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Modelling of Water Removal during a Paper Vacuum Dewatering Process Using a Level-Set Method

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Abstract

Water removal in paper manufacturing is an energy-intensive process. The dewatering process generally consists of four stages of which the first three stages include mechanical water removal through gravity filtration, vacuum dewatering and wet pressing. In the fourth stage, water is removed thermally, which is the most expensive stage in terms of energy use.

In order to analyse water removal during a vacuum dewatering process, a numerical model was created by using a Level-Set method. Various different 2D structures of the paper model were created in MATLAB code with randomly positioned circular fibres with identical orientation. The model considers the influence of the forming fabric which supports the paper sheet during the dewatering process, by using volume forces to represent flow resistance in the momentum equation.

The models were used to estimate the dry content of the porous structure for various dwell times. The relation between dry content and dwell time was compared to laboratory data for paper sheets with basis weights of 20 g/m² and 50 g/m² exposed to vacuum levels between 20 and 60 kPa. The comparison showed reasonable results for dewatering and air flow rates. The random positioning of the fibres influences the dewatering rate slightly. In order to achieve more accurate comparisons, the random orientation of the fibres needs to be considered, as well as the deformation and displacement of the fibres during the dewatering process.

Keywords: Simulation, Multiphase flow, Level-Set method, Porous media, Microstructure, Vacuum dewatering

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1. Introduction

Water removal in paper manufacturing is an energy-intensive process. The dewatering process generally consists of four stages, where the first stage comprises water removal due to gravity and low vacuum filtration to reach a dry content in the range of 6 %. In the second stage, higher levels of vacuum are applied through suction boxes and suction rolls. Most of the water in the fibre suspension is removed in the first two stages, since the dry content after vacuum dewatering ends at around 20 %. The third stage of dewatering is performed in the press section by compressing the paper in one or more press nips leading to a dryness of around 40 %. The final stage consists of thermal drying by which the remaining water is evaporated on steam-heated cylinders until the dry content has reached approximately 90 %, the remaining moisture being hygroscopically bound to the fibres.

The thermal drying section considerably exceeds the other dewatering stages in terms of energy use. Hence, increasing the incoming dryness to the thermal stage could lead to a major reduction of the production cost. The specific energy use of thermal drying is roughly 800 kWh/ton of produced paper. The electricity use of the vacuum systems of 27 fine paper machines varies widely between 38 kWh/ton and 132 kWh/ton (Lahtinen and Karvinen, 2010). Reduced dwell times, decreased vacuum levels as well as speed control and the correct choice of vacuum pumps are all measures to reduce the vacuum system's electricity use (Neun, 1994; Håkansson, 2010).

Several researchers have studied vacuum dewatering either by using laboratory equipment or by using pilot paper machines. They have investigated the influence of process parameters, such as applied vacuum, dwell time and basis weight, on the dryness (Britt and Unbehend, 1985; Neun, 1994; Räisänen et al. 1996; Granevald, 2003; Åslund et al., 2008; Pujara et al., 2008a and 2008b). It can be concluded from these studies that the dryness increases with an increased time spent in vacuum at a specific vacuum level. The dewatering rate is initially high and gradually decreases to zero, leading the dry content to reach an almost constant value. In order to increase the dry content further, a higher vacuum level must be applied. The initially present water consists of free water between the fibres, water associated with fibre surfaces, or water that is more tightly bound in the pores of the fibre walls. Only the water bound loosely enough for the vacuum to exceed the capillary pressure can be removed by means of vacuum dewatering. In standard operations in paper machines, the vacuum levels range from 15 kPa up to 80 kPa according to (Räisänen, 2000)

The mechanisms involved in vacuum dewatering have been described as web compression, displacement of water by air, and rewetting from the forming fabric (Ramaswamy, 2003; Åslund and Vomhoff, 2008a). Åslund et al., (2008) found the web compression during the suction pulse to be considerable. Pujara et al., (2008a and 2008b) found that the air flow through the sheet during vacuum dewatering increased with increasing dry content. The relative importance of the different dewatering mechanisms is not yet clear. It is, however, evident that both fluid phases, water and air, are present in the web during the suction pulse.

Gravity and low vacuum dewatering prior to the penetration of air have been modelled based on the filtration equation (Roux and Rueff, 2010). Approaches taken for the modelling of a dry content increase during high vacuum dewatering include data regression to an empirical equation (Neun, 1994, 1996), as well as the application of the instationary heat conduction equation to model the dry content increase as a function of time and penetration depth in the web (Räisänen et al. 1995). Kawka (2001) used the continuity equations for air, water and fibre as a basis when modelling vacuum dewatering assuming that the flow of water and air inside the sheet adheres to Darcy's law. This latter assumption is difficult to justify, considering the high flow rates and significant air density changes in the wet sheet.

Thus, our review of the existing literature presents no extensive research of numerical studies of a two-phase flow applied to vacuum dewatering processes for paper sheets. However, numerous researchers, such as Jackson and James (1986), have experimentally and theoretically studied single-flow mechanics in fibrous porous materials. Zhu et al., (1995) developed a mechanistic model to describe fluid permeation through compressible fibre beds. The model is based on micro-mechanical theories of fibre assemblies and showed good agreement with experimental results in terms of correlating fibre bed thickness, flow rate and hydraulic pressure drop. Nilsson and Stenström (1997) have focused on characterising permeability numerically in ordered and non-ordered fibre structures of various shapes.

