Phased Array System toolbox: An implementation of Radar System

A qualitative study of plane geometry and bearing estimation

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Abstract

In this thesis is the possibility to implement the Phased Array System toolbox provided by Mathworks within a current model used by SAAB Surveillance in Gothenburg investigated. By using the toolbox to create system objects for each component in the model one could compare it with the current model. A complete radar system with waveform generator, transmitter, antenna, and a receiver was created to simulate received data. The simulated received data was analyzed with provided signal processing tools such as matched filter, pulse integration, time varying gain, etc. After signal processing, detection of targets resulted in an interval in both range and angle. Creation of a system object resulted in an increased time duration with an order of magnitude in simulation time. Hence, the Phased Array System toolbox was not successfully implemented in the current model.

A new way has been developed to simulate a plane target that takes the geometrical shape into account, unlike a point target used today. The plane consisted of six points that resulted in that the plane gets a length, width, wing angle, and a rotation. Then, were both the new plane model and the point target used in the current model to compare the results. After a large number of simulations, no concrete conclusion could be made more than that the estimation in range and angle lie within the size of the plane.

Estimation in bearing angle has been investigated in a new way where more resolution cells have been used than before. By using not only the range bin where a local maximum is detected but also including the adjacent range bins, a more accurate estimation was desired. The current model was used for both point target and the new plane model in order to see the angle differences when more range bins were used. When the two adjacent range bins were used, a small shift in the angle could be observed compared to before (approximately 0.01 %). The standard deviation becomes lower for the plane target while it increased for the point target.

Sammanfattning


Uppskattningen av bearing vinkeln har undersöks på ett nytt sätt där fler upplösningsceller har tagit hänsyn till än förut. Genom att inte bara undersöka den range bin där det är ett lokalt maximum, utan även de två närliggande, önskades en bättre uppskattning av målet. Den nuvarande modellen användes för både punktmål och den nya plamodellen för att se vinkelskillnaderna då fler range bins användes. Då de två närliggande användes kunde man se en liten skiftning i vinkel mot förut (approximativt 0.01%). Standardavvikelsen blev lägre för plamodellen medan den ökade för punktmålet.
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Acronyms

ADPL  Air Doppler
CFAR  Constant False Alarm Rate
DOA  Direction Of Arrival
FFT  Fast Fourier Transform
FIR  Finite Impulse Response
I  In-phase
IF  Intermediate Frequency
LFM  Linear Frequency modulated
LLRT  Log-Likelihood Ratio Test
LNA  Low-Noise Amplifier
LO  Local Oscillator
LRT  Likelihood Ratio Test
MTI  Moving Target Indicator
NP  Neyman-Pearson
PAS  Phased Array System
PRF  Pulse Repetition Frequency
PRI  Pulse Repetition Interval
Q  Quadrature
RCS  Radar Cross Section
RF  Radio frequency
SNR  Signal-to-Noise Ratio
STAP  Space-Time Adaptive Processing
STC  Sensitive Time Control
TVG  Time Varying Gain
UCA  Uniform Circular Array
ULA  Uniform Linear Array
URA  Uniform Rectangular Array
Chapter 1

Introduction

In this section, the background, introduction, and main objectives of the thesis are described.

1.1 Background

This thesis is written for SAAB Surveillance in Gothenburg who are interested in the possibility to use the Phased Array System (PAS) toolbox provided by Mathwork. SAAB AB offers several radar models and solutions to ensure customers needs and "Sea giraffe" is one of the models based at sea to offer support. The radar system consists of several channels used to control the airspace, where one of them is the Air Doppler (ADPL) channel. The main purpose of the ADPL channel is to identify moving targets such as aircraft, missiles, helicopters, etc. in regions with heavy clutter. And will have a significant role in this work. It is of great interest to see if the PAS toolbox can be used together with the current radar model to increase the performance of the overall system. A short description of the toolbox from Mathworks homepage is

"Phased Array System Toolbox™ provides algorithms and apps for the design, simulation, and analysis of sensor array systems in radar, sonar, wireless communications, and medical imaging applications. The system toolbox includes pulsed and continuous waveforms and signal processing algorithms for beamforming, matched filtering, Direction Of Arrival (DOA) estimation, and target detection. It also includes models for transmitters and receivers, propagation, targets, jammers, and clutter."

