Teachers' choice of a challenging task through collaborative learning

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Challenging mathematical tasks are important for all students' learning processes, and the demanding job of finding and developing such tasks is preferably done through teacher collaboration. Using cultural-historical activity theory, we analyze three upper secondary teachers' collaborative learning process in choosing and rejecting tasks for a collection of challenging tasks they have agreed to develop. They collaboratively chose one task to fulfil the criteria of challenging tasks; one task was rejected as it did not fulfil the criteria, and another was temporarily rejected as not all the teachers knew how to solve it themselves. The analysis revealed a positive and open atmosphere among the teachers, with content-focused discussion highlighting mathematical content and teaching related to the tasks discussed. While their discussion showed several signs of collaborative learning, whether their work will result in changes to teaching practice remains to be explored.

Keywords: teacher collaboration, secondary school mathematics, mathematical enrichment

Introduction

Challenging mathematical tasks offer learning opportunities for all students (Nolte & Pamperien, 2017; Sheffield, 2003). However, textbooks usually offer a small proportion of challenging tasks (Jäder et al., 2020). Hence, teachers must find suitable tasks to offer their students. Searching for, analyzing, and developing challenging tasks is time-consuming work, which is preferably done together with other teacher colleagues (Mellroth, 2018). Swedish teachers' conditions for collaborative learning are often poor (Nordgren et al., 2019). But when the right conditions are provided, for example regularly scheduled times, collaborative learning benefits student learning (e.g., Timperley, 2011) and reduces stress among teachers (Nordgren et al., 2019). We assume that, under such conditions, teachers can work in a more structured way and focus on developing teaching activities together.

In this paper we report from an ongoing project aiming to improve mathematics education in terms of meeting students' different learning needs, including students with high abilities in mathematics. Eight mathematics teachers at a Swedish public upper secondary school participate in the project. They have decided to develop a collection of challenging mathematics tasks, with guidelines for classroom enactment and with clear connections to the Swedish national curriculum. We will present results from an activity in which three of these teachers worked together on choosing a task for further development. To frame, analyze, and understand the complexities of this collaborative learning process, we use Cultural-Historical Activity Theory (CHAT) (Engeström, 1999). Our guiding question is: What characterizes the collaborative learning process that leads to teachers' choosing or rejecting tasks for further development?

Literature review

Learning by failing is an accepted view among educators. Each student needs to be exposed to mathematical tasks that are challenging. Naturally, what is challenging will be different for someone who struggles with mathematics compared to someone with high ability in mathematics. As textbooks include few challenging problem-solving tasks (Jäder et al., 2020) and reasoning tasks (e.g., Bergwall, 2021), students with high ability are at risk of never encountering any challenges unless the teacher deliberately offers them. In addition, students with learning difficulties are at risk of mostly solving procedural tasks, as textbooks' enrichment tasks are often too hard for them (Jäder et al., 2020).

Designing mathematical tasks that offer challenges for all students is a demanding job (Mellroth, 2018). Sheffield (2003) has developed criteria that such tasks should fulfil. Four of the criteria are: (1) everyone should be able to start working with the task, (2) it should be possible to solve the task in several ways, (3) the task should be engaging, and (4) the task should offer an open end. Nolte and Pamperien (2017) found that tasks developed to be challenging for highly able students create learning opportunities for all students. Some researchers (Hoth et al., 2017) claim that the teacher must have deep mathematical knowledge to orchestrate teaching with challenging tasks. Mellroth (2018) showed that teachers themselves propose collaboration with teacher colleagues when they feel that they lack the mathematical knowledge themselves. It is also shown that in countries where it is part of the teacher culture to collaboratively reflect, discuss, and develop teaching, students perform better than in countries where it is more common for teachers to work alone (Hargreaves & Fullan, 2012). A school culture in which collective responsibility is taken for students' learning is necessary to create, or withhold, sustainable strategies for teaching development (Hargreaves & Fullan, 2012).

This paper reports results from one of several closely related projects carried out in the same municipality. All the projects center around *collaborative learning*, defined as follows: a group of teachers' jointly and systematically organized work, consisting of content-focused, inquiry-based, creative, and tentative communicative processes, that continuously change the work itself, and/or the group, and/or the individual teachers' professional practice and classroom practice (Andersson et al., 2019).

Research has shown that, to create sustainable collaborative learning processes, it is important that the participants themselves identify a problem they deem important to solve (Harvey & Teledahl, 2019). The communicative process of collaboratively working to solve the problem is a human activity with great complexity.

Theory

We use CHAT as our analytical framework, as it offers the possibility to understand a complex human activity system in relation to its context (Engeström, 1999). The theory is often presented in the form of the CHAT triangle, with six nodes that are of importance in interpreting the outcome of the activity system, see Figure 1.

