This is the published version of a paper published in *Early years*.

Citation for the original published paper (version of record):

Enhancing young children’s understanding of a combinatorial task by using a duo of digital and physical artefacts
*Early years*
https://doi.org/10.1080/09575146.2018.1501553

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:kau:diva-69133
Enhancing young children’s understanding of a combinatorial task by using a duo of digital and physical artefacts

Jorryt van Bommel & Hanna Palmér

To cite this article: Jorryt van Bommel & Hanna Palmér (2018): Enhancing young children’s understanding of a combinatorial task by using a duo of digital and physical artefacts, Early Years, DOI: 10.1080/09575146.2018.1501553

To link to this article: https://doi.org/10.1080/09575146.2018.1501553

© 2018 The Author(s). Published by TACTYC

Published online: 02 Aug 2018.

Submit your article to this journal

Article views: 41

View Crossmark data
Enhancing young children’s understanding of a combinatorial task by using a duo of digital and physical artefacts

Jorryt van Bommel\textsuperscript{a}\textsuperscript{*} and Hanna Palmér\textsuperscript{b}

\textsuperscript{a}Department of mathematics and Computer science, Karlstad university, Karlstad, Sweden; \textsuperscript{b}Department of mathematics, Linneaus University, Växjö, Sweden

ABSTRACT

In mathematics education, digital tools have been used to enhance young children’s understanding of specific subject matter. In such implementations, the digital tool can replace, amplify or transform ‘ordinary’ mathematics teaching. In an initial study, systematization and duplication were identified as critical when young children were to solve a combinatorial task. Therefore, a digital version of the task was developed and combined with a non-digital version, to introduce the use of dual artefacts. The digital version of the task enabled the children to visually explore systematization as well as the principle of completion. After using this digital version of the task, the children’s written records became more systematic and included fewer duplications. We conclude that the digital version of the task reinforced young children’s understanding of the combinatorial task and that the use of dual artefacts enhanced children’s understanding of what a combinatorial problem encompasses.

ARTICLE HISTORY

Received 29 March 2018
Accepted 14 July 2018

KEYWORDS

Duo of artefacts; combinatorics; mathematics; digital tools; representations

Introduction

The empirical material in this article is from an educational design research study (McKenney and Reeves 2012) exploring and developing a problem-solving approach in mathematics in the education of young children (six-year-olds) (Palmér, van Bommel, and Ebbelind 2015; van Bommel 2015). However, the focus of this article is only on the development of one of the tasks and not on the complete educational design research study. The task in question is a combinatorial task where the children are to consider how many different ways three toy bears can be arranged in a row on a sofa. To make the task meaningful for young children, it was presented as a conflict between the toy bears where the bears cannot agree on who should sit where on the sofa. One toy bear then suggests that they could change places every day. The task for the children was to find out how many consecutive days the bears could sit in different ways on the sofa. Thus, it is an enumerative combinatorial task where the children are supposed to count the permutations for \( n = 3 \).

CONTACT

Jorryt van Bommel\textsuperscript{*} jorryt.vanbommel@kau.se Department of mathematics and Computer science, Karlstad university, Karlstad, Sweden

\textsuperscript{*}Corresponding author.
The focus of this article is on the representations and systematizations the children spontaneously use when solving this combinatorial task and how both of these may be influenced by introducing a carefully designed digital version of the task as a prelude to tackling a paper and pencil version of the task. Thus, the digital version of the task, in combination with the paper and pencil version of the task formed what has been termed in the literature ‘a duo of artefacts’, such that use of the digital artefact complements use of the conventional material one (Maschietto and Soury-Lavergne 2013).

The digital version was developed based on an initial design cycle (Palmér and van Bommel 2018) where children seemed to struggle with systematization while working on the tasks using only paper and pencil. The forms of representation and systematization displayed by children directly tackling the paper and pencil version of the task influenced the design of a digital version of the task that would promote exploration of systematization.

In research, physical and digital artefacts are seldom both taken into account simultaneously in learning processes (Arzarello and Robutti 2010). However, a duo of artefacts as introduced by Maschietto and Soury-Lavergne (2013) includes both a physical and a digital artefact in learning processes. Their results indicate that the use of one artefact can add value to the use of another artefact, recognizing that it is a function of design rather than a technological certainty. It is not self-evident that the use of several artefacts will reinforce children’s learning processes without the combination and use of the artefacts being carefully designed. In this article we seek to establish in broad terms whether the duo of artefacts enhances young children’s solutions of the combinatorial task and, if it does, in what way?