Based on the description of the PAS toolbox shown above, an interest was raised to investigate the possibilities of implementing the toolbox together with the current system or replacing part of the current system with the PAS toolbox. An example of such improvement could be that the algorithms work more efficient than before, the predefined functions make the code easier to follow, simulate signals with different settings in order to test for future improvements, etc.

1.2 Introduction

Already the ancient Greeks were in need of good knowledge about the surrounding environment to ensure protection for the civilization. The enemies were spotted by placing people on high walls or lookout towers, by only using the naked eye the spotters could warn the city of a potential attack. A lot of developments have been made to improve the defence of society since
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the ancient Greeks, both in terms of artillery and the ability to detect hostile objects. One of the methods for detecting different types of vehicles is to use radar, a technology that is constantly being developed. Today are radar systems used in several different areas, both in civil and military applications. In the military, it can be used in air defence for detection of targets, target recognition and weapon control. It can be used strategically to identify the enemies position and gaining control over the area. Radar systems can also be used offensively in missile system to guide the weapon. Using these radar systems for a civil application could be for air traffic control, ground traffic control, and weather analysis. In air traffic control, the radar is used to control air traffic near airports, assist the aircraft to land in bad weather condition, and inform about the location of vehicles on the ground (at the airport). For ground traffic control, the first application one might think about is speed cameras and aiding traffic police. But with the development of technology, new uses have been found. For example, sensors for cars that analyse the surroundings and warn if other vehicles are approaching to reduce accidents. The radar system can also be used for education or scientific purposes e.g. by studying the planetary system, detect and track satellites, monitor the meteors, and guide space vehicle.

The word radar is an acronym for radio detection and ranging and is a technique using electromagnetic waves to obtain information about the surrounding. A great advantage of radar systems compared to optical telescopes is that it will not be affected by weather conditions to the same extent. The vision with an optical telescope is heavily reduced during the night or when it’s raining or are foggy. Since the radar system uses electromagnetic waves, the performance of the system won’t be affected in the same way by these conditions. By transmitting the waves in a given direction and then analyse the received echoes it is possible to extract different information about the object. As mentioned in the previous paragraph, a radar system can be used for several different applications and it is the purpose of the radar that decides what information that is extracted. For example, the traffic police measuring the velocities of passing vehicles will only be interested in the speed of the vehicle. But for surveillance purposes, the main task is to detect targets but also when it is detected one want to extract as much information as possible. Information that is desired for a foreign target could be: classifying the target (plane, helicopter, missile, car, ship, etc.), the position (distance, direction, and altitude), and properties such as velocity and travel direction, etc. In order to obtain the desired information from a target, one uses different types of signal processing on the received data.

1.3 Objective

The scope of this thesis includes the basic theory behind a radar system, the signal processing and comparison between current model and implementation with the PAS toolbox. A full description of the entire system used today is not possible within the scope of the thesis. The study will mainly be focused on the implementation of the PAS in the ADPL channel, but also use the toolbox to create, simulate, and analyse the signal used for the ADPL channel. It will also include construction of a new plane model compared to the point target used today. The comparison between point and plane target will be done in the current model together with a new way of estimating the bearing angle.
Chapter 2

Theory

In this chapter, an introduction to a radar system is presented together with the necessary theory for understanding the thesis. This includes a general theoretical background for a pulse radar, the signal processing, and the phased array system toolbox. Also illustrated how the new plane target looks like, and how the new way of estimating the bearing angle is performed.

2.1 Introduction to radar system and the basic principles

A great way to introduce a radar system is to think of a person screaming in a valley. The sound waves will be reflected on various objects and after a while will the echoes return back to the person. The same principle is applied to the radar system but instead of sound waves, electromagnetic waves are used. By illustrating the situation we need to replace the "scream" with a transmitter that transmits the wave and the ears receiving the echoes with a receiver for the electromagnetic signal. An illustration of this is presented in Figure 2.1.