Throughout recent years, along with the development of computer power, porous structures are being represented with various percolation models. An overview of computer modelling of fluid permeation through porous structures has been conducted by (Matthews, 2000). The focus of the paper is presenting the applicability and reality of the models representing the complexity of the void space. Issues involving in computer generated fibre spaces is the connectivity and the pore space distribution.

Fluid inertia has been studied for moderate Reynolds number in various fibre arrangements by Yazdchi and Luding (2012) and Koch and Ladd (1997), wherein Yazdchi and Luding used finite element simulations in 2D with an extension of non-linear corrections to consider inertial effects. Moreover, Yazdchi et al., (2011 and 2012) have analysed ordered and random fibre arrangements wherein random fibre generator algorithms have been implemented to create microstructure conditions in order to characterize spatial heterogeneity of the fibre structure and correlate it to macroscopic properties. Moreover, the influence on the permeability based on sample size, boundary conditions, homogeneity and isotropy were studied. The relations are used to predict permeability for various fibre arrangements and porosity levels. Similar research has been conducted by (Chen and Papathanasiou, 2008) where they simulated transverse Stokes flow around randomly placed fibres with the same orientation and correlated it with the permeability. The permeability model is extended with statistical modifications in which the permeability is correlated to a mean nearest inter-fibre spacing parameter which characterises the heterogeneity of the fibre network.

A review of the presented literature shows that the main focus has previously been on determining the permeability in various fibre-web arrangements under single-phase flow conditions. Our approach is to establish a numerical model of a vacuum dewatering process by using a two-phase flow model. In this study, a 2D model of the paper sheet is analysed numerically using a Level-Set (LS) method coupled with the continuity and momentum equations. No implementation of the LS method on dewatering in paper has been found. The LS method is, however, established in describing various two-phase flow phenomena in porous media such as Herreros et al., (2006) in which they analysed seepage through a dam involving an interface motion of immiscible fluids. Prodanovic et al., (2006) used a LS method to model pore scale displacement of immiscible fluids as an approach to improve predictions of pore network models. They use a progressive quasi-static (PQS) algorithm to handle complex curvature issues for throat drainage and pore imbibition.

The model of the paper sheet is composed of cylindrical fibres with a diameter and a bound water content typical of Kraft softwood. The initial moisture content corresponds to that obtained after gravity dewatering, and the influence of the forming fabric is described with volume forces in the momentum equation. The purpose of analysing vacuum dewatering numerically is to gain a physical understanding of the process and eventually to obtain better methods of dimensioning industrial equipment. The aim is to calculate the dry content and the flow rates of water and air as a function of dwell time and vacuum levels. Numerical data is compared to experimental data. The influence of temperature is excluded from this study as the experimental data is performed at standard operational conditions.

2. Governing equations

2.1 Level-set function

As the water is removed from the paper sheet, air penetrates the pores of the paper. In order to consider two different phases of fluids in the same system, the LS method is applied. The LS method is a numerical technique for tracking interfaces and shapes. The LS function is used to locate, in this case, air–water interface in which the function takes a positive or negative value depending on which side of the interface that is viewed:

$$\varphi(\vec{x}, t) = \begin{cases} > 0 \text{ if } \vec{x} \in \text{fluid 1} \\ = 0 \text{ if } \vec{x} \in \partial\Omega \\ < 0 \text{ if } \vec{x} \in \text{fluid 2}, \end{cases} \quad (1)$$

where $\varphi(\vec{x}, t)$ is the LS function and $\partial\Omega$ is the interface boundary. The interface motion is defined by an advection equation in an Eulerian representation:

$$\frac{\partial \varphi}{\partial t} + u_j \frac{\partial \varphi}{\partial x_j} = 0, \quad (2)$$

where the first term in this PDE is the local time derivative of the LS function, and the second term represents the advection of the interface. The LS function is defined as a signed distance function:

$$|\varphi(\vec{x})| = \min(|\vec{x} - \vec{x}_I|) \text{ for all } \vec{x}_I \in \partial\Omega, \quad (3)$$

where $\varphi(\vec{x}) = 0$ when $\vec{x}$ is on the interface boundary $\partial\Omega$. The signed distance function is approximated numerically in order to achieve smoothness of the solution. This is achieved by a first-order accurate smeared out approximation using a Heaviside function:

$$H(\varphi(\vec{x})) = \begin{cases} 0 & \varphi(\vec{x}) < -\varepsilon \\ \frac{1}{2} + \frac{\varphi(\vec{x})}{2\varepsilon} + \frac{1}{2\pi} \sin\left(\frac{\pi\varphi(\vec{x})}{\varepsilon}\right) & -\varepsilon \leq \varphi(\vec{x}) \leq \varepsilon \\ 1 & \varphi(\vec{x}) > \varepsilon \end{cases} \quad (4)$$

where ε is a parameter that determines the thickness of the numerically smeared interface. The value ε equals the maximum mesh element size divided by 4. The value of the smoothed LS function is 0.5 at the centre of the interface.

