_j, \Delta \underline{\chi}_j\rangle_{A_j} \otimes \prod_{k \neq j}^n |0, 0, X_k, \Delta \underline{\chi}_k\rangle_{A_k}, \quad x = 1, \dots, 2X; \end{aligned} \quad (32)$$

with

$$\begin{aligned} \Delta \underline{\chi} &= (\Delta \underline{\chi}_1, \dots, \Delta \underline{\chi}_n), \\ \Delta \underline{\chi}_k &= (\Delta \chi_k^{(0)}, \dots, \Delta \chi_k^{(2X-1)}). \end{aligned} \quad (33)$$

Here (a) is the initial state, (b) is the j th basis vector of the scattering state, (c) is the corresponding basis vector of the state from the first entanglement between μ and A ($x = 0$), in this case A_j (copying), to the final state ($x = 2X$). Here x is a measure of the extent of entanglement. We focus on the end states $|j, 2X, \underline{X}, \Delta \underline{\chi}\rangle$, assuming final absorption (irreversible registration) to take place from there.

We go now to an interpretation of the previously presented mappings of R-matrices. Above we considered mappings of R-matrices within the same linear space. We shall now consider mappings between different subspaces of the total Hilbert space. To begin with, we consider the set of stochastic parameters $\Delta\chi$ in (32) as fixed.

For given $\Delta\chi$, we then consider the R-operators $R^{(0)}(\Delta\chi)$ and $R^{(2X)}(\Delta\chi)$, defined as follows:

$$\begin{aligned} R^{(0)}(\Delta\chi) &= \sum_{j,k=1}^n R_{jk}^{(0)} \left| j, 0, \underline{X}, \Delta\chi \right\rangle \left\langle k, 0, \underline{X}, \Delta\chi \right|; \\ R^{(x)}(\Delta\chi) &= \sum_{j,k=1}^n R_{jk}^{(x)}(\underline{x}) \left| j, x, \underline{X}, \Delta\chi \right\rangle \left\langle k, x, \underline{X}, \Delta\chi \right|; \\ R^{(2X)}(\Delta\chi) &= \sum_{j,k=1}^n R_{jk}^{(2X)}(\underline{X}) \left| j, 2X, \underline{X}, \Delta\chi \right\rangle \left\langle k, 2X, \underline{X}, \Delta\chi \right|. \end{aligned} \tag{34}$$

We assume copying (from (b) to (c) in (32)) to occur with equal mechanisms in all channels. This is a simple (unrealistic but unessential) assumption, and then all $\underline{X}$ -dependence concerns propagation inside the subsystems (31) of A . The mapping goes from $R^{(0)}(\Delta\chi)$ to $R^{(2X)}(\Delta\chi)$ from where final absorption takes place. In (34), for even x , $\underline{x}$ is constructed like $\underline{X}$ in (22) for x instead of $2X$ steps, and $R_{jk}^{(x)}(\underline{x})$ accordingly (see (21)). Since final absorption takes place only from one given x ($x = 2X$), we can neglect the fact that along the way to absorption, we can have states that involve superpositions of several values of x .

The multiplicative mapping (20) can be interpreted as the result of a class of *final-state interactions* as follows. The outgoing particles of the scattering process interact independently, with small momentum and energy transfer, with their environment, subsystems of A_j , which they pass without being absorbed. These systems are assumed to be independent and to be macroscopic and metastable; they are acted on by the outgoing particles, but do not act on them except for the small energy-momentum transfer. As is very well known from infrared interaction in quantum electrodynamics [5], the scattering amplitudes are modified by factors describing the interaction between the outgoing particles and the subsystems of $A_1, A_2, \dots, A_n$ and the interaction within those systems themselves. The factors depend on the stochastic variables, and we are led to (20) and (21).

5. Why non-linearities in a linear unitary development?

The density matrices that have been used can be considered to be restricted to a Hilbert space where the total momentum/energy is given within a very narrow interval (almost plane-wave states). It is natural to ask the question why the density matrix for the final state $\rho_{jk}^{(2X)}(\underline{X})$ in (23) has such a strong non-linear behaviour in terms of the density matrix $\rho_{jk}^{(0)}$ which would obtain for the scattering state without any final-state interaction. The answer to this is that when

the scattering interaction within μ is taken into account together with the $\mu - A$ interaction, this has to be done to all orders in both kinds of interaction, although the $\mu - A$ interaction takes place later than the internal μ interaction.