![Figure 2.1: The basic concept for radar detection.](image)

However, the real situation is more complicated than what is illustrated in Figure 2.1. There are several different ways to construct a complete radar system and one possible block diagram for a simple pulsed monostatic radar is presented in Figure 2.2 [1].
In Figure 2.2 one can follow the process from the generated waveform to a detection on the display. Each component in the system will be introduced briefly [2]. The waveform generator generates a pulse waveform with desired appearance and properties. The transmitter modulates the generated waveform to desired Radio frequency (RF) and then amplifies it to a useful power level. In a monostatic radar system the same antenna used both for transmitting and receiving, and thus a switch for switching between transmitting and receiving is required. This is one of the functions of the duplexer, the other is to protect sensitive equipment from the high power transmission that otherwise would destroy the receiver. The Low-Noise Amplifier (LNA) amplifies a low power signal without disturbing the Signal-to-Noise Ratio (SNR) noticeably. It is designed to increase the power of both signal and noise, and minimize additional noise. The Local Oscillator (LO) and the mixer convert the RF signal to an Intermediate Frequency (IF) with a centre frequency and a bandwidth. The IF amplifier is designed with a matching filter such as the frequency response, $H(\Omega)$, can be maximized compared to the noise. This happens when the magnitude of the frequency response function is equal to the magnitude of the echo signal spectrum $|S(\Omega)|$. The signal processor can be both analogue or digital, but modern radar is processing the signal digitally and includes all steps necessary for the radars application.

The most common applications for a radar system includes detection, tracking, or imaging [1]. One of the most fundamental properties of interest for a target is to identify the distance between the radar and target. The radar transmits a pulse at some time, $t_1$, and the echoes from an object return to the receiver at time $t_2$. By taking the time difference, $\Delta t = t_2 - t_1$, is it possible estimate the range, $R$, to the object with the distance formula

$$R = \frac{c \Delta t}{2}, \quad (2.1)$$

where $c$ is the speed at which the wave travels in a given medium (assumed to be the speed of light), and the factor $1/2$ comes from that the signal goes two ways. Hence, the range measured
is a straight line from the radar system to the target. The position, $P$, of a target is described in a spherical coordinate system illustrated in Figure 2.3.

\[ P = (R, \theta, \phi) \]

Figure 2.3: The spherical coordinate system used for radar measurements with a target at point $P$ and the range $R$. The radar system is placed at the origin $O$.

The range $R$ only tells the absolute distance to a target and nothing more specific. For example, targets can have the same range from the radar system but still be located very differently, which Figure 2.4 is illustrating.
2.2 Derivation of radar equation

To understand how radar works as a concept are it necessary to study how the radar equation describes the relationship between transmitted power, $P_t$, and the received power $P_{rec}$. One of the numerous ways to derive a general equation is to start from the power density as a function of distance in space $[2]$. The power density of an isotropic source follows

$$\text{Power density} = \frac{P_t}{4\pi R^2}. \quad \text{W/m}^2$$  \hspace{1cm} (2.2)

If one uses an isotropic source it is only possible to tell if a target is present or not. For most of the cases, one desire more information about the target than only if it is nearby. Therefore, a surveillance radar transmits the electromagnetic waves in one specific direction. How much the radiation intensity increases in that specific direction compared to an isotropic antenna is called antenna gain, $G$. Thus, equation (2.2) can be rewritten to

$$\text{Power density from directive antenna} = \frac{P_t G}{4\pi R^2}. \quad \text{W/m}^2$$  \hspace{1cm} (2.3)

Figure 2.4: Circular trajectory where same range resulting in different vertical distance from antenna.

One can see how all targets on the dashed line will result in the same range while the vertical distance to the antenna is different. Therefore, it is more appropriate to describe the position in terms of other quantities. The position can instead be described by a vertical distance, an angle in the plane azimuth/bearing angle, $\theta$, and elevation angle $\phi$. Furthermore, there are a lot of other properties that can be of interest of a foreign object such as velocity, geometric shape, etc. The velocity of a target is estimated by calculating the Doppler shift.
Equation (2.3) describes the partition of the transmitted energy, at a given range, that interacting with an eventual target. The electromagnetic waves will interact with the target and scatter in various directions before it acts as a source reflecting the waves back to the antenna. The power reflected from the target will be