As Eq. (2) is solved during a simulation process, the motion of the fluid–fluid interface changes the LS function, which causes the smeared interface thickness to vary. The achievement of a constant interface thickness minimises numerical error due to smearing. Consequently, Eq. (2) is extended with a re-initialisation term:

$$\frac{\partial \varphi}{\partial t} + u_j \frac{\partial \varphi}{\partial x_j} = \gamma \frac{\partial \varphi}{\partial x_j} \left(\varepsilon \frac{\partial \varphi}{\partial x_j} - \varphi(1 - \varphi) \left| \frac{\partial \varphi}{\partial x_j} \right|^{-1} \right), \quad (5)$$

where γ is the re-initialisation parameter and $\varphi = H(\varphi(\vec{x}))$. Both the density and the viscosity is a function of the LS function according to

$$\rho = \rho_{air} + (\rho_{water} - \rho_{air})\varphi \quad (6)$$

$$\mu = \mu_{air} + (\mu_{water} - \mu_{air})\varphi. \quad (7)$$

The normal vector and the curvature at the air/water interface are determined with the LS function

$$n_j = \frac{\partial \varphi}{\partial x_j} \left| \frac{\partial \varphi}{\partial x_j} \right|^{-1}, \quad (8)$$

where the spatial derivative points in the direction of an increasing value of the LS function. The curvature is the divergence of the normal unit vector:

$$\kappa = \frac{\partial}{\partial x_j} \left(\frac{\partial \varphi}{\partial x_j} \left| \frac{\partial \varphi}{\partial x_j} \right|^{-1} \right), \quad (9)$$

where the parameter κ is positive for convex regions and negative for concave regions.

2.2 Continuity and momentum equation

In the two-phase system, the governing equations for mass and momentum are calculated. A direct numerical simulation with the Navier-Stokes (NS) equations with compressibility on air was implemented in the CFD software, COMSOL MultiPhysics 4.2a:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0, & \text{Air} \\ \frac{\partial u_i}{\partial x_i} = 0, & \text{Water} \end{cases} \quad (10)$$

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \mu \left[\frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] - \rho g_i + F_i + F_{st,i}. \quad (11)$$

Note in Eq. (11) that the two last terms represent sources in the NS equations. The first term is a momentum source term that represents the flow resistance exerted by the forming fabric. The last source term represents the surface tension between air/water and the fibre walls, see Eq. (12):

$$F_{st,i} = \frac{\partial}{\partial x_j} (\sigma (I_{ij} - n_i n_j) \delta), \quad (12)$$

where δ is a Dirac delta function, I_{ij} is the unit matrix, and σ is a surface tension coefficient.

3. Paper sheet

The 2D-model of the paper sheet is composed of cylindrical fibres with a diameter and bound water content typical of Kraft softwood fibres. Vacuum dewatering was performed in a bench scale laboratory setup (Granevald, 2005) that had been validated against data from the industrial process with good agreement in terms of the achievable dry content, as well as the time constant of the process. Once-dried and bleached softwood Kraft pulp was disintegrated

and beaten at 2000 revolutions in a PFI mill to yield a dewatering resistance in fair agreement with pulp used in the industry.

Sheets with dry basis weights of 20 and 50 g/m² were formed on a commercial forming fabric. Dewatering trials were performed at dwell times of 0.83, 1.43 and 2.00 ms for sheets with a basis weight of 20 g/m² and at 0.83, 1.67, 2.50, 3.33, 4.17 and 5.00 ms for sheets with a basis weight of 50 g/m².

The vacuum level was set to 40 kPa when dewatering the 20 g/m² sheets. For the 50 g/m² sheets, three vacuum levels 20, 40 and 60 kPa were used. The dry content was determined gravimetrically for the dewatered sheets, as well as for sheets not exposed to vacuum. The dry content of the sheet prior to vacuum dewatering was measured as 6 % and the achievable dry content during vacuum dewatering at 40 kPa was measured as 22 % for 20 g/m² sheets and 24 for 50 g/m² sheets. The difference is due to rewetting from the forming fabric. The amount of rewetted water is constant so that its influence on the dry content is greater for low basis weights. The pressure in the tank was logged during the pulse and the pressure increase was used to determine the air flow.

The fibre characterisation of the beaten pulp was carried out with an L & W FiberTester. The coarseness (length density) was found by Kullander et al., (2012) to be 0.168 mg/m, the fibre length and width were 2.10 mm and 29.6 µm, respectively. Assuming the volumes of dry matter and remaining water to be additional, the diameter of the water-containing fibre can be calculated as

$$D = \sqrt{\frac{4}{\pi} \left(\frac{\rho_l}{\rho_c} + \frac{\rho_l(1-DC_f)}{\rho_{water}DC_f} \right)}. \quad (13)$$

where ρ_l is the length density (coarseness) of the fibre and DC is the dry content. This is in good agreement with the diameter measured using the FiberTester; a value of 32 µm was calculated and used when constructing the model. Assuming no air is present, the porosity of the paper model can be estimated as

