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Abstract: Studies on high ability in mathematics rarely take a teacher's perspective. The purpose of our study is to add such a perspective, which we will do by using positioning theory to analyze elementary teachers' discussions on mathematical tasks, aiming to challenge all pupils, including the highly able. The study is conducted in the context of a two-year long teacher professional development program on high ability. Teachers expressed both their teaching and the mathematical tasks as helpful in orchestrating teaching suitable for highly able pupils. They highlight the opportunities given by some tasks as well as the importance of guiding highly able pupils to go further in such tasks. However, they expressed their own limited mathematical knowledge and time needed for pupils with learning difficulties as obstacles to orchestrate teaching for the highly able. The results show that it is important to, in close cooperation with teachers, further explore how to orchestrate teaching that challenges highly able pupils.

Keywords: High ability, Mathematics, Positioning theory, Professional development, Teachers

1 Introduction

> All children and pupils should be given the guidance and stimulus they need for their learning and personal development in order to develop as far as possible in accordance to the educational goals. (The Swedish Education Act, SFS 2014:458, p. 2)

The literature claims that highly able pupils develop their academic skills best in ability groups with like-minded (e.g., Idsøe, 2014; Wolfensberger, 2012). However, in Sweden there is no differentiation in compulsory school (grades 1-9, age 6-16), resulting in diverse classrooms. The quote above illustrate the high expectations of schools and teachers to develop and conduct mathematics teaching aiming for a wide range of pupils, as each of them have equal rights to develop their knowledge. The focus in previous teaching development have to a large extent been on the pupils with learning difficulties, but resent studies (e.g., Pettersson, 2011; Szabo, 2017) show the important role of teaching in our study when it comes to support highly able pupils. Leikin and Stanger (2011) indicate that teachers in diverse classrooms do not give the mathematically highly able pupils sufficient opportunities to develop their mathematical proficiency, and how to nurture highly able pupils in the

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diverse classroom has been a largely neglected field of study (Shayshon, Gal, Tasler, & Ko, 2014). In addition, studies on the education of mathematically highly able pupils usually take the perspective of the pupil (see Szabo, 2017).

Szabo (2017) showed in his review that, for example, the use of challenging tasks is assumed to have a beneficial effect on highly able pupils' development in mathematics. The use of challenging tasks with easy entries and open ends to develop mathematical learning for highly able pupils is further supported by, for example, Benölken (2015), Nolte (2012) and Sheffield (2003). With challenging tasks, we mean that each pupil will come to a stage where they must struggle, meaning that they don't automatically know a suitable solution process on forehand. Such tasks can also be described as mathematical problems (see e.g. NCTM, 2000; Lithner, 2017).

In a diverse classroom it is reasonable to assume that pupils' conceptual understanding in mathematics is very wide spread. It is also reasonable to assume that a mathematically highly able pupil either has extensive pre-knowledge, or learn, and can adapt, new procedures in a faster speed than most other pupils in the class. Therefore, the use of challenging tasks with an open end offers several levels of complexity so that also a mathematically highly able pupil must struggle. The easy entry in those tasks offers possibility for all pupils to start working on the task. Activating the pupils in challenging tasks can be a way to create an environment where detours and failures in the learning process are accepted. Pupils should feel safe to take intellectual risks in order to develope their mathematical proficiency and their creative thinking skills (Luria, Sriraman, Kaufman, 2017).

The construction of challenging tasks suitable for mathematically highly able pupils is a relatively well researched field (see e.g. Benölken, 2015, Nolte & Pamperien, 2017). But very seldom are questions addressed regarding how teachers' teaching experiences can contribute to operationalizing what earlier research has showed mathematically highly able pupils need to develop their learning.

The purpose of this study is to probe in-service teachers' discussions on how to orchestrate teaching of highly able pupils in the diverse classroom. In our study, we use positioning theory to search for patterns on what teachers express as possibilities and obstacles regarding orchestrating teaching to support highly able pupils to develop their learning in mathematics. The study is conducted in the context of a professional development program (PD) on high ability in mathematics. We analyzed discussions on challenging mathematical tasks among a group of teachers. Thereby, we include the teacher perspective to the field of research on education of highly able pupils.

2 Analytical framework

Positioning theory is a tool to illuminate "explicit and implicit patterns of reasoning that are

realized in the ways that people act towards others" (Harré, Moghaddam, Pilkerton Cairnie, Rothbart, & Sabat, 2009, p. 5). It employs the triad *speech act – storyline – position* (Harré & van Langenhove, 1999; Harré & Moghaddam, 2003), to capture the social meaning of human actions, and describes how meaning and understanding is organized. By using concepts and principles based on local moral appearing (e.g., as beliefs), the theory displays what persons "may do and may not do" (Harré et al., 2009). The explanation of the cognitive process supporting actions, for example speech acts, is hence seen to depend on formal rules of reasoning they also discern in actions of others and give to what someone would do him- or herself. Harré and his colleagues describe that people are momentarily bound to clusters of moral presumptions of what they, in a specific context, believe, or hold on to.

When using positioning theory to analyze practices, the analyst may display positions that are immanent in an interaction, related to a context with certain norms (Harré et al., 2009). For example, in a study of pre-service teachers' development of professional teacher identity Mosvold and Bjuland (2016) investigated the role of interactive and reflexive positioning. They used personal pronouns to identify reflexive respective interactive positions and through the further analysis they revealed the pre-service teachers' view of themselves as teachers as well as their confidence in the subject, mathematics. Two other examples are when Markström and Simonsson (2011) used positions to characterize pre-school girls as 'ordinary girls' or 'girlish girls', and Vanassche and Kelchtermans (2014) characterize teacher educators as educators of 'pedagogues', 'reflective teachers', or 'subject teachers'.

In our study, the participants were, due to the selection process of the PD, prepositioned as in-service mathematics teachers. The teachers' preposition gave them right to express themselves on how to orchestrate teaching. It also gave them duties, for example, explicitly given by the Swedish Education Act (SFS 2014:458), other steering documents, and the content of the PD-program, or, more tacitly, by school traditions. Through the PD the teachers studied literature on education of mathematically highly able pupils, and they also actively discussed that topic. This is assumed to have placed them in situations where they had to negotiate with their preposition as in-service mathematics teachers.

In our analysis process we searched for patterns where the teachers express opportunities and obstacles on how to orchestrate teaching of mathematically highly able pupils. Through the lens of positioning theory, the utterances on opportunities and obstacles expressed by the teachers are connected to the meaning-making (in relation to their teaching practice), and to perceived position as teachers. In the following section, we will describe how speech acts, storyline and position are operationalized.