\[ P_{\text{ref}} = \frac{(\text{Power density at range } R_1)G\sigma}{4\pi R_1^2}, \]

where \( \sigma \) is the radar cross section. Using the target as a source like equation (2.2) we can write the power density in space from the target in the same way

\[ \text{Power density reflected from target } = \frac{P_{\text{ref}}G}{4\pi R_2^2}, \]

where \( R_2 \) is the distance from the target. The received power will depend on equation (2.2) and the effective antenna aperture \( A_{\text{eff}} = AK_{\text{eff}} \), where \( A \) is the geometric antenna area and \( K_{\text{eff}} \) is the efficiency. We now have the first expression for the received power, \( P_{\text{rec}} \),

\[ P_{\text{rec}} = \frac{P_{\text{ref}}AK_{\text{eff}}}{4\pi R_2^2}. \]

Inserting equation (2.4) into (2.5) yields

\[ P_{\text{rec}} = \frac{P_tG\sigma}{4\pi R_2^2}AK_{\text{eff}}, \]

\[ P_{\text{rec}} = \frac{P_tG\sigma AK_{\text{eff}}}{(4\pi)^2 R_1^2 R_2^2}. \]

Using that the distance \( R_1 \) and \( R_2 \) will be approximately the same, we write \( R_1 = R_2 = R \). Then, we use an equation that describes the gain in terms of the wavelength \( \lambda \) and solve for the area

\[ G = \frac{4\pi AK_{\text{eff}}}{\lambda^2} \quad \Leftrightarrow \quad A = \frac{G\lambda^2}{4\pi K_{\text{eff}}}, \]

before inserting it in equation (2.7), we obtain

\[ P_{\text{rec}} = \frac{P_tG\sigma K_{\text{eff}}}{(4\pi)^2 R^4} \frac{G\lambda^2}{4\pi K_{\text{eff}}} = \frac{P_tG^2\lambda^2\sigma}{(4\pi)^3 R^4}. \]

Equation (2.9) is for an ideal case, i.e. no losses, and in order to take these into account we introduce a loss term \( L \), and the following formula is obtained

\[ P_{\text{rec}} = \frac{P_tG^2\lambda^2\sigma}{(4\pi)^3 R^4 L}. \]
Equation (2.10) can also be written in range form when solving for $R$

$$R = \left( \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 L} \cdot \frac{P_t}{P_{rec}} \right).$$  \hspace{1cm} (2.11)

From equation (2.10) it is clear that the received signal depends heavily on the range. It is also possible to calculate the maximum detectable range using equation (2.11), if the minimum ratio between transmitted power and received power in for the radar system is known.

A pulse radar transmits electromagnetic waves during a short time period resulting in a pulse width $\tau$. As mention in Section 2.1, the duplexer turns off the receiver while transmitting which leads to a dead time. The pulses are transmitted in intervals which are described as either Pulse Repetition Interval (PRI), or by its reciprocal Pulse Repetition Frequency (PRF). In Figure 2.5 it is illustrated how pulses are transmitted separated by a constant time interval.

From Figure 2.5 it is possible to see how targets located at a multiple of the PRI cannot be detected (due dead time while transmitting). Another effect caused by the PRI/PRF is that the echo from one signal can be received after the next pulse has been transmitted, which gives rise to an unambiguous range

$$\text{Unambiguous range} = \frac{c}{2\text{PRF}}.$$ \hspace{1cm} (2.12)

### 2.3 Resolution cells

The concept resolution cell is the volume in space where all collected echoes contribute to at any sampling time [1]. If a pulse with constant a frequency is used for simplicity, the resolution in range dimension is illustrated in Figure 2.6.