First, we present previous research on young children and combinatorics. In particular, we focus on representation and systematization as these areas are important when working on combinatorial tasks. This introduction is followed by a more detailed presentation of the different parts of the study, and finally, the results of each part of the study are discussed.

**Finding permutations**

The patterning and structural skills of children have been analyzed in several studies. For instance, Blanton and Kaput found that through scaffolding specific concepts they could promote children’s early algebraic reasoning (2005), but likewise, an intensive intervention with a focus on mathematical patterns produced similar results (Papic, Mulligan, and Mitchelmore 2011). Finding and creating patterns can be seen as preludes to finding permutations. English (1991, 1996, 2005) has also shown that a proper and meaningful context makes it possible for young children to work effectively to find permutations in combinatorial situations. Systematic variation, constancy, exhaustion, and completion are four important principles that must be understood to be able to solve combinatorial tasks successfully (English 1996). The principle of systematic variation means that a different combination will occur if at least one item is varied systematically. The principle of constancy means that a different combination will occur if at least one item is kept constant while at least one other is varied systematically. The principle of exhaustion means that a constant item is exhausted when it no longer generates new combinations when the other items are varied and the principle of completion means that when all constant items have been exhausted all possible combinations have been found.
**Representations**

When the children in this study were asked to find out how many days in a row the toy bears could sit in different ways on the sofa, in the initial design cycle they were offered only paper and a number of coloured pencils to work with. As they documented possible combinations, the children used a number of different representations. In research, representations have most often been investigated in close connection with numbers. Here, we present and compare the theories presented in three landmark studies on the graphic representations of young children: Piaget and Inhelder (1969), Hughes (1986), and Heddens (1986).

Piaget and Inhelder (1969) used the notion of symbols and signs to describe children’s representations. They defined symbols as representations with resemblance to the object they referred to—for example, pictures or tally marks—while signs had no resemblance to the object they represented but instead were based on the conventions of society—for example, numerals. Hughes (1986) distinguished between idiosyncratic, pictographic, iconic, and symbolic representations. Idiosyncratic representations are defined as irregular representations, pictographic representations as pictures of the represented item, iconic representations as (tally) marks, and finally, symbolic representations are defined as those instituted by the conventions of society, such as numerals. Heddens (1986) focused on connections between the concrete (objects) and the abstract (conventions of society) and identified two types of representations that fall in between these two extremes: semi-concrete and semi-abstract. Semi-concrete representations are defined as pictures of objects and semi-abstract representations as symbolic representations of objects where the representation does not look like the object it represents.

The theories of Piaget and Inhelder, Hughes, and Heddens on representations can be synthesized as pictured in Figure 1. The figure shows the resemblance between semi-concrete and semi-abstract representations as described by Heddens and the pictographic and iconic representations as described by Hughes. Piaget and Inhelder’s notion of symbol encompasses Hughes’ pictographic and iconic representation as well as Heddens’ semi-concrete and semi-abstract representation. Finally, the figure shows that Piaget and Inhelder’s notion of sign encompasses Hughes’ symbolic representation and Heddens’ abstract representation.

English (2005) has found that young children use pictographic, iconic, and symbolic representations when they work on combinatorial tasks, which is why all of these are of importance in our further analysis of the results of this study (Figure 2).

---

**Figure 1.** A synthesis of Piaget and Inhelder (1969), Hughes (1986) and Heddens (1986) notions on representations (Palmér and van Bommel 2017).
Systematization

When exploring and documenting possible combinations of the different ways the three toy bears can be arranged in a row on a sofa, the systematization of the permutations becomes an issue. According to English (2005), the major difficulty for young children working on combinatorial tasks, is to systematize their representation. This systematization includes the problem solution as well as the acquisition of domain-specific knowledge of combinatorics structure (English 1996). An early awareness of the existing structure and patterns of the task might enhance children’s understanding (Mulligan and Mitchelmore 2013); moreover, the strategies children use might facilitate their understanding of the structure. Hierarchical strategies of young children’s systematization within combinatorial tasks were first identified by English (1991) and later classified into three stages (English 1996): the random stage, the transitional stage, and the odometer stage. In the random stage, permutations are found through trial and error, and checking answers is a crucial step for success at the task. In the odometer stage, an organized structure for the selection of combinations is adopted: keeping one item constant and varying all other items systematically. This stage requires knowledge of the principle of exhaustion and principle of completion. When the written record shows signs of an organized structure but the systematization is not maintained throughout the task, the written records are classified as being part of a transitional stage. At this stage, knowledge of the principle of systematic variation and of constancy is required. The stages are hierarchical in such a way that the odometer stage is more effective when it comes to finding all possible combinations compared to the transition stage and trial and error stage.