2.1 Speech act

In our study, a speech act is an excerpt of what is said during teachers' discussions; often it is

a piece of the discussion that makes a unit, most often a sentence. Speech acts can also involve gestures and body language (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015); we only involve what is said. Herbel-Eisenmann et al. write: "*speech acts* are the meaning that those words/actions have for participants" (2015, p. 187). We interpret that they mean that the social meaning of what has been said can be interpreted through analysis of the speech act together with the connected storyline and position. The context of the speech acts' belonging is determined by the associated storyline.

2.2 Storylines

Storylines are mutually agreed context for the conversation (Harré & Moghaddam, 2003), in our case *Differentiation, Mathematical problems* and *Mathematical abilities*. Naming of storylines can indicate what is expected in a speech act (Herbel-Eisenmann et al., 2015), for example in our study the name *Mathematical abilities* on a storyline means that it is expected that the context in the speech act addresses theories by Krutetskii (1976). To explore what the teachers express on content related to education of mathematically highly able pupils in the diverse classroom, we used predefined storylines, *Differentiation, Mathematical problems* and *Mathematical abilities*. A speech act was judged to belong to one of the three storylines when it addressed the described content. To give a picture of what the teachers might talk about when a speech act was judged to belong to a specific storyline, we briefly describe some background for the three storylines.

2.2.1 Differentiation

Several studies report that when highly able pupils are not given learning situations that challenge them, they easily get bored and are at risk of giving up the learning in school (e.g. Lassig, 2003; Rubenstein, Siegle, Reis, McCoach, & Burton, 2012). According to literature, highly able pupils sometimes need to be placed in groups where they meet like-minded peers (e.g. Nolte & Pamperien, 2017; Rogers, 2007; Vogl & Preckel, 2014). However, as most of the time they are placed in the diverse classroom, they should be given opportunities and stimulation to develop as far as possible, just like all pupils should. One way to meet the needs of all pupils in a diverse classroom is to differentiate the teaching (MacLeod, 2005; Tomlinson, 2001; Vialle & Rogers, 2012). Teaching can be differentiated in content, process, results and learning environment (Tomlinson, 2001).

2.2.2 Mathematical problems

The storyline *Mathematical problems* is about mathematical tasks and the way these should be constructed to give opportunities for all pupils to be challenged. Sheffield (2003) means that when teachers actively give opportunities to all pupils to practice mathematical abilities, mentioned for example by Krutetsii (1976), all pupils benefit and develop their mathematical skills. To give those opportunities, the problem-solving situation should be designed. The construction of mathematical tasks is important both to give pupils the possibility to

demonstrate their abilities and to give teachers the possibility to observe those abilities in their pupils' work. Such tasks, for example rich learning tasks, are known to be challenging and stimulating for mathematically highly able pupils (Nolte, 2012; Sheffield, 2003) as well as for all pupils (Nolte & Pamperien, 2017; Sheffield, 2003; Taflin, 2007). However, the task itself is not enough, neither to challenge and stimulate pupils, nor for teachers to observe the pupils' mathematical abilities. The way the teachers orchestrate the problem-solving situation is also important (Nolte & Pamperien, 2017).

2.2.3 Mathematical abilities

In the third storyline, *Mathematical abilities*, the focus is on mathematical abilities. There are several definitions of mathematical high ability and most of them have similarities with the definition made by Krutetskii (1976). Krutetskii defines several abilities that mathematically highly able pupils often show when working with problem solving. Some of those abilities are the ability for logical thought, the ability to make fast and rapid generalizations, the ability to have a flexible mind, and a more general one – a mathematical cast of mind.

According to Krutetskii (1976) the abilities characterizing mathematically highly able pupils can only be observed when a pupil is active in a mathematical activity, preferably problem-solving. Creativity, curiosity and perseverance are other abilities associated with mathematically highly able pupils (Sheffield, 2003). Krutetskii's (1976) definition is, compared to Sheffield's (2003), more theoretical, while Sheffield's is more operational, and more connected to teaching and mathematical tasks.

2.3 Position

A position that someone either consciously or unconsciously takes in a conversation can be described through a character or a metaphor (see e.g. Markström & Simonsson, 2011; Mosvold & Bjuland, 2016; Vanassche & Keltermans, 2014). Positions are not permanent, but are dynamic and flexible, and are not freely chosen since they depend on other participants and the position taken by them (Evans, Morgan, & Tsatsaroni, 2006). What is logically possible for a person to say or do is implicitly limited by the position that person has in a certain context (Anderson, 2009; Harré & Moghaddam, 2003).

There are at least two positions to study in a speech act: interactive positioning when someone positions someone else (Davies & Harré, 1990; Mosvold & Bjuland, 2016), for example a pupil or another teacher, and reflexive positioning when someone positions himself/herself (Davies & Harré, 1990; Mosvold & Bjuland, 2016). According to Vanassche and Kelchtermans (2014), the way teacher educators reflexively position themselves is closely related to their normative assumptions about good teaching that are the ground for decisions for actions in special situations. We assume also that teachers' reflexive positions are closely related to their decisions for teaching highly able pupils.

In this study, we use positions to explore how the teachers position themselves and others in the speech act with regard to the different storylines. We therefore identify interactive and reflexive positions mainly through a focus on used subject-words and personal pronouns. The interest in our study lies in how the teachers express themselves, and in line with Harré and Moghaddam (2003), we believe that the context influences how and what they express; meaning that what the teacher expresses is authorized by the PD, their interactions with their colleagues as well as their own self-authorization. The used position can reveal if possibilities and obstacles are expressed differently between different storylines.

3 The study

3.1 Participants and design

This study was conducted on data coming from a PD for in-service teachers in mathematics. The PD lasted for two academic years, from autumn 2015 until summer 2017, and 15 of 24 teachers completed the whole PD. The first author of this article was also facilitator for the PD, which will be further elaborated in the discussion. The PD aimed to develop teachers' competence to orchestrate teaching of mathematically highly able pupils in the diverse classroom. Materials from an online course on gifted education in general (UNSW, 2004) were used as literature in the PD. In addition, literature specifically on mathematical high ability was added (e.g. Benölken, 2015; Sheffield, 2003).

An invitation for teachers to participate in the PD was send to all public schools in the municipality. In the invitation an approximate time schedule and approximate working hours were given. To fulfil the demands to participate, an applying teacher needed to have a teacher certificate for teaching mathematics. In addition, the teachers' principal needed to sign the application as a proof of that the teacher were supported by the principal. Through this application procedure it is assumed that the teachers who participated in this PD participated voluntarily. There was one school in the municipality where the principal specifically encouraged the staff to participate in the announced PD, from this school a group of five teachers chose to participate, one other school had two participating teachers, from the each of the other schools one single teacher per school participated.

