Teachers' characterizations of challenging introductory and enrichment tasks

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Developing tasks for use in mixed-ability classrooms presents teachers with several dilemmas. By making one such dilemma an explicit object of inquiry, this study aims to capture characteristics for challenging tasks suitable for introduction or enrichment. It is based on eight teachers' collaborative and retrospective analysis of challenging tasks developed in a combined research and school development project. Among the results are the observation that introductory tasks should have an easy entry level and not require pre-knowledge of the upcoming concept, while an enrichment task should require relatively deep conceptual pre-knowledge. It is suggested that attention to seemingly contradictory features of introductory and enrichment tasks can fuel collaborative learning processes so that they include several important aspects of tasks aimed at challenging all students. Teachers' verbalization of task characteristics is one outcome of such a process.

Keywords: task design, mathematical enrichment, mathematical introduction, upper secondary school

Introduction

Mathematics tasks serve many purposes in mathematics classrooms. A suitably designed task has the potential to provide all students in a mixed-ability classroom – those with learning difficulties as well as those with high abilities – with opportunities to develop their conceptual knowledge. To do this, the task should challenge all students. It should offer them opportunities to struggle with important mathematical ideas (Hiebert & Grouws, 2007), require them to put effort into their work, and involve some level of confusion (Bobis et al., 2021). In the mixed-ability classroom, tasks with a "low floor" (i.e., an easy entry level), or with enabling prompts, can help engage students who otherwise have difficulties with challenging tasks. When the same task also has a "high ceiling", an open end, or extending prompts, students with high ability in mathematics can also be challenged (Bobis et al., 2021). However, the guidance included in the task formulation must not turn into funneling – that is, leading the student around the difficulties and avoiding the struggle (Bauersfeld, 1998) – as this would place effective learning at risk. Tasks designed to let students create their own solutions lead to better conceptual learning, compared to when a task instructs students to apply ready-made methods (Russo et al., 2020; Sullivan et al., 2015).

In the professional development project that we report on in this paper, teachers have repeatedly decided whether a task is suitable for introduction or enrichment, whether it offers sufficient guidance without funneling, or whether it is specific enough while also offering opportunities for general reasoning (Mellroth et al., 2021). In this particular paper, we focus on the distinction between introductory and enrichment tasks. We argue that this distinction is beneficial when designing tasks

for a mixed-ability classroom, and aim to capture some useful characteristics for these types of tasks and elaborate on teachers' notions of introductory and enrichment tasks. More precisely, we aim to answer the following research question: What characteristics do experienced upper secondary teachers attribute to challenging mathematics tasks suitable for introduction and enrichment, respectively?

Literature review and theoretical considerations

Teachers themselves emphasize that they need challenging tasks, especially when introducing a new concept and when they want to help students deepen their knowledge (Mellroth et al., 2021). However, textbooks tend to offer few such tasks (Jäder et al., 2020). Finding tasks that are appropriate is difficult and time-consuming (Mellroth, 2018) and involves a wide range of didactic considerations (Bergwall & Mellroth, 2021), as tasks often needs to be selected and (re)designed to fit discrepancies to learning goals (Jäder, 2019). But teachers who participate in long-term professional development on developing challenging tasks for all students become competent in differentiating tasks in order to provide each student with appropriate challenges (Mellroth, 2018; Mellroth et al., 2021)

Teachers' collaborative efforts to find, develop, or adapt challenging tasks for use in their classrooms can be conceptualized as a collaborative learning process (Mellroth et al., 2021) situated in an activity system (Engeström, 1987). In this perspective, tensions and contradictions within and between different elements of the activity system are what drive the learning processes. One way such contradictions can manifest is as dilemmas (Engeström & Sannino, 2011). In the context of differentiating mathematics instruction for a mixed-ability classroom, teachers are faced with the dilemma that, on the one hand, all students must be able to participate in joint classroom activities, and on the other that they need to be offered support and challenges tuned to their individual needs. This dilemma can take many different forms.