CHAT offers opportunities to study how *subjects* in a *community* influence each other towards joint development. In line with Engeström (1999), we consider the collective processes by focusing on the subjects' object, mediating artefacts, and rules. More precisely, the *object* is the subjects' agreed goal.

The *community* is everyone who belongs to the same environment and shares the object. *Mediating artefacts* are the tools that are used or developed by the subjects in order to reach the object. *Rules* are explicit and implicit norms for actions and interactions between the subjects in the community that may influence the outcome. *Division of labor* includes hierarchical power structures in the community (Harvey & Teledahl, 2019). The rules, community, and division of labor nodes place individual and collective actions in their context. Thereby, the subjects' joint actions can be explained in the context to which they belong. In practice, most individuals are part of several different activity systems with their own objects, rules, artefacts, etc.

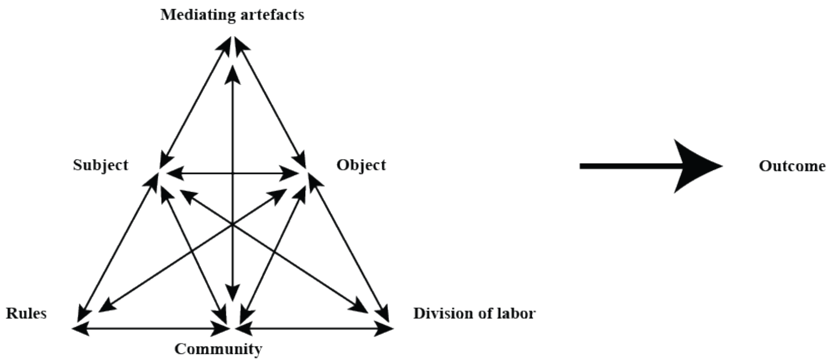

Figure 1: Activity system, illustrated by Engeström (1999)

In this study, we analyze the joint work of three of the participating teachers when they choose or reject tasks for further development. In the terminology of CHAT, these teachers are the *subjects* of this study. The other five teachers participating in the project are part of the subjects' *community*, along with the researchers, other colleagues at the school, the principal, politicians, and others who make decisions regarding the project's conditions and share its goals. Regarding the *division of labor*, the teachers participate on equal grounds in the following sense: They have all volunteered for the project, have no reduction in other tasks to make room for project work, and none of them have been appointed a particular role in the project. It is assumed that they will collaborate on project tasks. The principal supports the project, and has dedicated conference time for the teachers to work on it.

The other three CHAT nodes (object, mediating artefacts, and rules) are only partially determined by the project's design. Early in the process, the participants decided to create a collection of challenging problems, including introductory tasks as well as tasks that offer enrichment. This collection, which the participants refer to as the problem bank, is the subjects' general object. The collaborative learning process during the work with the problem bank is also a means to achieve the project's long-term object that teachers should become better prepared to meet the differing needs of all students, including those with high ability in mathematics. The meetings, seminars, and literature on collaborative learning and highly able students, and a task analysis protocol with criteria for challenging tasks are mediating artefacts provided by the researchers. Explicit rules include the assumption that the subjects will participate in meetings and activities as well as cooperate in the creation of the problem bank. The transcript analysis aims to reveal further details related to these three nodes.

Regarding rules for actions and interactions in the community, these include norms for the interplay between subjects as well as norms for what content they find worthwhile to discuss. We refer to the first kind of norms as *social rules* and the second as *mathematically oriented* or *didactically oriented*, depending on whether it is purely mathematical aspects that are discussed or if the discussion concerns teaching or learning.

Method

This study is part of an ongoing, 2.5-year long project aiming to improve mathematics education in terms of meeting students' different learning needs. It is explicit that this should include the needs of highly able students, that is those who quicker and easier reach the learning goals. The project is a collaboration between the local university and the municipality. Two researchers (the authors of this paper) serve as facilitators. Eight mathematics teachers who teach technology students at a public upper secondary school have volunteered to participate. They have been informed of the intention of the related research, and have all signed an informed consent form. The teachers expressed that they lacked a source of tasks that are ready to use when students (including the highly able ones) need extra challenges. Therefore, they decided to focus on developing a problem bank of challenging tasks and that the problem bank should involve tasks that introduce new concepts as well as tasks for enrichment.