At the announcement of the PD the applicants were informed about the connected research. The respondents were fully informed about the aim with the research, the methods involved, and that they would be anonymized in all texts connected to the research. All respondents agreed and gave their consent to participate. Furthermore, they were explicitly reminded that their participation in the research was voluntary and that they could end their participation at any time. In addition, the ethical issues of the study were discussed during a formal seminar with senior researchers.

During the first year of the PD, the participants had eight whole-day meetings, each seven

hours long. The meetings in the PD incorporated seminars, discussions and workshops in big groups, i.e. all participants, or in smaller groups. Between each meeting the participants prepared themselves through readings and assignments. Data for this study is obtained from one group of five teachers participating in the PD. The object of study is some specific discussions during workshops in which the teachers analyzed and developed mathematical tasks aiming to challenge and stimulate all pupils in their classrooms. The teachers were through the PD instructed to give extra focus to analyzing and developing the tasks to include challenges for the mathematically highly able pupils.

The five teachers in the group of interests for this study all taught in grades 1-3 at the same school, but they did not teach together. One of the teachers had less than five years teaching experience, one had between 6 and 10 years and the other three had more than 11 years of teaching experience. The workshops were distributed over five meetings from September 2015 to April 2016, the number of participating teachers varied between two and five, i.e. not all of them participated in all data collections. However, all assignments were available online through a platform for the PD. Each group made notes and comments on this platform during their assignments. If a participant was not able to join during a physical meeting in the PD, he or she was encouraged to follow online. The assignment for the teachers on those workshops was to collaboratively analyze and develop mathematical tasks aiming to challenge all pupils in the diverse classroom, including the mathematically highly able.

The five workshops aimed to give the teachers opportunities to combine their own experiences from teaching with the given frames. There were given frames to relate to in the workshops: criteria of rich learning tasks (Sheffield, 2003), abilities shown by mathematically highly able pupils (Krutetskii, 1976) and differentiated instructions (Tomlinson, 2001). The given frames were connected to the storylines used in the process of analysis.

For the PD the final aim with the mathematical tasks was to offer challenges for all pupils in the classroom. Therefore, the idea was to find or develop tasks that had an easy entry so that all pupils could understand what the task was about and would be able to start working on the task. With this purpose the teachers constructed what they called *a base question* for many of the chosen tasks. To offer enough complexity for the mathematically highly able pupils, the teachers constructed *extended questions* for the tasks and connected those to higher order thinking.

One example of a task they chose was the ghost house, we here present the 'final' task, after the teachers had analyzed it, developed it and implemented it in class in three periods with further discussions and development in between. During the discussions that are data for this study, the ghost house was not fully developed. For detailed description of the development process of tasks in the PD, see Mellroth (2018).

3.1.1 The ghost house

Base question

The ghost house has four floors. Four ghosts live on the top floor. On each floor lives twice as many ghosts as on the floor above. How many ghosts live in the house?

Figure 1 The ghost house, adapted from Mellroth (2018).

The pupils were told to use practical material if they wanted to when working on the task. They were also given a picture of a ghost house, Figure 1.

Extended questions

- How many ghosts live in the house if there are five floors? Six floors?
- How many ghosts live in the house if there are 5, 6, 7, ghosts living on the top floor?
- What if the number of ghosts are tripled, quadrupled on each floor?
- Can you formulate a rule, that works for each height of the house, that you can use to calculate the number of ghosts in the house? Or can you create an own task where the number of ghosts (or other creatures) are changing in a similar way?

Another task the teachers worked with during the PD was the folding task, its mathematical and didactical possibilities are well described by Nolte and Pamperien (2017). We refer the interested reader to their article for more description of tasks like those used in this PD.

3.2 Analysis procedure and data source

All meetings in the PD were video-recorded, and selected sections of the workshops chosen as data were transcribed verbatim. The time scale of the data used for further analysis is shown in Table 1.

Table 1 Summary of the amount data in time, and time scale

Data source	Time
All meetings 1st year in the PD	Approximately 90 hours
The chosen group, discussions on tasks	9 hours 17 sec
Sections containing storylines	4 hours 42 min 12 sec

To select sections for transcription, the video recordings were carefully looked at using NVivo (QSR International, 2014), nodes were placed on subsections that had some sort of connection to the given frames which in this study are the predefined storylines. Thereafter, those nodes were transcribed verbatim as object of further analysis. However, materials connected to the meetings in the PD (i.e. the full video recordings, the written assignments for each selected workshop and the participants' notes connected to the assignments) have served to clarify information to specific transcripts.

Positioning theory was used for further analysis of the transcripts. The three authors met

during a five-hour workshop session to ‘calibrate the analysis process’. Example (1) shows how a storyline and the position is detected in this way:

(1) Well it is not **very creative**

This is an example of a speech act within the storyline *Mathematical problems*, and the used word ‘creative’ is directly related to the criterion that a rich learning task should encourage creativity (Sheffield, 2003). ‘It’ relates to the task as a subject, therefore the position is interactive.

In this way the three authors together negotiated and agreed on marked storylines used in the proceeding analysis process.

Further, the position for each speech act was analyzed to be interactive or reflexive through studying used, or underlying, personal pronouns or subject words. Used words for identification of the position could not be seen in isolation. A holistic perspective was needed since some personal pronouns (e.g. *you*) were used for both interactive and reflexive positions. Our procedure divides the speech acts into two subgroups for each storyline (see Table 2).

Table 2 The six groups as object of further analysis

	Interactive	Reflexive
Differentiation	Differentiation - Interactive	Differentiation - Reflexive
Mathematical problems	Mathematical problems - Interactive	Mathematical problems - Reflexive
Mathematical abilities	Mathematical abilities - Interactive	Mathematical abilities - Reflexive

The six groups, Table 2, were further analyzed to explore what the teachers expressed as possibilities and obstacles to orchestrating the teaching to challenge all pupils in the diverse classroom.

4 Results

There were 668 speech acts analyzed in this study. As some (n=10) speech acts addressed two storylines, 678 triads (speech act - storyline - position) were further analyzed.

As shown in Figure 2, *Mathematical problems* was the most frequent storyline (n=326) and *Mathematical abilities* the least frequent (n=133). It is also seen that the teachers most often used an interactive position (n=429) compared to a reflexive one (n=249).

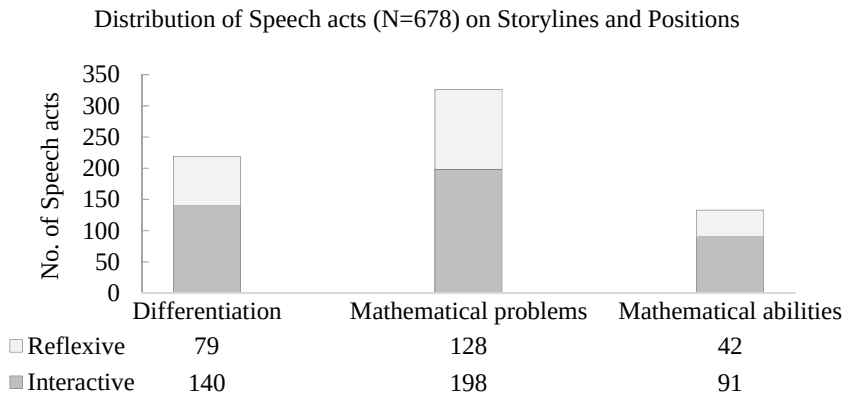

Figure 2 Distribution of the speech acts into the six groups that were further analyzed.