The study

As mentioned, the combinatorial task focused on in this article was part of an educational design research study aiming at innovating and improving classroom practice (Cobb and Gravemeijer 2008). The children were all around 6 years old, and approximately 50% were boys and 50% were girls. The study conformed to the ethical regulations for research in Sweden, and all participating teachers, children, and guardians approved their participation (Swedish Research Council 2017).

The combinatorial task has been explored in three design cycles with the digital version of the task being developed in the third. Different preschool classes and thus
different children and teachers participated in each design cycle. In cycle 1, the authors of this article acted as teachers in 6 classes with a total of 87 children. In cycle 2, 4 new classes with a total of 36 children and their teachers were involved and researchers instructed the teachers regarding the task but the teachers themselves taught the lesson. During the third and final cycle, 5 new classes with a total of 61 children were involved and, once again, the teachers taught the lesson. By conducting the lessons ourselves in cycle 1, we were able to focus on the development of the task and exercise greater control in the design. Based on this cycle a detailed lesson plan was developed, which was then provided to the teachers in cycle 2 and 3. Before the teachers implemented the lesson they were introduced to the overarching idea of the project and to the lesson plan in question. The task focused on here was the third problem-solving task the teachers conducted within the design research study. Before the implementation we discussed issues of implementation, expectations concerning the lesson and children’s written records and their role as a teacher during the task. Small adjustments were made regarding the colour of the bears and the emphasis on the context of the problem in the explanation. The mathematical problem itself remained the same for all three stages; however, in cycles 1 and 2 only one artefact (paper and pencil) was used whereas in cycle 3 both a digital and a paper/pencil version were used as a duo of artefacts.

Since the results from cycle 1 and 2 were the source for the development of the digital version of the task, the details of their design will be described later when cycle 3 is presented. In mathematics education, digital tools can have the different roles of replacement, amplification or transformation (Hughes, Thomas, and Scharber 2006). The aim of the digital tool that was developed within this study was amplification which implies that the digital task amplified the current instructional practice while the original task fundamentally was the same. Thus a duo of artefacts was used—not a replacement (digital tool instead of paper and pencil) or a transformation (transformation of the paper and pencil part of the task) of artefacts.

When using a duo of artefacts, the question of coherence between the two artefacts regarding continuity versus discontinuity is of importance (Maschietto and Soury-Lavergne 2013). While the artefacts must have some common characteristics enabling conversion there also have to be differences to enhance development. Further, the use of dual artefacts in the classroom must be organized in such a way that the teaching can take advantage of the duality.

‘Such a duo of artifacts can enlarge and improve the learning experience of the students’ (Maschietto and Soury-Lavergne 2013, 960).

**Implementation**

When introducing the task in cycles 1 and 2, the children were shown three small plastic bears in different colors. In the introduction, they were told about the conflict between the bears. After that, the children worked individually: they were given plain paper and pencils in different colors but were given no instructions about what to record on the paper or how to do so. After working alone for about 20 minutes, they were to compare their written records in pairs to identify similarities and differences. They did not, however, change their written records at this point. It is the written records produced by the children that were
analysed. Finally, the children were gathered for a joint discussion based on their written records.

In cycle 3, after being told—as before—about the conflict between the bears, the children were instructed to work with the digital version of the task. They were given time to explore the digital version of the task on table computers but were not asked any specific questions about the content. After this—as before—they proceeded to the original paper and pencil task implemented in the same way as in cycles 1 and 2.