There are two targets separated with $\Delta R = c\tau/2$ placed at a distance $R_0$, and $R_0 - \Delta R$ from the radar respectively. If a pulse is transmitted at some time $t$ with duration $\tau$ seconds, the received echoes from the first target will be at some time $t_0$. The scattering from each target
2.3. RESOLUTION CELLS

\[ \Delta R = \frac{ct}{2} \]  

Figure 2.6: The interactions when a rectangular pulse envelope is reflected from two stationary targets.

are overlapping which is shown in the lower part of Figure 2.6. Hence, two targets could only be distinguished if they are separated more than \( \frac{ct}{2} \) m. The quantity is called \textit{range resolution}.

\[ \Delta R = \frac{ct}{2}. \]  

(2.13)

The same principles are valid for angular resolution, where scattering from two targets from same range but different azimuth (or elevation) angles are illuminated. One usually set the width of the main lobe to 3–dB and let two targets be at the edges of the main beam illustrated in Figure 2.7. The distance between the two targets are called \textit{cross-range resolution} and is the dashed line. It is given by

\[ \Delta CR_0 = 2R_0 \sin \left( \frac{\theta_3}{2} \right), \]  

(2.14)

if the angle is small, which usually is the case for pencil beam antennas.
The entire radar resolution cells volume $\Delta V$ is approximately the product of the two angles (azimuth and elevation) and the range resolution. If not a completely symmetric beam is assumed the azimuth and elevation beam widths are $\theta_3$ and $\phi_3$, combination of equation (2.13) and (2.14) gives

$$\Delta V = \pi \left( \frac{R_0 \theta_3}{2} \right) \left( \frac{R_0 \phi_3}{2} \right) \Delta R = \frac{\pi}{4} R_0^2 \theta_3 \phi_3 \Delta R. \quad (2.15)$$

As mention in Section 2.2, it is desired to transmit in one particular direction. The beam could be steered in the desired direction by rotation or electronic steered with a phased array [3]. Instead of using only one antenna element an array could be used to change the radiation pattern. The antenna geometries can be for example Uniform Linear Array (ULA), Uniform Rectangular Array (URA), or Uniform Circular Array (UCA). When the echoes return to the antenna array the signal will be received at different times for different elements. In Figure 2.8 it is shown how a signal is incoming with an angle $\theta$ reaches the antenna element at different times (travels a distance of $d \sin(\theta)$ longer for the left element).
For a surveillance radar it is desirable to only get signals in the main lobe. In order to suppress contributions from the side lobes, a taper process can be used. There are different types of functions used to suppress side lobes, an example of that could be Taylor, Chebyshev, Hamming, Hann, Kaiser, or a custom. It is illustrated in Figure 2.9 how an applied function affects the antenna response.

Figure 2.9: The antenna response before and after tapering is applied (Taylor).
The antenna geometry for the illustration of tapering in Figure 2.9 is for a ULA with five antenna elements.

2.4 Waveform

Compared to the simple pulse used to derive equation (2.13) which only has two parameters, its amplitude $A$ and its duration $\tau$, usually a modulated pulse is used instead. A pulse can be modified in frequency or phase. A modulated pulse has a larger bandwidth compared to an un-coded pulse with the same duration. One of the types is a Linear Frequency modulated (LFM) waveform and is defined by

$$x(t) = \cos \left( \frac{\pi \beta}{\tau} t^2 \right), \quad 0 \leq t \leq \tau$$

(2.16)

where $\beta$ is the bandwidth. Equation (2.16) can also be written in its complex equivalent $x(t) = e^{j\pi \beta t^2/\tau}$. The instantaneous frequency of this waveform is the derivative of the phase function

$$F_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} = \frac{\beta}{\tau} t$$

(2.17)

The expected appearance of these two equations is plotted in Figure 2.10.
2.5. Sampling

It has been described in Section 2.1 how a radar system measures the reflected signal in a three dimensional coordinate system, i.e. distance, bearing angle, and elevation angle [1]. If the
received signals are going to be treated digitally, which is the case for most modern radars, one first have to answer how the sampling interval should be chosen in each dimension. The received signal is represented as a complex signal divided into two components, one In-phase (I) and Quadrature (Q) \[4\]. The I and Q are described by

\[
I = A \cos(\omega_1 t + \phi_0), \tag{2.18}
\]

\[
Q = A \sin(\omega_1 t + \phi_0) \tag{2.19}
\]

which gives the complex signal \( I + iQ \).