What the teachers expressed as possibilities and obstacles in orchestrating teaching for mathematically highly able pupils were revealed by further analysis of the six groups (Table 2). Patterns were discovered through the holistic view of the triad. The holistic view also incorporates an analysis of the triad in its context. For example, each speech act belongs to an episode of communication. In our results we therefore describe the episode and we show the speech acts before and/or after the specific speech act we use to describe the found patterns.

Some perspectives were recurring in the analyzed groups, although the perspectives were addressed in different ways. Those perspectives were: the tasks, the teaching as such, pupils and knowledge correlated to the PD. We will first present the possibilities expressed through the storylines and the respective position, thereafter the expressed obstacles are presented in a comparable way. Speech acts, numbered with their transcript number, are given together with an interpretation of their meaning to exemplify the analysis. The analyzed position and storyline is indicated with the speech act. Recall that the authors have reflected on the whole episode of communication in their analysis and interpretation of the speech acts. The sentence before the presented speech acts are meant to help the reader understand the analysis.

4.1 Quantitative summary of the analysis

In the further analysis, perspectives related to teaching, tasks and pupils were addressed within all three storylines. In Tables 3 - 5, we show for each storyline and position, interactive or reflexive, in which amount those perspectives were addressed and also in which amount they were connected to expressed possibilities or expressed obstacles. Several speech acts are analyzed to express both possibilities and obstacles. For example, in speech act (228) the teacher mentions the use of practical material, which is interpreted as an expressed possibility to help all pupils work with the task. While the comment of ‘difficulties to judge’ is interpreted as an expressed obstacle to analyze the task according to the given frames in the PD. Due to double existence of possibilities and obstacles in some speech acts the total number of speech acts in Table 3 – 5 is not the same as the sum of number of expressed

possibilities, obstacles and the number of speech acts without expressed possibilities or obstacles.

Table 3 Groups of speech acts addressing the storyline *Differentiation*; interactive position left, reflexive position right

Differentiation - Interactive 140 speech acts, of which 33 (24%) are neither possibilities, nor obstacles			Differentiation - Reflexive 79 speech acts, of which 15 (19%) are neither possibilities, nor obstacles		
	Possibilities (n=66)	Obstacles (n=51)		Possibilities (n=44)	Obstacles (n=29)
Teaching (No./%)	21/32%	15/29%	Teaching (No./%)	33/75%	19/66%
Task (No./%)	49/74%	31/61%	Task (No./%)	15/34%	9/31%
Pupils (No./%)	43/65%	44/86%	Pupils (No./%)	13/30%	13/45%
Other (No./%)	0/0%	0/0%	Other (No./%)	3/7%	0/0%

Table 4 Groups of speech acts addressing the storyline *Mathematical problems*; interactive position left, reflexive position right

Mathematical problems - Interactive 198 speech acts, of which 143 (72%) are neither possibilities, nor obstacles			Mathematical problems - Reflexive 128 speech acts, of which 70 (55%) are neither possibilities, nor obstacles		
	Possibilities (n=40)	Obstacles (n=15)		Possibilities (n=37)	Obstacles (n=21)
Teaching (No./%)	9/23%	2/13%	Teaching (No./%)	18/37%	6/29%
Task (No./%)	40/100%	14/93%	Task (No./%)	29/78%	13/62%
Pupils (No./%)	20/50%	9/60%	Pupils (No./%)	13/35%	6/29%
Other (No./%)	0/0%	0/0%	Other (No./%)	2/5%	1/7%

Table 5 Groups of speech acts addressing the storyline *Mathematical abilities*; interactive position left, reflexive position right

Mathematical abilities - Interactive 91 speech acts, of which 63 (69%) are neither possibilities, nor obstacles			Mathematical abilities - Reflexive 42 speech acts, of which 18 (43%) are neither possibilities, nor obstacles		
	Possibilities (n=22)	Obstacles (n=8)		Possibilities (n=14)	Obstacles (n=15)
Teaching (No./%)	1/5%	0/0%	Teaching (No./%)	6/43%	3/20%
Task (No./%)	16/73%	1/13%	Task (No./%)	7/50%	1/7%
Pupils (No./%)	6/27%	1/13%	Pupils (No./%)	5/36%	2/13%
Other (No./%)	5/23%	7/88%	Other (No./%)	3/21%	11/73%

4.2 Possibilities

4.2.1 Interactive position - addressing tasks

The type of tasks the teachers analyze and develop in the PD are by the teachers expressed as tasks that give the opportunity to challenge all pupils, with something challenging for all pupils to proceed on. Those tasks are referred to as an opportunity in all three storylines. Through taking an interactive position the teachers express, within the storyline *Differentiation*, that they believe pupils are interested in those tasks, the tasks allow different solution strategies and allow pupils to reach different levels in their results. The highly able pupils are especially addressed in combination with these tasks' possibilities to challenge, e.g. speech act (229).

> The episode of communication: the teachers discuss the use of practical material to help for all pupils to start, as well as allowing different solution strategies (see Sheffield, 2003) and different results (see Tomlinson, 2001).
>
> (228) It is difficult to say of course, but I'm (*reflexive position*) thinking that it is still a task, if you are using practical material, which everyone can get through. It is difficult to say how deliberate it is, and it differs a lot of course, but everyone can actually pick among whatever it is they choose. (*mathematical problems*)
>
> (229) And if you were to think the way she said, that those (*interactive position*) who are highly able, they can sort of take off with their ideas. (*differentiation*)
>
> (230) It is not a stop for them (*interactive position*). (*differentiation*)

Within the storyline *Mathematical problems*, and through taking an interactive position, the teachers express the possibilities with the easy entry and open end in those tasks, that pupils are encouraged by the challenges given on different levels and the possibility to work in different ways. Also, they positively express the possibility for pupils to choose solution strategies, e.g. speech act (665).

The episode of communication: the teachers discuss that different solution strategies should be possible for a task, one criterion of a rich learning task (Sheffield, 2003). Also, the teachers discuss how their formulations might steer the pupils' chosen strategies or not.