Analysis

The written records of the children in each cycle were analysed using a process and utility oriented analysis in order to be able to develop design theories related to learning (van den Akker et al. 2006). The analysis in each cycle of the study drew on appropriate theory, and the results informed the way in which the next cycle of the study was conducted (Table 1). In cycle 1, Hughes (1986) notions of pictographic and iconic representations were used to analyse the representations used by the children to find out if the selection of representation influenced children’s solutions. In cycles 2 and 3, Hughes’ notions were again used, but now complemented with English’s (1996) categories of trial and error, transition, and odometer for systematization. This analysis aimed at finding if there was any correlation between children’s choice of representation and the systematization in their solutions.

In the analysis we did not focus on the learning of individual children but instead we looked for regularities (Cobb and Gravemeijer 2008) between the task, the design of the lesson and children’s written records. Our analyses are based on categorizing the responses of individual children, regardless of their class. Bearing in mind that much of the task-based activity leading to these productions was conducted by children individually or in pairs, we think that this is a defensible approach, particularly for an exploratory study such as this one. In addition, although children were grouped in classes, their lessons had collective phases led by the class teacher, and the teacher also managed the phases of individual and pair activity. We have already described the steps we took to ensure comparability of approach in each class. Of course there were differences between the teachers and between the children in the classes but no teacher and no class diverged in a disproportionate way. In cycles 2 and 3, the lesson being conducted seemed to have been in line with the lesson plan, which was affirmed by the teachers and no notable differences were observed.

Results and analysis

This section is divided according to the three cycles described above. The analysis of cycle 1 focuses on the representations the children used. This analysis is based on 87

<table>
<thead>
<tr>
<th>Table 1. Overview of the study.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle 1</td>
</tr>
<tr>
<td>Taught by</td>
</tr>
<tr>
<td># children involved</td>
</tr>
<tr>
<td>Analysis</td>
</tr>
</tbody>
</table>
written records from cycle 1. The second analysis focused on both representation and systematization. Here, the 36 written records from cycle 2 are added to the previous 87. Finally, the third analysis describes the implementation of the digital version of the task and the analysis of the 61 written records made by the children in cycle 3 who used the digital version.

Thus, the representations used by the children were symbols (Piaget and Inhelder 1969) of two different kinds. For this reason, we chose to use Hughes (1986) definitions of pictographic and iconic representations when analysing the written records. However, we found it necessary to add a category that included both pictographic and iconic representations, as some children used both types. Table 2 shows the categorizations made on the 87 written records included in cycle 1.

As can be seen in Table 2, this was a challenging task for the children; only two of 87 children found six unique permutations. The majority (58 of 87) spontaneously used iconic representation; two of these were the ones who found six unique permutations. The majority of the children that used pictographic representation (15 of 18) produced only unique combinations while the majority (30 of 33) of the children who made duplicate combinations used iconic representation. Thus, even though the use of iconic representations implies a semi-abstract level (Heddens 1986)—which is more abstract than the use of pictographic representations (semi-concrete level)—the children who used pictographic representations made fewer duplications than the children who used iconic representations.

During analysis, we formulated three hypotheses to explain this somewhat unexpected result. First, that the time required to draw bears could have given the children

Table 2. Categorization of children’s written records from cycle 1 using Hughes (1986) (Palmér and van Bommel 2017).

<table>
<thead>
<tr>
<th>Category</th>
<th>Explanatory statement</th>
<th>Pictographic</th>
<th>Pic/Icon</th>
<th>Iconic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No new permutations</td>
<td>The child drew some toy bears or the combination shown by the teacher and then no further combinations.</td>
<td>3</td>
<td></td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Unique permutations A</td>
<td>The child drew unique combinations where the total number of combinations is less than six.</td>
<td>15 (4)</td>
<td>8 (3)</td>
<td>24 (10)</td>
<td>47</td>
</tr>
<tr>
<td>Unique permutations B</td>
<td>The child drew six unique combinations.</td>
<td></td>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Duplicate permutations</td>
<td>The child drew combinations where one or several combinations are duplicated.</td>
<td>3</td>
<td>30</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>21</td>
<td>8</td>
<td>58</td>
<td>87</td>
</tr>
</tbody>
</table>
more time to organize and compare the permutations. Second, that the connection to
real toy bears may have helped the children who used pictographic representations
better connect the task to reality and therefore they made fewer duplications. Third, that
the effort required to draw a further permutation using pictographic representation
motivated greater attention to whether it had already been included.

