An analytical framework for welfare analysis of trade

By

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Abstract: The purpose of this note is to develop a general model of trade between countries in which benefits and costs of free trade and restrictions to trade can be analysed. The model assumes a linear relationship in all of its components and the existence of a general equilibrium in prices in the post-trade markets determined by the world market price. Though the following situations are not examined in the note, the derived framework can be used to examine changes in social welfare from trade restrictions in the two countries, such as from a tariff or a quantity restriction on exports and imports and reciprocity, as well as industry subsidies, favourable production laws, lowering of environmental standards, and favourable corporate taxation.

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1. Introduction

This working paper examines trade between two countries with different pre-trade market conditions with the purpose of creating a framework for a welfare analysis of the consequences of opening markets for trade. To keep things simple we only look at trade between two countries, which both produce the same homogeneous product. We further assume that demand and supply can be described using linear functions.

2. Firm specific incentives to trade

So far, we have examined the country specific incentives to trade with other countries. It points in the direction of price differences between countries as a driver for trade. To elaborate on the effect of price differences it would be relevant to examine the conditions under which a single firm decides to sell its product abroad given that it also is selling the product at home.

The decision problem of the firm is that of price discrimination between to separate markets.

3. The pre-trade domestic market

Demand (D_d) in the domestic market is,

$$\text{Domestic demand, } D_d: p_d = A - bQ_d \Leftrightarrow Q_d = \frac{A}{b} - \frac{1}{b}p_d \quad (1)$$

Supply (S_d) in the domestic market is,

$$\text{Domestic supply, } S_d: p_d = B + aQ_d \Leftrightarrow Q_d = \frac{1}{a}p_d - \frac{B}{a} \quad (2)$$

Under these conditions, the pre-trade equilibrium in the domestic market is characterised by a market clearing price p_d^* and domestic demand equal to domestic supply $D_d = S_d$. Thus,

$$D_d = S_d \Rightarrow A - bQ_d = B + aQ_d \Leftrightarrow Q_d^* = \frac{A - B}{a + b} \quad (3)$$

Insert Q_d^* in either the demand or supply function to get,

$$p_d = A - b \frac{A - B}{a + b} \Rightarrow p_d = \frac{a}{a + b}A + \frac{b}{a + b}B \Leftrightarrow p_d^* = \frac{aA + bB}{a + b} \quad (4)$$

Define: $\alpha = \frac{a}{a+b}$ then $1 - \alpha = \frac{b}{a+b}$. Insert in equation (4) to get,

$$p_d^* = \alpha A + (1 - \alpha)B \quad (5)$$

Equation (5) tells us that the domestic price is an average from domestic conditions for demand and supply.

4. Social welfare in the pre-trade domestic market

The pre-trade social welfare is defined as the sum of producer and consumer surplus at the domestic market equilibrium quantity and price.

Social welfare is then,

$$W_d^* = CS_d^* + PS_d^* \quad (6)$$

Where consumer surplus is,

$$CS_d^* = \int_0^{Q_d^*} D_d(Q_d) dQ_d - p_d^* Q_d^* \quad (7)$$

Moreover, producer surplus is,

$$PS_d^* = p_d^* Q_d^* - \int_0^{Q_d^*} S_d(Q_d) dQ_d \quad (8)$$

Which then imply that social welfare is,

$$W_d^* = \int_0^{Q_d^*} D_d(Q_d) dQ_d - \int_0^{Q_d^*} S_d(Q_d) dQ_d \quad (9)$$

Inserting what we know about domestic demand and supply in the expression for social welfare to get,

$$W_d^* = \left[(A - B)Q_d - \frac{1}{2}(a + b)Q_d^2 \right]_0^{Q_d^*} \Leftrightarrow W_d^* = \frac{(A - B)^2}{2(a + b)} \quad (10)$$

5. The domestic, foreign and world market

Domestic and foreign purchasers and suppliers meet in the world market. Whenever the world market price is lower than the domestic market price domestic purchasers will find it more attractive to buy their product here. Although they would like to buy all they possibly can on the world market both market conditions in the domestic market and in the foreign market will limit the amount of the product that is supplied and demanded there.

Assuming that domestic market equilibrium price is higher than the foreign domestic equilibrium price, we can define the import demand from the domestic consumers and the export supply from foreign producers in the following way.

The demand for imports describes the difference between the domestic demand and domestic supply for prices in the world market lower than the domestic market equilibrium price and is denoted MD .

The supply of exports describes the difference between foreign supply and foreign demand for prices in the world market greater than the foreign market equilibrium price and is denoted XS .