(665) so it won't be incorrect even if they (*interactive position*) choose to do things in a different way (*mathematical problems*)

(666) The task is sort of free, that is what I (*reflexive position*) was thinking about if we with these steps and formulations will end up with 20 identical solutions. (*mathematical problems*)

Through an interactive position and within the storyline *Mathematical abilities* the teachers often give examples of how they think pupils will show the abilities defined by Krutetskii (1976), e.g. speech act (636).

The episode of communication: the teachers analyze how they can interpret and notice the ability to grasp the formal structure of a mathematical task and the ability to generalize (Krutetskii, 1976) in their pupils while working on the task.

(635) Ability to formalize mathematical material, separate form from content, operate between forms formal structures of relations and connections. I (*reflexive position*) think that one is difficult. (*mathematical abilities*)

(636) Well, they (*interactive position*) can actually diverge completely and drop the form and move directly to the content, if they think multiplication is a piece of cake, then they can skip certain things. (*mathematical abilities*)

(637) Ability to generalize mathematical material. Well, this is to generalize. (*interactive position*), (*mathematical abilities*)

4.2.2 Reflexive position - addressing tasks

When the teachers take a reflexive position within the storyline *Differentiation*, they express that they can use the variety in the task in their teaching, the entry of the task gives possibility for all pupils to start working on the task, and through the task they can guide mathematically highly able pupils to go further in the mathematics. They also express the possibility with those tasks in relation to having different expectations of pupils; who are not all expected to reach the same level. Further they express their possibility to change the tasks if needed for some pupils, e.g. speech act (60).

The episode of communication: the teachers are discussing a task and negotiate on how to make changes to make it more suitable for all pupils. Further they connect the task to a special case of one pupil.

(60) All those three, I (*reflexive position*) thought perhaps they weren't possible the way they are, but perhaps you could do a variant on them. (*differentiation*)

(61) And there is a lot there as well, because it isn't actually an answer (*interactive*

position). (*mathematical problems*)

(62) You got to use maximum three, but she (*interactive position*), I can't remember how she did it, but no matter how I tried I used two or three, but she only used two moves straight away. (*mathematical abilities*)

(63) She (*interactive position*) viewed it in a completely different way, sort of. (*mathematical abilities*)

Expressed possibilities through a reflexive position within the storyline *Mathematical problems* especially address that the entry of the task makes it possible to differentiate pupils and allow them to work with suitable challenges, e.g. speech act (170). The teachers also express the task as being engaging and address as a possibility, that how they formulate tasks makes a difference to whether or not the task becomes challenging for all pupils.

> The episode of communication: the teachers communicate on the development process of a task, they are focusing on how the teacher should act to make it possible for all pupils to start. Further, they discuss how pupils might start working.
>
> (170) Then you (*reflexive position*) really get it across that the teacher also points out that there are many ways to start. (*mathematical problems*)
>
> (171) Then they (*interactive position*) might come with suggestions on how you could try it. You don't give them the suggestion beforehand. But it doesn't actually belong there, but it's really good, it's important. (*mathematical problems*)

Analysis of the reflexive position within the storyline *Mathematical abilities* shows that the teachers express that the tasks give the pupils opportunities to demonstrate the abilities defined by Krutetskii (1976), e.g. speech act (137).

> The episode of communication: the teachers are analyzing a task regarding the mathematical ability to think logically and the ability to curtail mathematical reasoning (Krutetskii, 1976).
>
> (137) Sequential, logical, can separate … yes you (*reflexive position*) should be able to say that about this one, yes I think so. (*mathematical abilities*)
>
> (138) The ability to curtail reasoning, well we (*reflexive position*) can see that later, if we do the same type of task several times. (*mathematical abilities*)

4.2.3 Interactive position - addressing teaching

The analysis shows that the teachers address themselves as individuals or in collaboration with colleagues as a possibility to orchestrate teaching for mathematically highly able pupils in the diverse classroom. They address both practical things such as "how to do" as well as more "soft" issues such as what they as teachers allow and encourage.

Within the storyline *Differentiation*, and through taking an interactive position the teachers address the possibility to, in collaboration with colleagues, orchestrate teaching where each teacher's strength is used in the most effective way, for example speech act (91). The

possibility to group pupils is also expressed to orchestrate teaching in the diverse classroom. Further, as possibilities, they express their own teaching to guide the pupils forward in the task. And that they through their teaching can acknowledge differences among pupils, for example through giving the pupils access to, and allowing them to use, various kinds of practical material.

> The episode of communication: the teachers discuss grouping the pupils and cooperation with colleagues to be able to orchestrate the teaching.
>
> (90) Well I (*reflexive position*) don't have those type of groups, or I have problem solving in every group of course (*differentiation*).
>
> (91) I'm so lucky, my colleague is coming now. He (*interactive position)* takes that part and I do the other one. So now he will do a little more, he doesn't want to use the math books and then I step in (*differentiation*).

When the teachers take the interactive position within the storyline *Mathematical problems*, they express that it is important how teachers present these kinds of task and how they guide pupils through the work. For example, they express that teachers should guide the pupils to be open-minded to different solutions, e.g. speech act (123).

> The episode of communication: the teacher develops a mathematical task trying to make it fulfill criteria of a rich learning task; it should be possible for all pupils to start, there are many possible solution strategies, different starting points as well as ending points (Sheffield, 2003).
>
> (123) If you (*interactive position*) think that way, that you can fit that image as well, then there will be someone who thinks "I have to have blocks" (*mathematical problems*)
>
> (124) Someone says (*interactive position*) "I can draw!". "I can calculate!" (*mathematical problems*)
>
> (125) I'm (*reflexive position*) thinking that you might do task a then, not a, b, c, d, e, but that it might just be the first one. Then perhaps you continue on other occasions. (*differentiation*)
>
> (126) Well yes, you need to continue … with the way they (*interactive position*) should think and develop, right. (*mathematical problems*)
>
> (127) They (*interactive position*) are sort of going to make several steps on the same level, but then it is good if there is a continuation of course (*mathematical problems*)
>
> (128) Because then the first level might be enough for some (*interactive position*) and some might not even get through it anyway. (*differentiation*)
>
> (129) Because it gets too. While some (*interactive position*) do it without even building, some can simply picture it. (*differentiation*)

Expressed possibilities through the interactive position within the storyline *Mathematical abilities* are that the teachers through their teaching can give pupils opportunities to show the

abilities defined by Krutetskii (1976), e.g. speech act (79).