Then, based on the numbers in parentheses in the category Unique permutations a
fourth hypothesis was added. These written records contained three unique combina-
tions and no more, where each toy bear sat at each place once. Thus, these written
records indicated some kind of systematization, as each bear was drawn once at each
place on the sofa. This was done by 17 children (four using pictographic representation,
three using pictographic and iconic representation, ten using iconic representation). The
fourth hypothesis suggests that perhaps the explanation was not to be found in the
representation used (pictographic and/or iconic representations) but in the systematiza-
tion of the representations (Palmér and van Bommel 2017). This led to the research
becoming focused more specifically on the representation and systematization aspects
of children’s written records of the combinatorial task.

**Cycle 2—focus on representations and systematization**

In our second analysis, we investigated if there were any connections between the
systematization and representation used by the children and how they solved the
combinatorial task. Similarly to our first analysis, we used Hughes (1986) definitions of
pictographic and iconic representations when analysing the children’s representations.
When analysing systematization, we used English’s (1996) categorization of trial and
test, transition, and odometer as previously presented. Could we, for example, see that
one item had been kept invariant, or did the permutations seem to occur randomly? Of
course, it is impossible to interpret children’s intentions based on written records alone
which is why this analysis is made from an observational perspective only; it is possible
that children used systematizations that were not visible to us in their written records.
Table 3 shows the categorization that was made of the 123 written records from cycles 1
and 2. Nine written records were not possible to categorize regarding systematization as
they included only one permutation or a picture with more than three bears.

Of the 114 written records analysed at this stage, 82 had also been included in cycle 1
(the five written records with no new permutations (Table 1) were not possible to
analyse regarding systematization). A total of 35 children used pictographic representa-
tions, 71 children used iconic representations, and 8 children used both pictographic

<table>
<thead>
<tr>
<th></th>
<th>Pictographic</th>
<th>Pic/Icon</th>
<th>Iconic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial and error—no dup</td>
<td>19</td>
<td>2</td>
<td>9</td>
<td>37</td>
</tr>
<tr>
<td>Trial and error—all dup</td>
<td>2</td>
<td>9</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>Transition—no dup</td>
<td>7</td>
<td>3</td>
<td>14</td>
<td>24</td>
</tr>
<tr>
<td>Transition—all dup</td>
<td></td>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
<td>8</td>
<td>71</td>
<td>114</td>
</tr>
</tbody>
</table>
and iconic representations. Thus, the majority of the children spontaneously used an iconic representation. Four of 114 children found six unique permutations when they worked individually with the task. These four children all used iconic representations; two used a trial and error approach (a) and two used an odometer approach (b). Using a trial and error approach implies that these two children had to check each of the new permutations against all of their previous ones to figure out if each permutation was new or not.

Of the written records using a trial and error approach or a transition approach, 42 included duplications: over half of the iconic records (30/55), three of the five combined records, and about a third of the pictographic records (9/28) included duplications. In regard to the written records with unique combinations, two thirds of the pictographic records (19/28) consisted of unique combinations and less than half of iconic records (25/55) did.

Thus, there was less duplication in written records with pictographic representations which might lead to the conclusion that iconic representations did not generate a higher level of solution to the combinatorial task. However, a transition approach was visible more often in iconic (18) than in pictographic (2) records. This may indicate some connection between the development of systematization and the use of abstract representation as the use of iconic representations implies a semi-abstract level which is more abstract than pictographic representations (semi-concrete level) (Heddens 1986). When a trial-and-error approach was used, pictographic representations seemed to work best, which is in line with the previous hypothesis about time and connection to reality and context (English 1991, 2005). However, there were more (14) iconic representations on the odometer level than combined (3) or pictographic (7) representations, and if we compare the written records using a transition approach (with or without duplications) with those using an odometer approach, 20 of 22 children used iconic representation. According to English (2005), listing items systematically is the major issue for young children when solving combinatorial tasks and this second analysis indicated that the key issue was the systematization of the task. Further, the analysis indicated that the development of representations and systematizations seem to be somewhat synchronized. These results led us to the development of the digital version of the task (van Bommel and Palmér 2017a).

**Cycle 3—a duo of artefacts**

As indicated earlier, the results of the cycle 1 and cycle 2 analyses informed the design of a digital version of the task intended to create a duo of artefacts. In particular, what these analyses of the pen and paper task revealed about children’s difficulties provided the basis for our specification of design principles for the digital version, intended to provide particular types of cognitive support so as to structure children’s experience of the task. Offering a systematization as well as feedback on duplicates might help children to structure their solution process and help them to find all possible solutions.