The classification of tasks as introductory or enrichment tasks was introduced by the project's teachers in response to such a dilemma. However, the teachers' criteria for those two task categories were not a priori made explicit, and it was not clear if they represented opposing ends on a continuous scale, a dichotomy, or just two properties that a task can have in any combination.

Method

We will answer the research question by reporting on findings from a combined research and school development project in which eight Swedish upper secondary mathematics teachers develop challenging tasks and two researchers study their collaborative learning processes. The focus will be on the teachers' retrospective analysis of tasks they have classified as either introductory or enrichment tasks. The paper has been co-written by the researchers and two of the teachers in the project.

The context and the school development project

Swedish upper secondary school encompasses Grades 10 to 12 (students aged 16–19 years). Students choose from a variety of theoretical and vocational programs. The project's teachers teach mathematics within a technology program, a theoretical program with a high density of mathematics,

science, and technology, aimed to prepare students for tertiary education in STEM subjects. Five mathematics courses, building on each other, are offered within the program. For practical reasons, the teachers in the project formed two subgroups, one for those currently teaching the first two courses (here referred to as Group A) and one for those teaching the other three (Group B).

The school development project is a 2.5-year (Aug. 2019–Dec. 2021) project on collaborative learning in mathematics teaching. For collaborative learning to be successful, it is important that participants focus on developing some aspect of their practice that they themselves perceive as problematic. Early in the project the teachers decided to develop a collection of tasks, a problem bank. They felt that mathematics textbooks lacked tasks that can offer challenges to both students with difficulties and those who are highly able in mathematics (cf. Jäder et al., 2020). Their mutually agreed-upon aim for the problem bank was that it should include challenging tasks suitable for introducing new mathematical concepts (introductory tasks), and tasks that could be used to help students develop in-depth knowledge about one or several mathematical concepts (enrichment tasks). Thus, the decision was made early on to distinguish between introductory and enrichment tasks.

During the first two years of the project, the researchers and teachers read and discussed research-based literature on task design (e.g., Sheffield, 2003), differentiated instruction (e.g., Tomlinson, 2016), and high ability in mathematics (e.g., Szabo, 2017). Collaboratively, the teachers merged research findings with their own teaching experiences and developed (or adapted) 13 tasks for use in their own mixed-ability classrooms. The teachers adjusted the tasks to fulfil criteria of rich learning tasks (Sheffield, 2003), for example that (1) everyone should be able to start working with the task, (2) it should be possible to solve the task in several ways, (3) the task should be engaging, and (4) the task should offer an open end. They also tested a majority of the tasks (the COVID pandemic made classroom testing difficult), and then re-analyzed and revised some of them. Therefore, the teachers can be considered competent and experienced in task design as well as collaborative forms of educational development.

During the development and testing phase, the researchers took on the passive role of observers. To project meetings, the teachers brought tasks they found promising to develop to be challenging for all students. In the continued development process, the promising tasks were analyzed using a task analysis guide developed within the project. The criteria for rich learning tasks mentioned above formed part of the guide. Information was collected digitally, and was intended to be included in the problem bank as support for its future users. One item in the guide concerned whether a task was suitable for introduction or enrichment. Of the 13 developed tasks, one was classified as a pure introductory task, six as pure enrichment tasks, and three as both introductory and enrichment tasks. Three tasks were classified as neither introductory nor enrichment tasks, and are therefore out of the scope of this paper.