> The episode of communication: the teachers negotiate on how they could operationalize the mathematical ability for grasping the formal structure of a mathematical task, an ability defined by Krutetskii (1976).
>
> (79) but couldn't it be that you (*interactive position*) draw the task or that you build with blocks. Then you you both create the content but still separate it later, or? Am I getting it wrong? (*mathematical abilities*)
>
> (80) That sounds great, operating anyway. (*interactive position*), (*mathematical abilities*)

4.2.4 Reflexive position - addressing teaching

Within the storyline *Differentiation*, through the reflexive position the teachers express the possibility they have when clarifying explanations for pupils, to use the variety in the task in their teaching, to group pupils and collaborate with colleagues to orchestrate teaching. They specifically express that they think the tasks used in the PD can guide highly able pupils to progress further with the mathematics, e.g. speech act (53). Further they express the possibility to 'Ipad-record' some pupils of extra interest considering high ability when those pupils are working with the tasks, to be able to do a deeper analysis afterwards.

> The episode of communication: the teachers discuss how their pupils might work with a task they develop in the PD.
>
> (49) Because there are of course many (*interactive position*) who will only do, and not understand. (*differentiation*)
>
> (50) And others, those (*interactive position*) who want to solve this. (*differentiation*)
>
> (51) Everyone (*interactive position*) will of course see that something happens, that there will be a hole and a sort of pattern. Yes. (*differentiation*)
>
> (52) But those (*interactive position*) who can start to reflect, they might not be able to solve it but they might still be able to start thinking about that there must be connection. (*differentiation*)
>
> (53) And then it becomes exciting (*reflexive position*) to try this task, because then that kind of pupil can, perhaps you have that kind of pupil who wants to try, who won't stop. (*differentiation*)
>
> (54) Because it feels like it is a task that you (*reflexive position*) can try with everyone. (*differentiation*)

When taking a reflexive position within the storyline *Mathematical problems* the teachers express their possibility to guide and encourage the pupils to work with the tasks. In particular, the teachers' guidance to support pupils to be able to start working on the task and to go further is expressed, e.g. speech act (368). Collaboration between colleagues is also expressed as a possibility to orchestrate teaching that challenges all pupils.

The episode of communication: the teachers discuss how they believe the pupils will work with the mathematical task they have developed.

(368) I'm (*reflexive position*) thinking that this is the base, then you continue and give them the image and they are still going to have the same task. (*mathematical problems*)

(369) Then I'm (*reflexive position*) thinking that they will get to try to draw the task in several different ways. (*differentiation*).

Within the storyline *Mathematical abilities* and through a reflexive position, the teachers express that they through their teaching can guide pupils to show some of the abilities defined by Krutetskii (1976) while working with this kind of task. For example, the teachers express how they can notice those abilities in pupils through posing questions to pupils while they are active in working with a task, e.g. speech act (145).

The episode of communication: the teachers are analyzing a task regarding the mathematical ability to be flexible in the mental process in a mathematical activity (Krutetskii, 1976).

(145) also perhaps you (*reflexive position*) need to actually ask a few questions to see if they through the questions can look at the concept in a different way. I mean turn the reasoning around because it is not a given that they will do it automatically themselves. (*mathematical abilities*)

4.3 Obstacles

There are almost twice as many speech acts connected to possibilities compared to obstacles (see Tables 3-5). We choose therefore to present the expressed obstacles without separating the two perspectives tasks and teaching. Interactive and reflexive positions are presented separately.

4.3.1 Interactive position

Within the storyline *Differentiation* in relation to obstacles, pupils with learning difficulties are addressed. The teachers express their concern for those pupils, their difficulties to enter the task, to see patterns, solve the task without teacher guidance, and to reach higher level in the task, e.g. speech act (314). The teachers also express that it is time consuming to guide pupils with learning difficulties and they worry about not having that time. They specifically address the difficulty of orchestrating teaching for pupils with learning difficulties when the mathematics becomes abstract. Finding and adjusting suitable tasks is raised as an obstacle, for example to give enough, but not too much instructions. They find a risk in that pupils copy each other's strategies despite being encouraged to choose a strategy on their own. The teachers lack confidence in their own mathematical knowledge, e.g. speech act (56). In addition, they address the difficulty of keeping the class gathered when working with tasks that allow differentiation.

The episode of communication: is a part of a longer episode where the teachers discuss some

difficulties they believe pupils with learning difficulties might encounter with the mathematical tasks they develop in the PD.

(313) Because otherwise there will be utter confusion right from the start (*interactive position*), (*differentiation*).

(314) But otherwise they (*interactive position*) don't have a chance, no. (*differentiation*).

The episode of communication: the teachers discuss how some tasks might lead a pupil showing the mathematical ability to grasp the formal structure of the task, one of the abilities defined by Krutetskii (1976).

(55) Who just discover, that there (*interactive position*) has to be a reason for this. It has to be connected and that alone might be an indication that (*mathematical abilities*)

(56) But that is exactly what I mean. This is what is so difficult, I mean this thing with tasks and finding them for these children (*interactive position*), when we ourselves can't do it. (*differentiation*)

When addressing the storyline *Mathematical problems*, obstacles correlated to tasks are mainly mentioned (see Table 4). For example, the teachers raise criticism of the construction of the tasks, meaning that the pupils are sometimes given too many instructions so it becomes an obstacle to the open approach, e.g. speech act (170). The teachers mention that the tasks are not enjoyable for all pupils, and that there is a risk that pupils follow each other's solution strategies instead of choosing their own.

The episode of communication: the teachers discuss the development process of a task. They are focusing on how pupils can use different solution strategies when working with the mathematical task.

(170) In this task they (*interactive position*) should. The task is that they should use the cars and experiment, right? It's not something they choose? (*mathematical problems*)

(171) No but, I'm thinking like this, that it (*interactive position*) doesn't have to mean that you have actually cars, but perhaps you have a few symbolic cars in your classroom. (*mathematical problems*)

(172) Or perhaps you (*reflexive position*) have colored pieces of paper that you can replace the cars with, or you might have blocks, or something like pieces of lego. But that that is not something we should actually give them. Because just like Elisabet demonstrated, the older ones can visualise it straight away, to perhaps only have three different colored pens. (*differentiation*)

(173) Sort of do it with them all the way. They (*interactive position*) need, I mean some children won't need the practical, while others only use the practical. (*differentiation*)

When using an interactive position within the storyline *Mathematical abilities* the teachers mainly address the difficulties to interpret Krutetskii (1976), see e.g. speech act (387). They also raise as a problem that some pupils will have difficulties showing the abilities defined by

Krutetskii (1976), see e.g. speech act (385).