6. Deriving the import demand function

The import demand function is derived using the demand and supply functions of the domestic market. That is by using,

$$\begin{aligned} \text{Demand: } p_D &= A - bD_d \Leftrightarrow D_d = \frac{A}{b} - \frac{1}{b}p_D \\ \text{Supply: } p_S &= B + aS_d \Leftrightarrow S_d = \frac{1}{a}p_S - \frac{B}{a} \end{aligned} \quad (11)$$

The size of import in the domestic market is the difference between domestic demand and domestic supply or,

$$\text{Domestic import} = D_d - S_d \quad (12)$$

Using this we can derive the import demand function for the world market. Thus,

$$\begin{aligned} D_d - S_d &= \frac{A}{b} - \frac{1}{b}p - \frac{1}{a}p + \frac{B}{a} \Leftrightarrow D_d - S_d = \frac{A}{b} + \frac{B}{a} - \left(\frac{1}{b} + \frac{1}{a}\right)p \Leftrightarrow \\ p &= \frac{aA + bB}{a + b} - \frac{bc}{a + b}(D_d - S_d) \Rightarrow p_{MD} = \frac{aA + bB}{a + b} - \frac{bc}{a + b}Q_{MD} \Rightarrow \\ Q_{MD} &= D_d - S_d \end{aligned} \quad (13)$$

From the derivation of the domestic market demand we can see that the intercept term in the above expression is equal to the domestic market equilibrium market price, i.e. to p_d^* .

Inserting this in the equation, we get the import demand function,

$$MD: p_w = p_d - \frac{bc}{a + b}Q_w \quad (14)$$

7. The pre-trade foreign market

Just like in the domestic market the incentives to export depends on the foreign market situation. Assume that demand and supply in the foreign market is characterised in the following way,

$$\begin{aligned} \text{Foreign demand, } D_f: p_f &= C - dQ_f \Leftrightarrow Q_f = \frac{C}{d} - \frac{1}{d}p_f \\ \text{Foreign supply, } S_f: p_f &= D + cQ_f \Leftrightarrow Q_f = \frac{1}{c}p_f - \frac{D}{c} \end{aligned} \quad (15)$$

The equilibrium in the foreign market is when $D_f = S_f$. Thus,

$$\begin{aligned} C - dD_f &= D + cS_f \Leftrightarrow Q_f^* = D_f = S_f = \frac{C - D}{c + d} \Leftrightarrow \\ p_f^* &= \frac{cC + dD}{c + d} \end{aligned} \quad (16)$$

Define: $\beta = \frac{c}{c+d}$ then $1 - \beta = \frac{d}{c+d}$. Insert in equation (16) to get,

$$p_f^* = \beta C + (1 - \beta)D \quad (17)$$

Equation (17) tells us that the domestic price is an average from foreign conditions for demand and supply.

8. Social welfare in the pre-trade foreign market

The pre-trade social welfare is defined the same way as for the domestic market as the sum of producer and consumer surplus at the foreign market equilibrium quantity and price. Social welfare in the foreign market is,

$$W_f^* = \int_0^{Q_f^*} [D_f(Q_f) - S_f(Q_f)] dQ_f = \frac{(C - D)^2}{2(c + d)} \quad (18)$$

9. Deriving the export supply function

Producers in the foreign market will supply products to the world market as long as the world market price is higher than the price the producer can get from exclusively sell their product to consumers in the home market. This implies that if foreign producers sell on the world market the production on the home market is greater than the demand for the products at home. Thus,

$$\text{Foreign market export} = S_f - D_f \quad (19)$$

Using this together with what we know about the demand and supply conditions in the foreign market allow us to derive the export supply function, XS.

If foreign demand and supply is given by,

$$\begin{aligned} \text{Foreign demand, } D_f: p_d &= C - dD_f \\ \text{Foreign supply, } S_f: p_s &= D + cS_f \end{aligned} \quad (20)$$

Then export is,

$$S_f - D_f = \frac{1}{c}p - \frac{D}{c} - \frac{C}{d} + \frac{1}{d}p \Leftrightarrow S_f - D_f = \frac{d+c}{cd}p - \frac{D}{c} - \frac{C}{d} \quad (21)$$

With $Q_w = S_f - D_f$ the world market price is,

$$p_w = \frac{cd}{c+d}Q_w + \frac{cC+dD}{c+d} \quad (22)$$

From the derivation of the equilibrium price in the foreign market, we can rewrite the equation to get the export supply function. Thus,

$$XS: p_w = \frac{cd}{c+d}Q_w + p_f^* \quad (23)$$

10. Equilibrium in the world market

The equilibrium in the world market will determine the post-trade equilibrium in the domestic as well as in the foreign markets. Following the derivation of the MD and XS curve above, we have to utilize the information in each functional expression to derive the world market equilibrium. The import demand and the export supply is,

$$\begin{aligned} MD: p_w &= p_d - \frac{ab}{a+b}Q_w \\ XS: p_w &= \frac{cd}{c+d}Q_w + p_f \end{aligned} \quad (24)$$