For pulse radars, it exists several distinct and independent sampling intervals. Two of them are time sampling intervals. The first is related to the PRI, i.e. the time between each transmitted pulse. The other is the sampling time of each antenna channel which is done at a much higher rate than the time between pulses. Therefore, one sometimes name these fast time and slow time. One dimension is related to the antenna geometry, recall from Figure 2.8 that different elements receive the signal at a different time. This is a spatial sampling. In order to ease work, analyse, and understand the received data one modulate it as a datacube. The datacube stores the received complex signal from one sampling as a block and stores it as a matrix.

![Datacube with stored information.](image)

Time samples are generally taken at a spacing no greater than the time resolution of the radar pulse, i.e. for a simple pulse the pulse length \( \tau \) \[3\]. The sampling will then occur at every \( \tau \) second from some initial time \( t_i \), to some final time \( t_f \). Since the returned echo from one target will also have the length \( \tau \) (recall Figure 2.6) it will be in one time sample. If the signal is strong enough in that time sample to be detected, there will be a time delay, \( t' \), that corresponds to an
2.6. CLUTTER

Clutter can be defined as any unwanted echo from various sources. It can be divided into surface clutter and volume clutter. Example of surface clutter are reflections from trees, vegetation, ground terrain, man-made structures, and sea surface. Volume clutter usually includes rain, birds, insects, and chaff [5].

2.7 Signal processing

When the signal has been obtained it will be analysed with various filters and techniques, e.g. matched filter/pulse compression, Moving Target Indicator (MTI) and pulse Doppler filtering,
Signal processing involves techniques such as coherent and non-coherent integration, Space-Time Adaptive Processing (STAP), and Constant False Alarm Rate (CFAR) [3]. These methods are primarily implemented using Finite Impulse Response (FIR) digital filters, correlation, Fast Fourier Transform (FFT), and matrix-vector algebraic operations. The goal is to extract useful information from the received signal by filtering out unwanted noise and clutter. The received signal typically consists of contributions from various sources, including targets, clutter, atmosphere, gravitational effects, and internal noise from the radar system.

A matched filter is used to increase the Signal-to-Noise Ratio (SNR) [1]. The question arises at what frequency on the receiver the response \( H(\Omega) \) will be maximized? We write the spectrum of the receiver output as

\[
Y(\Omega) = H(\Omega)X(\Omega),
\]

where \( X(\Omega) \) is the spectrum of the waveform. By maximizing the SNR at a specific time, \( t_M \), the power of the signal component of the output at that time is

\[
|y(t_M)|^2 = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega)H(\Omega)e^{i\Omega t_M}d\Omega \right)^2.
\] (2.20)

An expression for when the SNR maximized is obtained by dividing the spectrum of the receiver output with the total output noise power. Using Schwarz inequality one gets that the SNR is maximized when

\[
H(\Omega) = \alpha X^*(\Omega)e^{-i\Omega t_M} \quad \text{or} \quad h(t) = \alpha x^*(t_M - t)
\] (2.21)

The filter is called matched filter since it is matched with the radar waveform. One can see in equation (2.21) that the complex conjugate, \( X^* \) and \( x^* \), to the transmitted waveform is present. The general matched filter could be described in a block scheme.

![Figure 2.13: Block diagram of matched filter.](image)

The signal strength will suffer losses from range (recall equation (2.10)) and to reduce the influence from close range clutter Sensitive Time Control (STC)/Time Varying Gain (TVG) can be used [3]. One take advantage of that clutter falls off as \( R^{-2} \) or \( R^{-3} \), while targets falls off as \( R^{-4} \). It is mention in Section 2.2 how the radar system transmits multiple pulses in an interval (pulse train) which one can use to increase the detection probability [6]. The SNR can be improved by integrating over multiple pulses, because the returns from a target will be summed while the statical clutter will cancel each other (not completely but enough to improve the SNR). If the received signal is denoted \( X \) and the integrated \( Y \) two different types of integration can be made, coherent and non-coherent. For coherent integration we have
\[ Y_i = \sum_{j=1}^{N} X_{ij}, \quad (2.22) \]

and for the other case when non-coherent integration

\[ Y_i = \sqrt{\sum_{j=1}^{N} |X_{ij}|^2}. \quad (2.23) \]