$$\theta_{paper} = \frac{\left(\frac{1-DC_s}{DC_s} - \frac{1-DC_f}{DC_f} \right) \frac{\rho_l}{\rho_{water}}}{\frac{1-DC_s}{DC_s} \frac{\rho_l}{\rho_{water}} + \frac{\rho_l}{\rho_c}}, \quad (14)$$

where the numerator represents the volume of removable water and the denominator is the total volume of the structure. The basis weight of the model is estimated as the ratio between the mass of dry substance and the surface area of the sample. N is the number of fibres, whereas S and Z are the width and the length of the sample, respectively:

$$\text{Basis Weight} = \frac{N \cdot Z \cdot \rho_l}{S \cdot Z}. \quad (15)$$

4. Computational domain

A MATLAB-code was developed to create and position fibres throughout the domain that represents the paper. The code was set to create and place fibres in a quasi-random manner in which the fibres have a square array arrangement. Based on Eq. (15), the amount of fibres (N) was set to 45 and 105 comprising a basis weight of 20 and 50 g/m² respectively. The porosity was regulated at a value of approximately 0.72 for either basis weight, see Eq. (14). Eighteen different structures were created in MATLAB in which a standard minimum gap space between two fibres was set to 3.73 µm between fibre boundaries. The effect of the minimum gap space was tested by varying the parameter to 2.93, 3.33 and 4.13 µm.

Fifteen structures with a basis weight of 20 g/m² and three with a basis weight of 50 g/m² were created. These models were compared to each other in order to investigate the significance of the positioning of fibres.

The paper is supported by a forming fabric during the vacuum pulse. The influence of the fabric on the flow process is described by volume forces. Displacement and deformation of the fibre geometry over time are disregarded in these simulations. The web configuration has a complex structure with fibres entangled with isotropic orientation in the plane. In the simulation models, the fibres are aligned in the same direction, which is a requirement for 2D simulations, see Fig. (1). To achieve similarity to experimental measurements, the porosity is fixed according to Eq. (14)

The dewatering process occurs under high vacuum pressures that cause extensive velocity variations as the dimension in the computational domain is in the order of 100 µm. Even though the Re-number is relatively low, large velocity gradients occur at the curvature of the fibre walls at narrow regions. The local Re-number varies depending on the location in the domain. The maximum Re-number based on fibre diameter is roughly 400 and 200 for air and water respectively. Taking the characteristic length as the largest gap between two fibres, the maximum Re-number for air and water is roughly 500 and 300. The global Re-number is calculated based on a volume-averaged velocity and the sample width of the domain, including the porosity due to the volume-averaging. Based on these calculations, the global Re-number is 600 and 350 for air and water respectively. These values provide an overestimation of the Re-number as the maximum velocity in stationary single-phase simulations of air and water are used. Moreover, the solution is time-dependant which results in a continuous variation of the flow field and, in turn, boundary layer development. Hence, these Re-numbers are rarely reached in the transient simulations.

The advective terms in Eq. (11) will cause disturbances in the solution due to large velocity gradients. There are techniques applicable for handling numerical instabilities without having to refine the mesh structure further. In the case of a Galerkin finite element method (GFEM), which is used in COMSOL, an artificial diffusion coefficient can be applied to COMSOL's generic scalar convection-diffusion transport equation. When the NS equations are discretised, the Péclet (Pe) number indicates instability when the value surpasses a certain value for numerical stability. The quantity is defined as the ratio of the rate of advection of a quantity to the rate of diffusion of the same quantity. Hence, high advective values as well as a coarse mesh structure results in numerical instability. In order to minimise these disturbances, an artificial diffusion parameter is applied in order to stabilize the solution of the NS equations. Hence, an isotropic diffusion term is implemented in the Pe-number by adding artificial viscosity. It is desirable to reduce the value of the isotropic diffusion as high values could deviate the numerical solution due to excessive smoothing. The value of the isotropic diffusion is 0.2 in our simulation. Lower values resulted in numerical convergence issues.

4.1 Initial and boundary conditions

Initially, the 2D representation of the paper and the forming fabric is filled with water. The grey zone in Fig. (1) represents air and the blue zone represents water. The boundary that connects these zones represents the initial LS interface.

Fig. (1). A 2D representation of the paper sheet on top of a forming fabric, structure 7.

The boundary conditions for the 2D-model in Fig. (1) is calculated as follows:

Pressure, no viscous stress:

$$\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) n_i = 0, \quad p = p_0; \quad (16)$$

Symmetry:

$$n_i \cdot u_i = 0, \quad t_j \left[-\frac{\partial p}{\partial x_i} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] n_i = 0; \quad (17)$$

Wetted wall:

$$n_i \cdot u_i = 0, \quad F_{i,fr} = \frac{\mu}{\beta} u_i. \quad (18)$$

Eq. (16) was applied at the inlet and outlet boundaries where the vacuum level was set at 40 kPa for all cases with 20 g/m² sheets at the outlet and the reference pressure was set at 0 Pa at the inlet. Symmetry was set at the vertical boundaries. The symmetry condition indicates no mass flux across the boundary which is approximate as fibres enhance normal mass transportation close to the symmetry boundary. This condition is tested by increasing the system size by a half domain width at each symmetry boundary, resulting on further extending the symmetry boundary away from the original width. Twelve different quasi-random fibres were generated at the extend domains in order to analyse the influence of the symmetry boundary and, hence, determine the importance of the sample size. The wetted wall function was applied at the fibre walls to allow for capillary forces and also providing greater numerical stability than the no-slip conditions. Eq. (18) indicates a slip condition at the fibre

walls. In addition, the condition adds a frictional force where β is the slip length. It is the length in which the extrapolated tangential velocity is set to zero at the distance β from the surface. The slip length is equal to the local mesh element size.