In quantum field theory, the ordering theorem (Wick's theorem [6]) gives the time-ordered products in terms of normal-ordered products with Feynman diagrams and the corresponding mathematical expressions as a result. A particular feature of this is that particle propagators go both forward and backward in time, i.e., they go forward and backward between interaction within μ and $\mu - A$ interaction, so that $\rho^{(0)}$ can be found to all orders in $\rho^{(2X)}(\underline{X})$. This was described in more detail in ref. [3], using a Feynman-diagram method for the R-matrix [7].

One function of this mechanism is that unitarity is maintained. Clearly, the denominator of (23) functions as a renormalization factor which is necessary to keep normalization.

6. Conclusion

The description of measurement suggested here deals with the combined system of the quantum system μ and the measurement apparatus A (for the observable F). After μ and A have become increasingly entangled with each other, the final state of the combined system $\mu + A$ (non-normalized) is $\underline{R}^{(2X)}(\Delta\chi)$ of the last equation (34). The normalized density matrix is (23).

It depends on the outgoing state $\rho^{(0)}$ of μ before measurement, and on the stochastic variables of A , represented by $\underline{X}$. This dependence is rather intricate. Looked at in another way, (23) is quite simple: rows and columns of $\rho_{jk}^{(0)}$ are multiplied by factors determined by the random walk, and the result of this is again normalized.

The ensemble density in $\underline{X}$ -space, the space of "hidden variables", i.e., $Q(\underline{X})$ of (24), describes the competing paths leading to the final result. It gets support only from n different, clearly distinguishable, narrow peaks. The l th peak with a height proportional to $\rho_{ll}^{(0)}$ corresponds to the measurement result f_l for F . For a single measurement, a single random walk with a value of $\underline{X}$ in one of the peaks leads to one of the corresponding results. In the thermodynamic limit, the distribution in terms of the macroscopic variables $\underline{z}$ is the sum (28) of δ -functions. The paths leading to other values of $\underline{z}$ are simply not competitive (too few combinations or too uncoordinated in terms of the stochastic enhancement/inhibition factors).

Philosophically, one can say that we have avoided a situation where the basic uncertainty in the epistemology of quantum measurement is allowed to make the ontology of quantum mechanics incomplete. Instead we have adopted a simple ontology where the quantum state of a system is considered to represent reality; then we derive the physics of quantum measurement and hence the epistemology of measurement. There is no classical separation possible between the process taking place within the quantum system μ and the process of interaction between μ and the measurement apparatus A ; they are strongly interwoven with each other.

One way to test experimentally the theoretical interpretation of the measurement process given here would be to simulate one mapping (or a set of mappings) in a measurement process and to

do this with a process that is not stochastic but can be controlled by the experimenter. One way would be the following.

An incoming photon is split into a superposition of two components of equal *a priori* probability amplitude travelling macroscopically separated ways. Each component meets a target of atoms, sufficiently extended and dense, so that there is no way for the photon to pass without a collision. Each target consists of one kind of atoms with one outer electron that can be knocked out by the photon, having sufficient energy for this. The atoms of the two targets are of the same kind but differ in energy, the outer electron being in a higher excited state in the atoms of one target than in the other. The main thing is to have a difference between the two targets in the phase space available to the knocked out electron.

If the ideas presented in this paper are correct, then such a difference in the downstream atom targets would change the probabilities for the photon's choice of path from a half-half situation to a situation in favour of the path leading to a larger phase space for the knocked out electron.

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Appendices

Appendix A. Derivation of (9)

We shall determine the second order term of ΔR in (5) for a hermitean $\Delta\Lambda$. The general form is

$$\Delta R = \Delta\Lambda R + R\Delta\Lambda + a(\Delta\Lambda^2 R + R\Delta\Lambda^2) + b\Delta\Lambda R\Delta\Lambda,$$

where a and b are real constants to be determined. To use the condition (7), we first note that

$$\begin{aligned}\langle\langle\Delta R_{jj}\rangle\rangle &= (2a + b)\eta_j R_{jj}, \\ \langle\langle\Delta w\rangle\rangle &= (2a + b)\sum_{k=1}^n \eta_k R_{kk}.\end{aligned}$$