Two difficulties were identified in the analysis of cycles 1 and 2; children did not seem to structure and systematize their permutations, and children produced duplicates of previous solutions and did not seem to recognize this. Further, analysis of cycles 1 and 2
indicated a relationship between the number of duplicates and the level of abstraction in the solutions (Palmér and van Bommel 2018).

This resulted in design principles (van den Akker et al. 2006) for the development of the digital version of the task. The children would work within a semi-concrete level (Figure 3: picture of bears) with pictographic representations (which had shown to produce fewer duplications) (van Bommel and Palmer 2017b). To address the issue of noticing duplicates, an option to save combinations was developed, and in the event of attempting to save a duplicate, the children would receive feedback showing that the combination had already been saved (Figure 3: red frame). This feedback served as an initiator for a reflection process around the concept ‘duplicates’. The children’s own solutions were visible (Figure 3: frames on the right hand side).

The crucial characteristic of the digital artefact, then, is the way in which it operationalizes the principles derived from the earlier analyses of children’s written records. The purpose of designing this digital artifact was to see if a synergic use of a duo of artefacts would address the difficulties identified in cycles 1 and 2. The digital version of the task that offered systematization as well as feedback on duplicates might help children to structure their later written records and help them in their search for possible solutions. As mentioned, coherence is of importance when using a duo of artefacts (Maschietto and Soury-Lavergne 2013) which is why both continuity and discontinuity had to be taken into consideration when developing the digital version of the task. By using the same context and the same task, ‘transfer and reinvestment from one to the other’ (Maschietto and Soury-Lavergne 2013, 960) is supported. The additional aspects of working on a semi-concrete level during the solution process, the exposure to the idea of duplicates and the possibility to a systematic overview all differ from the original task-setting and therefore meet the stipulation that the evolution of children’s knowledge is fostered by this difference.

A duo of artefacts was implemented in 5 classes with a total of 61 children (Table 4). The amount of duplicates in these children’s written records diminished in two ways. Only 12 out of 61 included duplicates in their written record. Additionally, the actual

Figure 3. Image of digital version of the task showing semi-concrete level, duplications and overview of own solutions.
number of duplicates in each such written record diminished as compared to those analysed in cycles 1 and 2. Whereas in cycles 1 and 2 several children had listed long lists of permutations, with as many as 33, the longest list of permutations made in cycle 3 only went up to 11 by one child (Table 4) and a list of 5–7 permutations in the remaining written records containing duplicates.

One could imagine that the children would not find the paper and pencil version of the task interesting or challenging after using the digital version, but the teachers reported no indications of this. Thus, the duo of artefacts seems to offer a balance between continuity and discontinuity (Maschietto and Soury-Lavergne 2013) leading to coherence in the lesson design. When looking at the total number of permutations, several children (13 out of 61) managed to list 5 of the 6 possible permutations—whereas previously only 5 (out of 123) children managed to do so. Almost a third of the children (17/61) managed to list all permutations, compared to only 2 of the 123 in cycles 1 and 2. However, even though the children who used a duo of artefacts found more permutations and made fewer duplications compared to the groups who had not worked with the digital version, still few children, only 13, ‘solved’ the task. Thus, the task still seemed to have been a challenging problem-solving task for the children, but the duo of artefacts seemed to help them with strategies for their paper and pencil written record.

**Discussion**

The focus of this article has been on the representations and systematizations young children use when solving a combinatorial task and how these may be influenced by the use of a duo of artifacts. As we stated before, two challenges were identified in cycles 1 and 2 of the study; systematization and structure of the solution process as well as a non-reflection on the existence of duplicates. These two challenges could be combined, where the non-systematization of the solution process could lead to non-structured records of possible solutions, which in its turn leads to a difficulty when comparing new solutions with ones obtained previously. When analysing children’s written records we do not know if it is the case that children do not see their duplicates, or if it is more about the fact that they do not compare their permutations. However, the latter one is more likely because there are written records where the same combination is repeated.