The simulation model is solved in a time dependent manner, where the CFD program chooses appropriate time steps depending on the relative numerical error. A stop criterion was implemented to end the simulation if the volume fraction of the remaining water reaches a level of 1 %.

4.2 Modelling of the forming fabric

The fluid resistance that the forming fabric exhibit is modelled as volume forces. Rough calculations of the characteristic Reynolds (Re) number indicate an influence of inertial forces. Hence, considering Darcy's law alone to volume average the FE-simulated flow field with the NS equations, the model will underestimate flow resistance. Instead, Forchheimer's equation is used to characterise flow resistance:

$$F_i = \alpha_{V_{ij}} u_j + \alpha_{I_{ij}} u_j |u_j|, \quad (19)$$

where F_i and u_j are vectors that denote the volume force (N/m^3) and velocity field (m/s), respectively. The $\alpha_{V_{ij}}$ and $\alpha_{I_{ij}}$ coefficients represent the viscous and inertial resistances, respectively. The viscous resistance coefficient represents energy losses due to viscous forces at the air-water/fibre wall interface. The inertial resistance coefficient is related to the geometric properties of the porous medium as it is related to inertial forces. The resistance coefficients in Eq. (19) are determined numerically by the geometric structure of the forming fabric and the boundary layer development in the resolved FE-simulations of the NS equations. The viscous and inertial terms are viewed as coefficient matrices. They are implemented in the porous model as constant domain properties, in which they act as flow resistance coefficients and are dependent of the direction of the flow velocity.

A 3-dimensional flow representative volume (FRV) was created for the yarn structure with a yarn diameter of 0.16 mm and a hydraulic diameter of 0.20 mm. Single phase flow of air and water was simulated in the FRV-model, in which a force and momentum flow balance was set up:

$$\sum F_{loss} = \int_{A_{out}} p_{out} dA + \rho \int_{A_{out}} u_{out}^2 dA - \int_{A_{in}} p_{in} dA - \rho \int_{A_{in}} u_{in}^2 dA, \quad (20)$$

where the term on the left hand side of Eq. (20) is the sum of momentum losses and pressures acting on the flux boundaries. The force acting on the flow from the forming fabric is determined by dividing Eq. (20) with the volume of the fluid domain so that the forces are inflicted over a volume unit:

$$\langle F_i \rangle = \frac{\sum F_{loss}}{V_{FRV}}. \quad (21)$$

Moreover, the velocity in the FRV-model is volume averaged in accordance with

$$\langle u \rangle_{FRV} = \frac{1}{V_{FRV}} \int_{V_{FRV}} u dV. \quad (22)$$

Eq. (19) is rewritten with volume averaged coefficients from Eq. (21) and Eq. (22):

$$\langle F_i \rangle = \alpha_{V_{ij}} \langle u_j \rangle_{FRV} + \alpha_{I_{ij}} \langle u_j \rangle_{FRV} \langle |u_j| \rangle_{FRV}. \quad (23)$$

Twelve velocity-force (u-F) correlations were established from the FRV-model for each phase. The twelve cases represent various mean velocity levels, resulting in different Re-numbers. The resistance coefficients are therefore valid in the range of the Re-numbers established in all cases. The maximum Re-number reached is approximately 1000 and 1200 for air and water respectively.

In Eq. (23), the resistance coefficients require a solution. The least square method was applied in a MATLAB code in order to achieve an approximate solution for these coefficients. Importation of a series of velocity vectors, $\langle u_j \rangle_{FRV}$ and $\langle u_j \rangle_{FRV} \langle |u_j| \rangle_{FRV}$, and the volume force vector $\langle F_i \rangle$ into MATLAB yields four coefficients for each phase.

When these coefficients are implemented in the paper model in Fig. (1), the LS function is used to distribute the value of the coefficients for each phase over the interface thickness:

$$\begin{cases} \alpha_{V,yx} = \alpha_{V,water} \varphi + \alpha_{V,air} (1 - \varphi) \\ \alpha_{V,yy} = \alpha_{V,water} \varphi + \alpha_{V,air} (1 - \varphi) \end{cases} \quad (24)$$

$$\begin{cases} \alpha_{I,yx} = \alpha_{I,water} \varphi + \alpha_{I,air} (1 - \varphi) \\ \alpha_{I,yy} = \alpha_{I,water} \varphi + \alpha_{I,air} (1 - \varphi). \end{cases} \quad (25)$$

The volume force sink term is implemented in the lower water domain of Fig. (1) as

$$F_y = \left(\alpha_{V_{yx}} u_x + \alpha_{V_{yy}} u_y \right) \frac{1}{\theta_{fabric}} + \left(\alpha_{I_{yx}} u_x + \alpha_{I_{yy}} u_y \right) \frac{|u_j|}{\theta_{fabric}^2}. \quad (26)$$