> The episode of communication: the teachers analyze how they could notice the ability to curtail mathematical reasoning (Krutetskii, 1976) in their pupils while they are working on the task.
>
> (384) Ability to curtail reasoning (*interactive position*), (*mathematical abilities*)
>
> (385) How should they (*interactive position*) curtail it then? (*mathematical abilities*)
>
> (386) Isn't it (*interactive position*) a curtailment in itself to say half. (*mathematical abilities*)
>
> (387) What does it mean, or should they (*interactive position*) be able to curtail that it is a half half or what is meant by curtail. (*mathematical abilities*)
>
> (388) Yes that one is a little difficult. (*reflexive position*), (*mathematical abilities*)

4.3.2 Reflexive position

Through a reflexive position within the storyline *Differentiation* the teachers express that they find the whole-classroom situation difficult for orchestrating the teaching. They mention that that they do not have enough time, they find it hard to find suitable material and tasks, they lack resources to implement larger tasks like the one in focus in the PD, and they also question whether grouping pupils is okay in elementary school. The teachers worry that pupils with learning difficulties will find the tasks too difficult, and that the task will affect them negatively, e.g. speech act (308). The teaching situation is addressed as difficult when some pupils find it hard to discover the mathematics in a task at the same time as other pupils 'rush away' too quickly, risking that they do not stop and think. The teachers express that the easy entry of the task might risk lowering the level of the task.

> The episode of communication: refers to the teachers' upcoming implementation of the task in their classrooms. They notice that it might bring trouble to some pupils. Further, they highlight that they as teachers can give instructions that help all pupils to work on the task.
>
> (308) Should we (*reflexive position*) give it to them straight away, or should we just give them a square? Because that could cause a problem for some straight away. (*differentiation*)
>
> (309) But I'm thinking that if you (*reflexive position*) give this, but still say, look at the whole image, the square I mean, we can draw on the board, I mean just the frame and show (*differentiation)*

Within the storyline *Mathematical problems* the teachers express that their own mathematical competence might be too low, for example they mention that they find it hard to solve the kind of tasks used in the PD, e.g. speech act (28), and especially in different ways. They criticize the criteria of a rich learning task that say that a task should promote competence and enjoyment; the teachers mean that a task is not necessarily enjoyable just because it promotes competence. The teachers also express that some of the criteria are difficult to judge without

interfering with pupils. The teachers mention that there is a risk that pupils do not use their own solution strategies and that they might copy a strategy from a classmate instead. They also find it hard to formulate good questions for the task without steering the pupils too much.

> The episode of communication: the teachers discuss if a task fulfills the criteria, that a rich learning task should have important and useful mathematics embedded (Sheffield, 2003). The excerpts are interpreted as if the teachers believe the task might fulfil the criteria of Sheffield, but their mathematics knowledge is too low to judge it.
>
> (27) But perhaps it (*interactive position*) is important or illustrates something which is useful in mathematics even if I don't. (*mathematical problems*)
>
> (28) Even if we (*reflexive position*) don't understand. (*mathematical problems*)
>
> (29) That I (*reflexive position*) don't know. (*mathematical problems*)

Addressing the storyline *Mathematical abilities* through reflexive position the teachers express difficulties with interpreting Krutetskii (1976); this is mentioned in the majority of the speech acts related to obstacles, see e.g. speech act (94). The teachers express that they should plan their teaching to be able to give pupils the opportunity to show the abilities defined by Krutetskii (1976), which is also analyzed as a possibility. To be able to observe if a pupil shows those abilities the teachers express that they want to sit one-to-one with the pupil and that they need to observe them during a long-term process. In one speech act they mention that pupils with learning difficulties will find it hard to express those abilities.

> The episode of communication: the teacher discussion is about analyzing how pupils might show the mathematical abilities defined by Krutetskii (1976) through a mathematical task.
>
> (93) That is how we (*reflexive position*) are going doing it with the groups anyway. He does, it's great, problem solving, he takes the strong ones sometimes and then the weaker ones sometimes (*differentiation*)
>
> (94) You know these abilities, we (*reflexive position*) have to find out what they mean. (*mathematical abilities*)

4.4 Neither possibilities nor obstacles identified

In all the storylines, there were some speech acts that could not be interpreted as either possibilities or obstacles; this was especially noticeable within the storylines *Mathematical problems* and *Mathematical abilities* (see Tables 3-5). Although the aim of this study is to search for patterns in expressed possibilities and obstacles, we find it interesting to present what the teachers communicated in those speech acts.

Within all three storylines, using both interactive and reflexive positions, the teachers mainly addressed issues related to their assignment in the PD, i.e. analyzing and developing tasks. For example, they discussed how to construct extended questions, how to adjust the entry of the task and what pre-knowledge pupils needed.

When addressing the storyline *Mathematical problems* they also discussed the meaning of the criteria of a rich learning task and how pupils might work with the specific tasks they discussed in the PD. When addressing the storyline *Mathematical abilities*, they discussed how they should interpret Krutetskii in the specific task they work on in the PD. They also discussed what pupils might do during a solution process when he or she shows the abilities defined by Krutetskii (1976). In both the storylines *Mathematical problems* and *Mathematical abilities* the teachers strategically analyze the tasks they worked on according to the criteria for a rich learning task, with regard to the abilities defined by Krutetskii (1976).

5 Discussion

The teachers in this study have, through analyzing, developing and discussing challenging tasks, showed understanding on how to orchestrate teaching for mathematically highly able pupils, differentiated education is expressed as a possible pedagogical strategy to use. Some misconceptions occurred which led to some problems for the teachers to create understanding. One explanation to this might be that the teachers do not feel that their mathematical knowledge is sufficient. However, to focus on challenging tasks in a PD aiming to improve teachers' competence to orchestrate teaching for mathematically highly able pupils in the diverse classroom has shown to be successful in this study.

Through the analysis we showed what the teachers expressed as possibilities, and also revealed that their knowledge of teaching mathematically highly able pupils is coherent with what research has shown as important. For example, they reflect over situations where pupils are working with challenging tasks and discuss the importance of guiding pupils further through questions instead of giving them answers or just confirming response. Therefore, those teachers have knowledge of how to support pupils working with challenging tasks in a similar way that for example Nolte and Pamperien (2017) mean tutors in fostering programs for mathematically highly able pupils do, in contrast to teachers in diverse classrooms.

Further, the teachers show that they have good knowledge in the construction of challenging tasks. For example, they verbalize the open ends as important specially to give mathematically highly able pupils challenges, which is coherent with previous literature (Benölken, 2015; Nolte & Pamperien, 2017; Sheffield, 2003). The teachers also see possibilities in those tasks' suitability for all pupils in the classroom, which is supported by Taflin (2007). The easy entry is highlighted by the teachers as important for pupils with learning disabilities. We interpret the discussions as if the easy entry is perceived as a crucial factor for the tasks possibility to be used in the diverse classroom. Through the teachers understanding of how challenging tasks can offer possibilities for learning opportunities for mathematically highly able pupils, we interpret that the teachers also understand that those pupils need to be offered challenges to develop in learning. Thereby an implication of

teachers' participation in a PD with a focus on challenging tasks might lead to improved teaching skills in offering mathematically highly able pupils learning opportunities. Teaching skills that Leikin and Stanger (2011) showed were lacking among teachers in diverse classrooms.