**Table 4. Categorization of written records cycle 3 based on representation and systematization using Hughes (1986) and English (1996) (Palmér and van Bommel 2018).**

<table>
<thead>
<tr>
<th></th>
<th>Pictographic</th>
<th>Pic/Icon</th>
<th>Iconic</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial and error—no duplications</td>
<td>1</td>
<td>1a</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Transition—no duplications</td>
<td>2</td>
<td>2a</td>
<td>3b</td>
<td>7</td>
</tr>
<tr>
<td>Odometer—not all solutions</td>
<td>9</td>
<td>1</td>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>Odometer—all solutions</td>
<td>3</td>
<td>1</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>20</td>
<td>7</td>
<td>34</td>
<td>61</td>
</tr>
</tbody>
</table>

aOf which one child obtained all solutions.
bOf which one child obtained all solutions.
cOf which two children obtained all solutions.
sequentially. Whichever, based on the results of cycle 3, a duo of artifacts seemed to influence the children’s written record regarding both systematization and duplication. This is in line with previous studies by Maschietto and Soury-Lavergne (2013). But, what caused this?

Maybe it was the fact that children were exposed to the problem over a longer period of time that influenced their outcome and not the digital version and the duo of artifacts as such? The duration of an activity was addressed by Jupri, Drijvers, and Van Den Heuvel-Panhuizen (2015). In their case, the intervention lasted for more than 5 h. However, the students in their study did not use a duo of artifacts but either a digital or a physical artifact. Their study showed that students using the digital artefact outperformed their peers using the physical artefact. Despite our much shorter intervention we got results similar to those of their students who used the digital artefact, maybe as a result of our use of a duo of artefacts. However, as we have not studied children who have used only the digital artefact we can only record how the digital version of the task influenced the written records in combination with a paper and pencil version of the task and not how the digital version of the task might function on its own in a different educational setting.

Another possible explanation of the results in cycle 3 is the issue of proper and meaningful context when young children work on combinatorial tasks, which has been addressed by English (1991, 2005). In cycle 3 the children first were given the opportunity to make sense of the context of the task by using the digital version without an actual task being posed to them. Maybe this led to the context of the later task being more understandable when they were asked to work with paper and pencil and explain the changes shown.

Even taking time and context into consideration, the duo of artefacts seems to have had impact. While being exposed to a duo of artefacts, the children made fewer duplications and more children managed to find more permutations. We argue that exposure to a possible structure contributed to this improvement. This aligns with Maschietto and Soury-Lavergne (2013) when they state that the artefacts forming the duo both have to have common characteristics and have to differ in order to improve a learning experience.

Nevertheless, it could be asked whether another paper and pencil artefact might not be as effective as this digital artefact in providing this amplification: for example, if the children had been given a sheet with 6 boxes or 6 outline drawings of bears in a row for them to colour. However in such a situation feedback regarding the concept of duplicates would not automatically come up. Further, several children had less than 6 solutions — including their duplicates — when only working with paper and pencil, indicating that providing six boxes would not necessarily have helped them. Systematic variation and constancy (English 1996) are a natural part of the digital version of the task whereas these principles are not always possible to discern when working on the task with merely paper and pencil.

When using the digital version of the task the children do not need to use systematization, but systematization is offered in the form of a structured way to look at solutions. Also the principle of completion (and consequently also the principle of exhaustion) is incorporated in the digital version, as it indicates when all permutations are found. The concept of duplications is introduced visually to
the children working with the digital version of the task while they get the opportunity to explore systematization and duplication further when working with paper and pencil. Thus, the duo of artefacts is coherent and offers both continuity and discontinuity, where the digital version makes children aware of systematization and duplication.

We are aware that such exposure would be possible with paper and pencil; however, a digital version adapts the time of such exposure to the process of the user and makes it therefore more effective. When the children later work with paper and pencil they themselves have to find a way to systematize their permutations that (hopefully) helps them to notice duplicates. The outcome of fewer duplications along with more successful solutions argues that children might have come to a better understanding. We conclude that the duo of artefacts offers an opportunity to enhance children’s understanding of what a combinatorial problem encompasses.

**Acknowledgement**

We thank Prof K. Ruthven for comments that greatly improved the manuscript.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**ORCID**

Jorryt van Bommel [http://orcid.org/0000-0001-6525-9871](http://orcid.org/0000-0001-6525-9871)

Hanna Palmér [http://orcid.org/0000-0003-2217-6624](http://orcid.org/0000-0003-2217-6624)

**References**