The porosity is used to scale the velocities in the volume force domain so they are correlated to the velocities in the FRV-model:

$$\langle u \rangle_{FRV} = \frac{\langle u_{porous} \rangle}{\theta_{fabric}} \quad (27)$$

and

$$\langle u \rangle_{FRV}^2 = \frac{\langle u_{porous} \rangle^2}{\theta_{fabric}^2}. \quad (28)$$

4.3 Mesh grid

An unstructured mesh grid containing triangle elements was created for the paper models, see Fig. (2). The grid is solved in a finite element space where a set of basis functions are used to create piecewise linear relations between the mesh elements and convert them to weak formulations for them to be solved. A direct linear system solver was used coupled to a time-dependent second order BDF solver. The direct solver is called PARDISO and handles sparse linear systems using LU factorisation to compute a solution. More information on the solver is found in the COMSOL documentation. The BDF solver performs time stepping using a backward differentiation with a maximum order of accuracy of 2, which is the degree of the interpolating polynomial.

Fig. (2). Grid resolution of a paper model on top of a forming fabric, structure 7.

The same mesh grid resolution was used for all 20 g/m² models, see first row in Table (1). The mesh parameters that were tested are the maximum mesh element sizes, the minimum element sizes, the resolution of narrow regions and the element growth rate. The sensitivity of the grid was tested on structure 7 by varying these element specifications, which resulted in eight cases see bold text in Table (1) for each case.

Table (1). Grid element parameters tested on structure 7. Bold text indicating change of the specific mesh element parameter for the particular case.

	Maximum element size [μm]	Minimum element size [μm]	Element growth rate	Resolution of narrow reg.
Case Standard	16.5	7.15	1.15	1
Case 1	12.5	7.15	1.15	1
Case 2	10.0	7.15	1.15	1
Case 3	16.5	3.57	1.15	1
Case 4	16.5	1.43	1.15	1
Case 5	16.5	7.15	1.05	1
Case 6	16.5	7.15	1.025	1
Case 7	16.5	7.15	1.15	3
Case 8	16.5	7.15	1.15	5

5. Results and discussion

Fig. (3) presents the volume fraction of air and water in structure 7 where blue colour represents water and red colour represents air. The colour spectrum in between these fluids represents the LS function, which is described as a straight line at the start of the process. Due to the high vacuum pressure and the fibre positioning, the transportation of the LS function is complex. The re-initialisation term, however, retains the interface thickness at a constant level. Air displaces water fairly quickly in the coarse spaces in the paper model, which can be

seen when $t = 0.1$ and 0.2 ms. Dense regions of fibres, however, cause low local permeability, which results in water being retained in narrow regions as the air displacement in such regions is insignificant due to capillary and viscous forces. This is shown when $t = 0.3, 0.4$ and 0.5 ms. This process indicate that the dewatering rate reaches a peak early in the process and gradually decreases as the dry content increases. A paper model with a more dense fibre arrangement should reduce the average permeability and, in turn, decrease the dewatering rate.

Fig. (3). Volume fraction of water and air for structure 7 at various time steps.

The dry content over time is compared between three selected structures and experimental data, see Fig. (4). At the beginning of each process, the dewatering rate is zero as the water is accelerated from a standstill. The dewatering rate changes with time due to various fibre arrangements, see Fig. (5). Due to minor variations in dewatering rates, the maximum dry content is reached for dwell times of great variation. The difference in dry content, however, is minor at a time of 0.5 ms according to Fig. (4). This is a result of the retained water in the dense regions as air is continuously leaking through the paper, resulting in minor changes of the dry content at the end of the vacuum process. This indicates that quasi-random fibre positioning has an insignificant effect on the dry content during high vacuum levels. As expected, basis weight and porosity are the paper elements that dictate the result. The experimental data in Fig. (4) indicate a lower dewatering rate than the calculated data. The time resolution of the measured solids content, however, is not high enough to draw definite conclusions. Further extending the sampling of experimental data in Fig. (4) will result in a final dry content at 22% as was mentioned in chapter 3.

Fig. (4). Relation between dry content and dwell time for three different 20 g/m² models and experimental data at 40 kPa.

Fig. (5). Relation between dewatering rate and dwell time for three different 20 g/m² models at 40 kPa.

The influence of the isotropic diffusion term was verified, as five different values in the range of 0.2–0.4 were tested on a 20 g/m² model. The model shows that the dwell time when the stop criterion is reached varies by approximately 1–2 % between these values. The effect

of the isotropic diffusion coefficient is insignificant due to low Re-numbers during a long stretch of the dewatering process. As the volume of air is continuously growing in the paper model, the velocity magnitude and gradients increase rapidly. Under these conditions, a numerical disturbance in the model occurs and the isotropic diffusion coefficient dampens the solution. Hence, lower values than 0.2 could not be achieved, as the solution becomes unstable in the end of the dewatering process.