The equilibrium world market price and quantity is then,

$$\begin{aligned} MD = XS &\Rightarrow p_d - \frac{bc}{c+b}Q_w = \frac{nm}{m+n}Q_w + p_f \Leftrightarrow \\ p_d - p_f &= \left(\frac{nm}{m+n} + \frac{bc}{c+b} \right) Q_w, \text{ define: } \Delta p \equiv p_d^* - p_f^* \text{ to get,} \\ Q_w^* &= \frac{\Delta p}{\frac{nm}{m+n} + \frac{bc}{c+b}} \Leftrightarrow Q_w^* = \frac{\Delta p(m+n)(c+b)}{nm(c+b) + cb(m+n)} \end{aligned} \quad (25)$$

Inserting in MD or XS to find the equilibrium world market price, i.e. p_w^* ,

$$p_w^* = p_d^* - \frac{bc}{c+b} \left(\frac{\Delta p(m+n)(c+b)}{nm(c+b) + cb(m+n)} \right) \Leftrightarrow \quad (26)$$

$$p_w^* = p_d^* - \frac{bc(m+n)}{nm(c+b) + cb(m+n)} \Delta p$$

Define $\mu \equiv \frac{bc(m+n)}{nm(c+b) + cb(m+n)}$ with $0 \leq \mu \leq 1$ to get,

$$p_w^* = p_d^* - \mu \Delta p \quad (27)$$

This expression implies that for trade to take place we must have,

$$p_f^* \leq p_w^* \leq p_d^* \quad (28)$$

Rewriting equation (28) to get,

$$p_w^* = (1 - \mu)p_d^* + \mu p_f^* \quad (29)$$

The expression tells us that the world market price is a constructed price determined by the weight of domestic to foreign price.

11. Derivation of the world market equilibrium price

To find the exact world market price we need to insert the elements that we know for domestic and foreign market pre-trade equilibrium prices.

The domestic market pre-trade equilibrium price is,

$$p_d^* = \frac{aA + bB}{a + b} \quad (30)$$

The foreign market pre-trade equilibrium price is,

$$p_f^* = \frac{cC + dD}{c + d} \quad (31)$$

As found above the parameter μ is,

$$\mu \equiv \frac{ab(c+d)}{cd(a+b) + ab(c+d)}; 0 \leq \mu \leq 1 \quad (32)$$

Which mean that $1 - \mu$ is,

$$1 - \mu = 1 - \frac{ab(c+d)}{cd(a+b) + ab(c+d)} \Leftrightarrow 1 - \mu = \frac{cd(a+b)}{cd(a+b) + ab(c+d)} \quad (33)$$

Inserting the above expressions in the expression for the equilibrium world market price to get,

$$p_w^* = (1 - \mu)p_d^* + \mu p_f^* \Rightarrow$$

$$p_w^* = \frac{cd(aA + bB) + ab(cC + dD)}{cd(a + b) + ab(c + d)} \quad (34)$$

12. Consequences of trade for domestic and foreign markets

When trade is possible, the world market prices will affect the price charged as well as the quantity produced in the domestic and foreign markets. Due to the ‘new’ level of competition from foreign products the producers in the domestic market will adjust their prices downwards to accommodate the change in the competitive environment from cheaper foreign products. Foreign producers on the other hand seize the opportunity to sell at a higher price than at the pre-trade market price resulting in an increase in output as well as an increase in the price for the home market. All markets will be at equilibrium when prices are equalized across markets, implying that the world market price becomes the determining price for the quantity produced and demanded in the domestic as well as the foreign market. Thus, a general equilibrium prevails when,

$$p_d = p_w^* = p_f \quad (35)$$

To find the resulting domestic market supply and demand in the domestic and foreign market we use the world market price we found above together with our knowledge about the domestic market demand and supply conditions.

Domestic market post-trade demand and supply

To find the post-trade demand (D_d^T) in the domestic market we insert the world market price in the demand function, i.e. p_w^* in D_d to get,

$$D_d^T(p_w^*) = \frac{A}{b} - \frac{p_w^*}{b} \quad (36)$$

As $p_d^* \leq p_w^*$ then it must follow that,

$$D_d^T(p_w^*) \geq D_d^*(p_d^*) \quad (37)$$

If we insert p_w^* in the demand function of the domestic market we will get the general equilibrium demand in the market. Thus,

$$D_d^* = \frac{A}{b} - \frac{p_w^*}{b} \Rightarrow D_d^* = \frac{A}{b} - \frac{1}{b} \left(\frac{cd(aA + bB) + ab(cC + dD)}{cd(a + b) + ab(c + d)} \right) \Rightarrow$$