Data selection and analytic procedure

The results presented in this paper are based on the teachers' retrospective analysis of a subset of the tasks they classified as introductory or enrichment tasks (or both). This new task analysis was conducted during the last meeting of the project's second year. Prior to the meeting, the teachers voted on which tasks they found to be of most interest to analyze according to their suitability for

introduction and enrichment, respectively. At the meeting, and based on the votes, the two groups singled out one task each from each category. Group A chose Colored Cube for introduction and The Ant's Walk for enrichment, while Group B chose Ferris Wheel for introduction and Disease Spread for enrichment. Of these four tasks, Ferris Wheel was the only one which had been classified as suitable for both introduction and enrichment. In the next step of the analysis the teachers focused on its use as an introductory task only. English translations of the four tasks are presented in Figures 1– 3. Due to space limitations, the descriptions have been somewhat shortened.
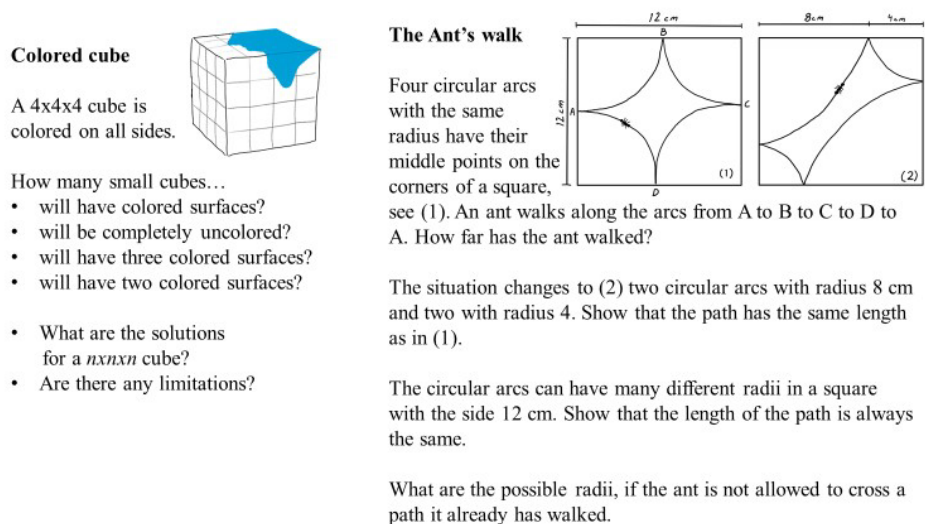
Colored cube

A 4x4x4 cube is colored on all sides.

How many small cubes…
- will have colored surfaces?
- will be completely uncolored?
- will have three colored surfaces?
- will have two colored surfaces?

- What are the solutions for a *nxnxn* cube?
- Are there any limitations?

The Ant's walk

Four circular arcs with the same radius have their middle points on the corners of a square, see (1). An ant walks along the arcs from A to B to C to D to A. How far has the ant walked?

The situation changes to (2) two circular arcs with radius 8 cm and two with radius 4. Show that the path has the same length as in (1).

The circular arcs can have many different radii in a square with the side 12 cm. Show that the length of the path is always the same.

What are the possible radii, if the ant is not allowed to cross a path it already has walked.

Figure 1: The two tasks chosen for retrospective analysis by Group A

Ferries Wheel

The diameter of a Ferries Wheel is 60m. Its center is placed 40m above the ground. One lap takes 4min. Assume that the wheel begins to rotate when a passenger is at the bottom.

1. Sketch a graph of the passenger's height above the ground at different times.

2. A mathematical model for the movement is given by $h = a + \cos(ct)$. Use the attached GeoGebra file and investigate how a, b and c affect the model. Determine a, b and c for your graph.

3. Sketch a graph for a passenger's position in the x-direction at different times. The movement in x can be described by $x = a + b\sin(ct)$. Determine a, b and c.

4. The movement in the x-direction can be described by $x = a + b\sin(ct)$. Determine a, b and c.

5. Describe the movement in part 2 with a sine function. You may need another constant.

6. Derive the equation of a circle by using the functions found in part 2 and 4.

Figure 2: The introductory task chosen for retrospective analysis by Group B

Disease Spread

Students are asked to implement the model and investigate how changing the constants in the model effects the spread of infection.