The analysis show that the teachers perceive it as a possibility to differentiate their teaching through the challenging tasks, i.e. through the component *content*, in differentiated instruction (Tomlinson, 2001, 2016). In a study on differentiated education (Gaitas & Martins, 2017), it was found that of the four components in differentiated instructions, *content, process, product* and *learning environment* (Tomlinson, 2001, 2016), the teachers found content to be the most difficult to differentiate. This study has shown that the participating teachers understand the principles of how to differentiate challenging tasks according to content, further research is needed to probe how they manage to implement it in practice.

The teachers express the theories of Krutetskii (1976) to be difficult to understand. Indications of misunderstanding his theories is that they are discussing how pupils with learning difficulties can be able to show the mathematical abilities Krutetskii describes. Although, Krutetskii very clearly describe those abilities as such abilities that are possible to detect in mathematically highly able pupils and this distinguish those pupils from others. This misconception amongst the teachers might also be an explanation to why they find it hard to understand Krutetskii.

In this study, we have used positioning theory (Harré & van Langenhove, 1999; Harré & Moghaddam, 2003) as an analytical tool to extract what teachers express as possibilities and obstacles regarding orchestrating teaching for mathematically highly able pupils. In their study, Herbel-Eisenmann et al. (2015) discovered that researchers in mathematics education usually fail to use all three components of the positioning theory. Further they found that most often the *storyline* and the *speech act* are not addressed or are considered of less importance than the *position*. We have focused in particular on expressed possibilities and obstacles when analyzing teacher discussions through a holistic approach using all three components of positioning theory.

Our analysis thus reveals the meaning in the teachers' discussions based on their perceived view of their professional task. We believe that their perceptions are affected both by their prepositions as in-service teachers and by their participation in the PD. In line with Harré et al. (2009), the professional task perceived by the teachers influence what they may or may not say. Thereby it is our view is that we capture the teacher perspective in our study.

It is rare that research on education of mathematically highly able pupils addresses these pupils in the context of the diverse classroom (Shayshon et al., 2014). The in-service teachers' perspective on how to orchestrate this teaching is lacking. The study by Leikin and Stanger (2011) is an example of this, although their results indicate that teachers in diverse classrooms

do not give the mathematically highly able pupils opportunities to develop their knowledge.

Current research using interactive and reflexive positions within positioning theory (e.g. Mosvold & Bjuland, 2016; Vanassche & Kelchtermans, 2014) argue of the importance of using those two positions in the analysis. The study by Vanassche and Kelchtermann (2014) demonstrated that for teacher educators the reflexive position is a crucial factor in understanding their practices as well as for their understanding of students' learning. Mosvold and Bjuland (2016) used both interactive and reflexive positions when observing conversations between a mentor teacher and pre-service teachers. Through their study Mosvold and Bjuland, for example, revealed the importance of the mentor as well as the pre-service teachers' confidence in themselves as teachers. Our results verify that the reflexive position reveals in-service teachers' understanding of possibilities and obstacles regarding orchestrating teaching for mathematically highly able pupils, which is especially seen within the storyline *Differentiation* (see Table 1). In addition, our results indicate that the interactive position is of importance to reveal teachers' understanding of the used material, in our case mathematical tasks.

From our analysis we noticed, for all three storylines, that through the reflexive position the teachers expressed themselves about teaching to a higher extent compared to the interactive position (see Tables 1-3). In comparison, we noticed that through the interactive position the teachers expressed themselves more about the tasks compared to when taking a reflexive position (Tables 1-3). Further analysis revealed both the expressed possibilities and obstacles.

We learn from the analysis of our data that, in the context of a PD, possibilities are expressed approximately twice as often compared to obstacles (see Tables 1-3). We could however reveal one exception: Within the storyline *Mathematical abilities* and through the reflexive position the expressed possibilities were equal to the expressed obstacles. In our further analysis, we see that the teachers find it hard to operationalize the theory of Krutetskii (1976) which is expressed through the reflexive position.

We believe that when teachers themselves choose to participate in a PD they become positively minded towards the content. Therefore, they are more likely to search for ways to implement the content into their practice, rather than to criticize it, leading to a higher extent of expressed possibilities. To decrease the influence of the facilitator, she actively stayed out of the rooms where the teacher discussed and worked with the assignments in the PD. She only entered on the rare occasions when the teachers actively asked her to come, for example to clarify a concept or an assignment in the PD.

We argue that we have revealed important aspects viewed as possibilities and obstacles to orchestrating teaching of mathematically highly able pupils, from the teacher perspective. We

have, through the PD and predefined storylines in our analysis, connected the teacher perspective to well established theories in the field of education of mathematically highly able pupils. That is, to use challenging tasks (Sheffield, 2003), the importance of noticing mathematically highly able pupils – for example by observing their mathematical abilities according to Krutetskii (1976) – and to have a pedagogic strategy allowing the inclusion of all pupils in teaching and learning, for example through differentiated instruction (Tomlinson, 2001).

6 Conclusion

Our results show that teachers express the tasks used in the PD as suitable for all pupils and that the tasks give opportunities for the highly able to be challenged. The teachers highlight the importance of guiding both pupils with learning difficulties and highly able pupils to go further in such tasks. Those results are in line with the study of Nolte and Pamperien (2017) who found that challenging tasks constructed for mathematically highly able pupils with success also can be used in diverse classrooms. The teachers do not express that they find the highly able pupils difficult to orchestrate teaching for. However, Nolte and Pamperien show that mathematically highly able pupils in homogeneous groups needed less time on challenging tasks and achieved better results for example in generalization. This indicates a need for further research and development of teacher practice.

We suggest future research to explore how mathematically highly able pupils are included in teaching and learning when in-service teachers are encouraged and challenged: to use mathematical tasks with easy entries and open ends for all pupils, observe mathematical abilities defined by Krutetskii (1976) in their pupils' mathematical activities and to use differentiated instructions. Design research is an example of methodology we believe can be used to further investigate and develop teachers' knowledge. It seems promising to increase teacher competencies on differentiated instructions. Such studies, also may reveal why and when teachers express obstacles regarding teaching pupils with learning difficulties. Further, we suggest the use of positioning theory, and the interactive and reflexive positions, as a tool in the analysis to probe what can be developed in the teaching and what can be developed regarding external aspects such as used material, flexible grouping and resources.

We believe that our results contribute to the field of high ability in mathematics through revealing what teachers express as possibilities and obstacles to orchestrating teaching that includes all pupils, including the highly able. In practice, our results can be used when school development projects on high ability are designed, so that participating teachers can focus on finding solutions to the obstacles and further develop the possibilities. Our results can therefore contribute to helping all pupils receive the guidance and stimulation they need for their learning and to develop as far as possible in accordance with their potential, as required in the Swedish Education Act (SFS 2014:458).

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