$$D_d^* = \frac{cd(A - B) + a(c + d)(A - 1)}{cd(a + b) + ab(c + d)} \quad (38)$$

To find the post-trade supply (S_d^T) in the domestic market we insert the world market price in the supply function, to get,

$$S_d^T(p_w^*) = \frac{p_w^*}{a} - \frac{B}{a} \quad (39)$$

From the same condition, $p_d^* \leq p_w^*$ it must follow that,

$$S_d^T(p_w^*) \leq S_d^*(p_d^*) \quad (40)$$

Inserting p_w^* in the domestic supply function will give us the general equilibrium supply. Thus,

$$S_d^* = \frac{p_w^*}{a} - \frac{B}{a} \Rightarrow S_d^* = \frac{cd(A - B) + b(c + d)(1 - B)}{cd(a + b) + ab(c + d)} \quad (41)$$

Foreign market post-trade demand and supply

To find the post-trade demand (D_f^T) in the domestic market we insert the world market price in the demand function, i.e. p_w^* in D_f to get,

$$D_f^T(p_w^*) = \frac{C}{d} - \frac{p_w^*}{d} \quad (42)$$

As $p_w^* \geq p_f^*$ it must follow that,

$$D_f^T(p_w^*) \leq D_f^*(p_f^*) \quad (43)$$

If we insert p_w^* in the demand function of the foreign market we will get the general equilibrium demand in the market. Thus,

$$\begin{aligned} D_f^* &= \frac{C}{d} - \frac{cd(aA + bB) + ab(c + d)}{cd^2(a + b) + dab(c + d)} \Leftrightarrow \\ D_f^* &= \frac{cd[a(C - A) + b(C - B)] + ab(c + d)(C - 1)}{cd^2(a + b) + dab(c + d)} \end{aligned} \quad (44)$$

To find the post-trade supply (S_f^T) in the domestic market we insert the world market price in the supply function, to get,

$$S_f^T(p_w^*) = \frac{p_w^*}{a} - \frac{B}{a} \quad (45)$$

From the same condition, $p_w^* \geq p_f^*$ it must follow that,

$$S_f^T(p_w^*) \geq S_f^*(p_f^*) \quad (46)$$

Inserting p_w^* in the foreign supply function will give us the general equilibrium supply. Thus,

$$S_f^* = \frac{cd[aA + bB - (a + b)D] + ab(c + d)(1 - D)}{c^2d(a + b) + cab(c + d)} \quad (47)$$

13. Social welfare in the pre-trade domestic market

To understand the impact on social welfare from trade we need to calculate the change in social welfare in the domestic as well as in the foreign market and add the two to get the total change. The figure below will aid us in the task of doing so for the domestic market.

In the pre-trade market, the consumer surplus is the area Δabg and the producer surplus is the area Δgbe . In the post-trade market, consumer surplus is Δacf and producer surplus is Δfde . From this it is clear to see that the difference between the post-trade and pre-trade social welfare is the area Δbcd . Thus, the area is calculated as,

$$\Delta W_d = \frac{1}{2}(p_d^* - p_w^*)(D_d^* - S_d^*) > 0 \quad (48)$$

The difference in domestic demand which is also the amount of domestic import is,

$$D_d^* - S_d^* = \frac{(c + d)[aA + bB - (a + b)]}{cd(a + b) + ab(c + d)} \quad (49)$$

In addition, the difference in domestic price is,

$$p_d^* - p_w^* = \frac{ab[(c + d)(aA + bB) - (a + b)(cC + dD)]}{(a + b)[cd(a + b) + ab(c + d)]} \quad (50)$$

This gives us the total change as,

$$\Delta W_d = \frac{(c + d)[aA + bB - (a + b)][ab[(c + d)(aA + bB) - (a + b)(cC + dD)]]}{2(a + b)[cd(a + b) + ab(c + d)]^2} \quad (51)$$

To calculate the impact of free trade on the foreign social welfare we proceed in the same way as above by drawing the problem and using this as our aid.

In the pre-trade market, the consumer surplus is the area Δadf and the producer surplus is the area Δfde . In the post-trade market, consumer surplus is Δabg and producer surplus is Δgce . From this it is clear to see that the difference between the post-trade and pre-trade social welfare is the area Δbcd . Thus, the area is calculated as,

$$\Delta W_f = \frac{1}{2} (p_w^* - p_f^*) (S_f^* - D_f^*) \quad (52)$$

Inserting the elements found in the previous sections enables us to calculate the change in social welfare, which in this case is a cumbersome calculation best left for the reader to attempt to find.

14. Conclusion

The condition for trade and the incentives to trade between countries lies in price differences for similar products. In turn, price differences is linked to differences in the terms of production such as the ability of some countries to take advantage of production in scale or simply being able to produce more at a lower price from using a superior technology.

References

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