The population in a society is divided in three groups:
a) *Susceptible* (S): The proportion who have not yet been infected.
b) *Infected* (I): The proportion who are ill.
c) *Recovered* (R): The proportion who have recovered and become immune.
In this model, everyone who has recovered will be immune.
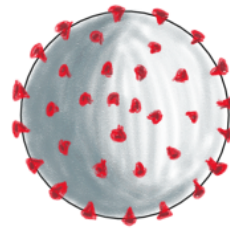
The SIR model for how the disease spreads can be described with the following differential equations:

$$\frac{dS}{dt} = -\beta IS \qquad \frac{dI}{dt} = -\beta IS - \gamma I \qquad \frac{dR}{dt} = \gamma I$$

β describes how contagious the disease is. γ describes the rate of recovery.

In this model the proportion of susceptible population decreases proportionally to the product of S and I. Those who leave Group S go to Group I. The proportion of those who leave Group I is proportional to I. Those who leave Group I go to Group R.

Examples of values for a disease are $\beta = 0.5$, $\gamma = 0.14$, $S(0) = 1$, $I(0) = 1 \cdot 10^{-6}$, $R(0) = 0$.

One way to compare different diseases is to find the R-number definded as $R = \frac{\beta}{\gamma}$.

Figure 3: The enrichment task chosen for retrospective analysis by Group B

All eight teachers, four from each group, participated in the analysis. We agreed on the following procedure: Each group would analyze their introductory task and enrichment task in parallel and answer two questions for each task. For the introductory task: a) What, above all, makes the task suitable as an introductory task? b) What, above all, makes it less suitable as an enrichment task? For the enrichment task: a) What, above all, makes the task suitable as an enrichment task? b) What, above all, makes it less suitable as an introductory task?

The time for analysis amounted to 45 minutes, approximately split as follows: 10 minutes for individual reflection, 20 minutes to compare and discuss individual reflections within the group and come to an agreement, 15 minutes to write a summary of the group's analysis. The discussions were audio-recorded for future reference, but the results presented below are based on the written summaries only.

From our perspective, those summaries are the results of the task analysis. However, for the presentation in the *Results* section, they have been translated to English and rearranged to have similar dispositions. This means that to some extent a content analysis has been conducted. All participating teachers have had opportunities to contribute to and influence the final formulations.

Results

First, as some tasks were categorized as neither introductory nor enrichment tasks, this categorization is not exhaustive. Neither is it a dichotomy, as some tasks were classified as suitable for both introduction and enrichment. Therefore, we conclude that from the teachers' point of view, a task can have both these properties in varying degree and in any combination. To cast further light on this, we now present the summaries of what it is that makes the four tasks suitable/not suitable as introductory and enrichment tasks, respectively. We start with Group A's analysis of Colored Cube and The Ant's Walk, and then Group B's analysis of Ferris Wheel and Disease Spread. Comparisons, similarities, and recurring themes are touched on in the *Discussion* section.

Group A considered Colored Cube to be primarily an introductory task. They found it suitable for this use because it is visual, has an easy start, and has a step-by-step increase in difficulty. It also offers opportunities for review later, and has a cliffhanger. Colored Cube satisfies the criteria for a rich learning task, and offers opportunities for the use of different solution strategies (here the group referred to Frank Lester). Group A found the task less suitable as an enrichment task because its first subtasks are too easy. They suggested that, for use as an enrichment task, the first four or five subtasks should be omitted, and the students should be asked to head directly for the general case.

The Ant's Walk was categorized as an enrichment task. Group A's main reason for this was that the task requires the student to produce a general, algebraic solution. The reason why they considered the task less suitable as an introductory task was that the step between providing a numerical solution and a general solution is too big.

Group B found Ferris Wheel to be a good introductory task because it refers to an everyday situation that students are familiar with, which helps turn the abstract theory into something concrete and tangible. Students can handle the problem even though they are unfamiliar with the underlying mathematical concepts/theory. In addition, they get a taste of what will be treated later within the trigonometry topic. The reason why Group B found this task less suitable as an enrichment task was that it would be too easy for someone who has understood the concepts/theory of trigonometry. Also, the second (and more difficult) part of the task is too similar to its first part.