The influence of the minimum gap space and the system size configurations are presented in Fig. (6). The configurations of the system size were conducted by adding a half domain width with quasi-random positioned fibres at each side of the original domain. The analysis was conducted on structure 7 and the figure indicates that the choice of the original sample size is valid as the dry content deviate with 6 % at the most. Due to high vacuum levels, the streamwise velocity largely exceeds the transverse velocity, resulting in low mass fluxes crossing the original width. Furthermore, the variation of the minimum gap space has minor effect on the dry content according to Fig. (6). The gap space influence the second peaks in the dewatering rate, see Fig. (7), as those peaks represent water removal from dense regions. The peaks, however, occur for low dewatering rates and are momentary which results in minor variations in the dry content.

Fig. (6). Dry content of structure 7 based on several system size and gap space configurations.

The differences in mesh case dwell times and simulation times are presented for structure 7, see Table (2).

Table (2). Comparison between mesh case dwell times and simulation times

	Dwell time [ms]	Simulation time [hour]	Number of grid points
Case standard	0.554	1.23	10104
Case 1	0.647	1.85	11215
Case 2	0.650	2.75	13172
Case 3	0.559	2.36	15979
Case 4	0.559	2.36	15979

Case 5	0.648	2.20	18000
Case 6	0.623	3.76	26987
Case 7	0.704	2.53	15012
Case 8	0.788	4.47	23352

As shown in the table, the dwell time increases as the mesh structure becomes denser. The motion of the air/water interface is determined by the number of grid points in which numerical interpolation is used to estimate the location of the LS function. The exact location of the LS function could hypothetically be determined with an infinite number of grid points. In this case, however, the dwell time is underestimated with coarser structures. It is noted that cases 3 and 4 have the same amount of grid points, which results in identical dwell and simulation times.

The dry content and the dewatering rate are presented for all cases in the mesh sensitivity study of structure 7, see Fig. (7). The figure indicates no major variation in dry content. The dewatering rate for all cases trail the same path along the time axis and reaches a maximum value at approximately $t = 0.135$ ms. The dewatering rate decreases afterwards as a result of air leaking through the outward boundary of the paper. The dewatering rate starts to fluctuate as the water content in the pore space is emptied. The second peaks start at $t = 0.255$ ms which is a result of water removal from dense fibre regions. These peaks fluctuate and are occur due to the reduced amount of water in the domain. The fluctuations of the peaks deviate between different mesh cases. They are temporary, however, and result in minor changes in the dry content. The determination of the dwell time is dominated by the amount of narrow regions in which the water is retained by capillary and viscous forces close to the fibre walls. Although Table (2) presents a wide variation in dwell times, Fig. (7) indicates that the paper model is not mesh sensitive, as the dry content changes marginally along the dwell time axis.

Fig. (7). Comparison between dry contents and dewatering rates for all mesh cases in structure 7.

Structures 16–18 have a basis weight of 50 g/m^2 . They were simulated and compared to data from our bench-scale laboratory setup and experiments conducted by Pujara et al. (2008b). The dry content and dwell time are related in Fig. (8), which shows that the simulated models underestimate the dwell time as the dry content reaches its limit value too fast. Overseeing small fluctuations, dry content and dwell time for all simulations seems to exhibit an exponential behaviour. The experimental data obtained in two different setups agree very well for low dwell times, although the sampling frequency is somewhat lower in the data from Pujara et al. (2008b). The final dry content reached is somewhat higher in our experiments. Pujara et al. (2008b) used a mixture of 50 % softwood and 50 % hardwood, where hardwood holds more water in the fibre.

Fig. (8). Comparison between experimental data and three different 50 g/m^2 models, structures 16–18.

Structure 18 was further analysed by simulating the dewatering process for three vacuum levels at 20, 40 and 60 kPa. Fig. (9) presents the dewatering rate for all vacuum levels in which the dewatering peak is reached at different time levels. The peak of the dewatering rate is delayed along the time axis for lower values of vacuum levels, indicating not only lower dewatering rates, but also a slower acceleration of the water. The magnitude of the dewatering rate is lower in the experimental data resulting in a dry content difference viewed in Fig. (8). The dotted lines in Fig. (9) are filtered in order to view the trend of the dewatering rate, hence, eliminating the fluctuating behaviour of the lines at the end of the process. Eventually, the process reaches a steady-state manner resulting in air flowing through the paper which has reached a maximum dry content.

Fig. (9). Relation between dewatering rate and dwell time. Comparison between experimental data, data from Pujara et al. (2008b) and structure 18.

The dewatering rate is plotted against the dry content and is presented in Fig. (10). It shows that the peak of the dewatering rate is reached at the same dry content independently of the vacuum level, indicating similar dynamics of the dewatering in the structure regardless of dewatering rate. Moreover, reducing the vacuum level results in lower final dry content which is evident for both simulations and experiments. At this stage of the process, air is leaking through the paper without increasing the dry content, making the operation less efficient. Fig. (10) shows that structure 18 at a vacuum level at 20 kPa has a final dry content of 18 % whereas 40 and 60 kPa simulations approximately end up with a dry content of 21 %. This is an indication that it is not necessarily efficient to increase the vacuum pressure to extensive levels.

Fig. (10). Relation between dewatering rate and dry content. Comparison between experimental data, data from Pujara et al. (2008b) and structure 18.