The condition (7) then gives

$$0 = \langle\langle\Delta R_{jj}\rangle\rangle - \frac{R_{jj}}{w}\langle\langle\Delta w\rangle\rangle = (2a + b)R_{jj}\left(\eta_j - \frac{1}{w}\sum_{k=1}^n \eta_k R_{kk}\right)$$

which must hold for general positive R_{jj} and η_k . Hence

$$b = -2a$$

In particular, we note that

$$\begin{aligned}\langle\langle\Delta R_{jj}\rangle\rangle &= 0, \\ \langle\langle\Delta w\rangle\rangle &= 0.\end{aligned}$$

The general form for ΔR is now

$$\begin{aligned}\Delta R &= \Delta\Lambda R + R\Delta\Lambda + a(\Delta\Lambda^2 R + R\Delta\Lambda^2 - 2\Delta\Lambda R\Delta\Lambda), \\ \Delta w &= 2\text{Tr}(\Delta\Lambda R),\end{aligned}$$

To determine the constant a , we have to compute the terms appearing in (8):

$$\begin{aligned}R\Delta R &= R\Delta\Lambda R + wR\Delta\Lambda + a(R\Delta\Lambda^2 R + wR\Delta\Lambda^2 - 2R\Delta\Lambda R\Delta\Lambda) = \\ &= \frac{1}{2}\Delta w R + wR\Delta\Lambda + a(R\Delta\Lambda^2 R + wR\Delta\Lambda^2 - \Delta w R\Delta\Lambda), \\ \Delta R R &= \frac{1}{2}\Delta w R + w\Delta\Lambda R + a(R\Delta\Lambda^2 R + w\Delta\Lambda^2 R - \Delta w\Delta\Lambda R), \\ R\Delta R + \Delta R R &= \Delta w R + w(\Delta\Lambda R + R\Delta\Lambda) + a(2R\Delta\Lambda^2 R + w(\Delta\Lambda^2 R + R\Delta\Lambda^2) - \Delta w\Delta R) = \\ &= \Delta w R + w\Delta R + a(2w\Delta\Lambda R\Delta\Lambda + 2R\Delta\Lambda^2 R - \Delta w\Delta R). \\ \Delta R^2 &= \Delta\Lambda R\Delta\Lambda R + R\Delta\Lambda R\Delta\Lambda + w\Delta\Lambda R\Delta\Lambda + R\Delta\Lambda^2 R = \\ &= \frac{1}{2}\Delta w\Delta R + w\Delta\Lambda R\Delta\Lambda + R\Delta\Lambda^2 R.\end{aligned}$$

We then get from (8),

$$\begin{aligned}0 &= R\Delta R + \Delta R R + \Delta R^2 - w\Delta R - \Delta w R - \Delta w\Delta R = \\ &= (a + \frac{1}{2})(2w\Delta\Lambda R\Delta\Lambda + 2R\Delta\Lambda^2 R - \Delta w\Delta R).\end{aligned}$$

In general, the second factor here is different from zero. Hence

$$a = -\frac{1}{2}.$$

Appendix B. Derivation of (14)

From (4) and (11) we get

$$\Delta\rho = \frac{R + \Delta R}{w + \Delta w} - \frac{R}{w} = \left(1 + \frac{\Delta w}{w}\right)^{-1} \left(\frac{\Delta R}{w} - \frac{\Delta w}{w}\rho\right),$$

i.e.,

$$\Delta\rho_{jk} = \rho_{jk} \left(1 + \frac{\Delta w}{w}\right)^{-1} \left[\frac{1}{2}(\Delta\chi_j + \Delta\chi_k) - \frac{1}{4}(\eta^2 - \Delta\chi_j\Delta\chi_k) - \sum_{l=1}^n \rho_{ll} \Delta\chi_l \right].$$

References

- [1] Gisin N 1984 *Phys. Rev. Letters* **52** 1657
- [2] Gisin N and Percival I C 1992 *J. Phys. A* **25** 5677
- [3] Ghirardi G-C, Rimini A and Weber T, 1986 *Phys. Rev. D* **34** 470
 Ghirardi G-C, P, Pearle and Rimini A 1990 *Phys. Rev. A* **42** 78
 Diósi L 1988 *Phys. Letters A* **129** 419
- [4] Eriksson K-E 2007 *ArXiv:0705.1649v1 [quant-ph]* 11 May
- [5] Jauch JM and Rohrlich F 1954 *Helv. Phys. Acta* **27** 613
- [6] Wick G 1950 *Phys. Rev.* **80** 268
- [7] Nakanishi N 1958 *Progr. Theor. Physics (Japan)* **19** 159