Finally, Disease Spread was considered a suitable enrichment task because it requires considerable pre-knowledge about differential equations. Students need to understand that they cannot solve the system of nonlinear differential equations analytically but instead need to invoke digital tools. The task is relevant (in light of the COVID pandemic) and interesting, and lets the students see mathematics in a complex context. The results offer opportunities for interesting discussions, and the task can easily be modified and extended. The high demand on pre-knowledge, the inclusion of differential equations on a (for upper secondary school) high level, and the use of digital tools were also reasons why Group B found Disease Spread to be less suitable as an introductory task.

Discussion

In this paper we have asked what characterizes a challenging task that is suitable for introduction and enrichment, respectively. We have answered this question from the viewpoint of an experienced group of upper secondary teachers who have participated in a school development project and designed, analyzed, tested, and revised challenging tasks for use in mixed-ability mathematics classrooms. During their work, the teachers have in various ways encountered the dilemma that, on the one hand, every student must be able to work with the same task and, on the other, the task must offer challenges for all students. This dilemma can be conceptualized as a manifestation of a contradiction between agreed-upon aims of the tasks (Engeström & Sannino, 2011). As contradictions in an activity system fuel collaborative learning processes (Engeström & Sannino, 2011), such dilemmas should not be avoided but rather made visible and an explicit object of inquiry. The analysis of the four tasks presented in this paper is the result of such inquiry. The teachers' verbalization of the respective characteristics of introductory and enrichment tasks can be seen as an outcome of a collaborative learning process.

We therefore believe that our study and its results can contribute to mathematics teaching practice and educational research in different ways. Here, we highlight three. The first is that the results highlight characteristics for introductory and enrichment tasks that can guide teachers in designing, assessing, or selecting material for classroom enactment. Even though the results are hardly surprising, they point to important dimensions along which a task needs to be assessed in order to determine whether it is suitable for introduction or enrichment, with the two kinds of tasks often representing opposite ends of the scale. An introductory task should have an easy start and be visual and concrete, while an enrichment task should not have too easy a start and should aim for general solutions. In introductory tasks, the gap between subtasks must not be too big, and in enrichment tasks not too small. In introductory tasks one should not head for general, algebraic solutions too quickly, while in enrichment tasks one can go directly to general solutions. Introductory tasks must not require pre-knowledge, while enrichment tasks should do just that. One can be tempted to conclude that introductory tasks are those with a "low floor" or with enabling prompts, while enrichment tasks are those with a "high ceiling" or with extending prompts. However, for effective learning, all students should be offered opportunities to struggle with mathematical ideas (Hiebert & Grouws, 2007). To offer appropriate challenges to students with difficulties as well as those with high abilities in mathematics, both introductory and enrichment tasks should be designed to have both a "low floor" and a "high ceiling" (Bobis et al., 2021). We therefore suggest another interpretation: Introductory and enrichment tasks offer different kinds of learning challenges. These differences require that, when designing introductory tasks, extra focus should be placed on ensuring a "low floor", while the design of enrichment tasks requires greater focus on providing a "high ceiling".

A second contribution is that our results show how attending to the question of whether a task is an introductory or an enrichment task also can serve as a catalyst for discussions of other important aspects of task design for differentiated mathematics instruction. The act of designing tasks suitable for introduction and enrichment will present teachers with other dilemmas, such as whether a task provides enough guidance without funneling (Bauersfeld, 1998), and whether it is specific enough without depriving students of opportunities for general reasoning. Thus, for the collaborative learning process, attending to the question of whether a task is an introductory or an enrichment task might be more important than deciding on absolute criteria for such tasks.

A third, and methodological, contribution to the research community is the method used to clarify an outcome of a collaborative learning process: by conducting a retrospective analysis related to a previously discovered dilemma – in this case, the dilemma of designing introductory and enrichment tasks that are challenging for all students in the mixed-ability classroom.

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