According to national steering documents, Swedish upper secondary schools offer five mathematics courses called Mathematics 1, 2, 3, 4, and 5. This paper shares the preliminary results from when three of the teachers teaching Mathematics 3–5 met to choose their first task for further development for the problem bank. Earlier in the same semester, the researchers had arranged three meetings, 90 minutes each. These included a mix of literature studies, seminars, and discussions, with the aim of increasing the participants' knowledge about collaborative learning (Harvey & Teledahl, 2019) and challenging mathematical tasks (Mellroth, 2018; Sheffield, 2003). All meetings were audio-recorded. To analyze, choose, and develop tasks, the researchers and teachers together developed a task analysis protocol with criteria for challenging tasks (e.g., Sheffield, 2003). Immediately before the analyzed activity, the participants analyzed one task together with the researchers to create a shared understanding of the protocol's different criteria.

The following procedure was followed to select and analyze the data for this paper: 1) All audio-recordings from the three teachers' meetings were listened through, and all episodes in which they gave reasons for choosing, rejecting, or revising a task were marked; 2) The marked episodes were listened through once more and by both authors; 3) Transcripts were produced on the parts still judged to give reasons for the choice or revision of a task; 4) Verbatim transcripts were produced from the whole episode in which the teachers started their process of developing tasks for the problem bank; 5) Each author separately identified nodes from the CHAT triangle in the transcripts; 6) The authors compared their analyses and discussed the parts they had different opinions on until agreement was reached; 7) The complete analysis of the outcome using CHAT theory was performed jointly by the two authors.

Analysis

The analysis is exemplified in three paraphrased transcripts (translated from Swedish to English) that follow the chronology of the analyzed meeting. The agenda for the analyzed meeting was to initiate the work of choosing, analyzing, and developing tasks for the problem bank (i.e. the subjects' general object). The analysis focuses on how the subjects address the CHAT nodes called object, mediating artefacts, and rules.

The extracts are from the first 30 minutes of the meeting. During this part, all the teachers present possible tasks for the problem bank and ask one another about what they have brought to the meeting. This indicates the existence of the social rule that all the teachers should (and do) contribute. Teacher T1 presents *Ferris Wheel*, a task in which the student is asked to investigate how a gondola's height above ground level varies as the Ferris wheel turns. The task is meant to serve an introduction to trigonometric functions, which relates to the object that the problem bank should include introductory tasks. In the episode below, the teachers discuss the difficulty of the task in relation to what is taught before it. That they deem it important to discuss such things is a didactically oriented rule.

69	T2:	It depends on whether you've introduced circular arcs and that kind of stuff.
70	T1:	Yeah, you usually do that first, don't you?
71	T3:	Yeah, that comes immediately before. Or at the beginning of the chapter.
73	T2:	I think it's after.
74	T3:	What comes first, then?
75	T2:	I think it starts with circular arcs, before radians.
77	T1:	The other way around, I think. You introduce radians, then circular arcs.
79	T1:	Yes, and then functions with radian.

Several social rules come into play here. It is accepted behavior to question one another (73, 77), ask for (74) and give (75) clarification, and express support (70, 71, 79). In addition, the discussion does not end until the ambiguities are sorted out and consensus is reached (after 79). The explicit textbook reference (71) shows that the textbook is a mediating artefact in this process.

In the next episode, the teachers discuss the possibility to develop (which in itself is an object) *Ferris Wheel* to be an enrichment task (which is another object). In the discussion, x and h refer to the horizontal and vertical positions, respectively, of a gondola on the Ferris wheel.

149	T2:	Here we only study h. What if we study x, so that you do the same thing as in part 1, but for x instead of h?
151	T2:	It's the same kind of task, but it becomes a two-dimensional motion. We call them $x(t)$ and $h(t)$, and get two different formulas that are rather similar. And then, maybe make a connection back to the unit circle. Provided that we place the origin at the center of the Ferris wheel.
153	T3:	Yeah, that can be part 3, and where you said $x(t)$, part 4 maybe. Changing a function's values when you move the center should be straightforward.
154	T2:	Finally, you could add these two: Can you go from your two functions to the equation of a circle? Both when the center isn't at the origin and … So, from

		the beginning we should've placed the center at the origin. That makes it a little easier.
155	T1:	Yeah, absolutely. It has potential, this task.
157	T3:	It gets extensive but that's good. Not everyone will have time to do it, but that's not the intention either.

Here we see how the social rules allow all teachers to contribute ideas, ask for opinions (149), and confirm each other's thoughts (153, 155). The mathematically oriented rules allow for a discussion of purely mathematical details (151), and didactically oriented rules are seen when the teachers express the belief that their students should find some steps "straightforward" (153) or that certain changes should make the task "a little easier" (154). The predetermined object that the tasks should also include challenges for students with high ability is seen in the last utterance (157). The reference to the unit circle (151) indicates that the ordering of content in the mathematics course serves as a mediating artefact when additions to the task are suggested.