At the end of the dewatering process, the airflow rate of the simulated structures 7 (20 g/m²) and 18 (50 g/m²) were compared to our experimental data and experimental data from Pujara et al. (2008b) for vacuum levels at 20, 40 and 60 kPa, see in Fig. (11). The figure presents similar trends in the evolution of the airflow rate with vacuum levels. Considering the complexity of the paper structure and the meager experimental data available in the literature, the agreement is more than satisfying.

Fig. (11). Air flow rate through paper with basis weight 20 and 50 g/m². Comparison of simulated structures 7 and 18 to experimental data and data from Pujara et al. (2008b).

6. Conclusions

A numerical model of a vacuum dewatering process was established as the LS method was applied to simulate two-phase flow phenomena in a 2D paper sheet model constructed from data for the fibre diameter and the volume fraction of removable water. The model is shown to perform well in terms of mesh and system size sensitivity and it illustrates several important phenomena observed or postulated by experimentalists in the field. These include; rapid initial dewatering as water in the larger pores is accelerated by applied vacuum, removal of water from smaller pores as air starts to penetrate the sheet; dry content approaching a constant value which increases with the applied vacuum level and air flow eventually reaching a constant value. Moreover, the influence of the minimum gap space on the dry content is insignificant as water removal from coarse pore regions dominates the rate of dewatering.

Operation of the wire section at an increased temperature should lead to more rapid dewatering due to the decrease in the viscosity of water with increased temperature. It should be possible to use the simulation model for predicting such effects. However, considerable investments are necessary for increasing the process temperature and such investments will only be considered if the paper mill has an excess of heat available.

When comparing calculated flow rates of water and air to measured laboratory data from two independent sources, the air flow rates are found to be in relatively good agreement whereas the calculated dewatering rates are a factor two higher than the measured. This discrepancy arises already in the first phase of dewatering where acceleration of pore water and friction against fibre surfaces determine the dewatering rate.

Previous literature points out the influence of fibre deformation on vacuum dewatering, as well as the effects on the permeability due to fibre orientation. Future work will aim at improving our description of the paper structure to include also 3D-models. A possible approach to analyse the influence of random fibre orientation in the plane is to make a LS 3D-

model, which would require a large amount of data resources. Implementation of volume forces is an additional method by which random orientation is considered. The approach is based on creating a flow representative volume of random fibre orientation in a 3D-environment and conducting single-phase flow simulations of water and air in order to create volume forces. These volume forces are then implemented in an LS model to replace the internal fibre structure. Local velocity gradients will be lost as a result. A more accurate numerical model could, however, be established in terms of achieving a valid relation of the solid dry content and dwell time.

Based on our knowledge of the field of vacuum dewatering, no one has attempted to apply a two-phase model in order to correlate dry content to dwell time for high vacuum levels. The established numerical model in this study represents the first step towards developing a more accurate model representation of water removal from paper sheets.

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Nomenclature

A	Area	[m ²]
D	Diameter	[m]
DC	Dry content	[%]
F	Force	[N]
g	Gravitational acceleration	[m/s ²]
h	Mesh cell length	[m]
H	Heaviside function	[-]
I	Identity matrix	[-]
$\dot{m}$	Mass flow rate	[kg/s]
N	Number of fibres	[1]
n	Normal of the interface	[-]
p	Pressure	[Pa]
S	Width of sample	[m]
t	Time	[s]
u	Local velocity coefficient	[m/s]
V	Volume	[m ³]
$\vec{x}$	Space coordinate	[m]
Z	Length of fibre/Sample	[m]

Greek symbols and mathematical operators

α_V	Viscous resistance coefficient	[Pa·s]
α_I	Inertial resistance coefficient	[kg/m ⁴]
β	Slip length	[m]
γ	Re-initialisation	[m/s]
δ	Direc delta function	[1/m]
ε	Interface thickness	[m]
θ	Porosity	[-]
κ	Curvature of the interface	[1/m]
μ	Viscosity	[Pa·s]
ρ	Density	[kg/m ³]

Σ	Summation	[-]
σ	Surface tension coefficient	[N/m]
φ	Level-Set function	[-]
$\bar{\cdot}$	Time averaged property	[-]
∂	Partial derivative	[-]
$\partial\Omega$	Interface boundary	
$\langle\cdot\rangle$	Volume averaged property	[-]

Subscript

0	Boundary
<i>air</i>	Air
<i>c</i>	Cellulose
<i>fabric</i>	Fabric
<i>f</i>	Final
<i>fr</i>	Friction
<i>l</i>	Interface
<i>I</i>	Inertial
<i>i,j</i>	Tensor indices
<i>in</i>	Inlet
<i>l</i>	Length
<i>loss</i>	Loss
<i>out</i>	Outlet
<i>paper</i>	Paper
<i>porous</i>	Porous
<i>s</i>	Start
<i>st</i>	Surface tension
<i>V</i>	Viscous
<i>water</i>	Water
<i>x,y</i>	Cartesian coordinates

Acronyms

CFD	Computational fluid dynamics
GFEM	Galerkin finite element method
FRV	Flow representative volume
LS	Level-Set
NS	Navier-Stokes
PDE	Partial differential equation
Pe	Péclet
PQS	Progressive Quasi-Static
Re	Reynolds