Teacher T2 presents four different tasks. The teachers agree that the most promising one is *The Beam*, which asks for the maximum length of a beam that should be possible to carry through a right-angle turn in a corridor. Finally, Teacher T3 presents *Cut Cones*, in which the student is to cut a circular sector from a circular sheet of paper and form a cone with maximal volume. The teachers agree that *Cut Cones* offers opportunities to use different representations. They refer to a seminar held earlier the same day; i.e., this seminar is a mediating artefact. They also suspect that the task will be too easy and not meet the object of being a challenging task. In the episode below, the discussion is coming to an end and the teachers decide which task to develop first.

264	T3:	I thought *Cut Cones* was kind of fun. You can actually build these things.
276	T1:	It's a very good task.
277	T3:	But it doesn't meet the criteria.
296	T2:	And I think the first step isn't very difficult.
298	T3:	The first step isn't difficult in *Ferris Wheel* either. But it's hard to make *Cut Cones* more difficult. It's possible to develop *Ferris Wheel*.
303	T3:	I think both *The Beam* and *Ferris Wheel* have potential.
310	T2:	Should we start with *Ferris Wheel*? We can analyze it and then solve *The Beam*.
311	T3:	Yeah, because I need to solve it first.

In this episode, several motives (relating to different objects) for choosing or rejecting tasks are presented. T3 considers that the fact that *The Cones* offers an opportunity to work with physical representations speaks in favor of this task (264). This might be an individual object of T3, who also refers to applicability on several occasions. The overall object of developing tasks that can challenge students on different levels is used against *The Cones* (277, 298) as well as in favor of *Ferris Wheel* (298, 303). The criteria from the task analysis protocol serve as a mediating artefact in this discussion (277, 310).

Results

What characterizes the collaborative learning process that leads the teachers to choose *Ferris Wheel* and reject the other tasks? They chose *Ferris Wheel* because they believed (1) it has a first step that is not difficult, (2) it can challenge students on several levels, and (3) it can be developed to become even more challenging. Thus, their choice was driven by their agreed object to find a task that can challenge all students, and that can be used as an introductory task but can also be developed to offer enrichment. Their discussions were mediated by tools supplied within the project, such as the task analysis protocol and its criteria for challenging tasks, but also by other tools such as the textbook, local teaching plans, national steering documents, and the teachers' shared understanding of their students' prior knowledge.

During their discussions, rules of various types came into play. The teachers expected everyone to contribute, and asked for and questioned each other's opinions. They prioritized discussions about mathematical details related to the task and students' common understandings and difficulties in relation to the material. They rejected (for the time being) a task they all liked, *The Beam*, as they wanted to first solve it on their own; they had no problems admitting to each other that at first glance they did not know how to do this.

Discussion

This study focuses on three teachers' collaborative learning process when choosing a task for a problem bank of challenging mathematical tasks, which are rare in textbooks (Jäder et al., 2020). The teachers analyze and develop the task to ensure that it meets the needs of students who struggle with mathematics as well as those who are highly able and need extra challenges. They do this by providing a low entrance level and combining it with a sequence of subtasks of increasing complexity and an open end. Leaning on Nolte and Pamperien (2017) and Sheffield (2003), *Ferris Wheel* is expected to challenge all students in the classroom. The analysis shows how Sheffield's (2003) criteria, operationalized as the analysis protocol, provide guidance and help the teachers in choosing, rejecting, and developing tasks. This is an example of how a mediating artefact helps to keep the work organized and content-focused, which are two defining features of a collaborative learning process (Andersson et al., 2019).

The study sheds further light on the importance of creating opportunities for teachers to collaborate (Nordgren et al., 2019). The analysis of the CHAT node called *Rules* (Engeström, 1999) shows that the discussion when choosing a task involves the sharing of mathematical and didactical knowledge. As Hoth et al. (2017) mention, deep mathematical knowledge is important, and the analysis indicates that this is true, at least among upper secondary teachers when working to develop challenging tasks. Although we have not explored how an individual teacher would choose or reject a task using the same process, in line with Mellroth (2018) we assert that choosing and developing challenging tasks benefit from teacher collaboration.

Thus far, the teachers' work has complied with the first part of our definition of collaborative learning (Andersson et al., 2019). They have worked jointly and systematically, and their communicative processes have been content-focused, inquiry-based, creative, and tentative. The second part of the definition of collaborative learning, relating to changes in their professional practice, is an important next step in this ongoing research project. However, in line with Harvey & Teledahl (2019) and Nordgren et al. (2019), we see promising results regarding how a clearly articulated and common goal, regularly scheduled time for collaborative work, and the provision of operationalized frameworks (such as the task analysis tool) can stimulate collaborative learning processes. On a practical level, our study shows how such processes can guide and support teachers in the demanding work of supplementing textbook materials with suitable tasks that can challenge all students.

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