In order to ignore returns from unwanted objects an MTI filter is used [7]. For a pulse radar, the returns from a stationary (or slow-moving) object will have zero, or a small frequency shift. The shift is calculated from pulses adjacent to each other in time. A lot of the techniques used by modern radars take advantage of the differences in this shift, Doppler shift, to identify characteristic returns for targets, clutter, and noise in order to separate contribution from each other in the received signal [8]. If there is a relative motion between the radar and the target it will be a frequency shift between the transmitted and received signal. The classical treatment of the Doppler effect is given by

\[ f_b = f_a \frac{c - v_b}{c - v_a}, \quad (2.24) \]

where \( f_a \) and \( f_b \) are the frequencies measured in the rest frames of the source \( S_a \) and observer \( S_b \). In this case the velocities \( v_a \) and \( v_b \) are assumed to be small enough compared to \( c \), then supersonic effects can be neglected. However, equation (2.24) is dependent on if the object or the source is moving and not the relative motion \( (v = v_b - v_a) \) between object and source. By setting \( v_b = v \) and \( v_a = -v \) it can be rewritten to

\[ f_b = \frac{f_a}{1 + v/c}, \quad (2.25) \]

Using a Taylor series approximation on equation (2.25), one obtains equation (2.26) where \( \Delta f \) is the Doppler shift

\[ \Delta f = \frac{2v}{c} f_a. \quad (2.26) \]

For a monostatic radar the Doppler shift is measuring the relative velocity between the radar and target, called radial velocity. Since the change in velocity is small it is more convenient to analyse the signal in the frequency domain instead of the time domain. The transition from time to frequency is done with a Fourier transform.

\[ F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} \, dt \quad (2.27) \]

If the returns from a radar resolution cell consist of signals from both a stationary target (clutter) and a moving target, the received signal will be a superposition of them. However, a
stationary object will be at the transmitted RF, \( f_0 \), while the signal from the moving target will be at \( f_0 + f_d \), where \( f_d = 2v/\lambda \) is the Doppler frequency shift and \( v \) is the radial component of the moving target. A positive Doppler shift represents an approaching target while a negative a receding target [7].

## 2.8 Detection

After signal processing, one wants to identify if there exist echoes from targets among the noise. Example of characteristic noise without any target present is presented in Figure 2.14. The simulated received noise at the sensor is for a ULA with 8 antenna elements, carrier frequency 0.3 GHz, propagation speed \( c \), and done with the Matlab command `sensorsig`.

![Simulated received noise](image)

Figure 2.14: Simulated noise.

One sees how the simulated received signal consists of several peaks, but not all of them will be of interest. In this case is none of the peaks of interest since there is only noise. However, if a target is present one wants to separate and compare the contribution from it with the noise. In order to be identified as a detection, the peak must exceed a certain threshold level. To identify if a target is present or not hypothesis testing is used [1]. For all cases, one can assume that one of these two hypotheses is true:

1. There is only contributions from interference.
2. There is contributions from both interference and echoes from a target.

The first hypothesis is denoted \( H_0 \) and is the null hypothesis while the second is \( H_1 \). The radar system tests each measurement to see which of the two hypotheses that fit best. Since the signals are described statistically decision theory is used to test the hypothesis. The threshold
2.9 Phased array system toolbox

Phased array system is a toolbox for Matlab which provides predefined algorithms and apps for the design, simulation, and analysis of sensor array system in radar, sonar, wireless communications, and medical imaging application. One can read a short introduction in Section 1.1 or the full description on Mathworks homepage. In PAS the radar systems is constructed with different system objects. This includes all the necessary components mentioned in Section 2.1 for creating a radar system, such as antenna, collector, radiator, receiver, sensor motion, transmitter, and waveform. If no information about an existing system is known, the requirement for detection is set by the desirable performance such as probability of detection and false alarm, the maximum unambiguous range, the minimum detectable RCS, and the number of pulses to integrate.

2.9.1 Introduction to the toolbox

A system object used in the PAS toolbox is, in general, created in the following way

```matlab
System object = phased.SystemObject('Property 1', value 1,...
                'Property 2', value 2,...
                'Property 3', value 3);
```

Figure 2.15: General code for create a system object in PAS.

The different properties in Figure 2.15 are of course depending on which system object that is used. Some parameters are used for the entire system, e.g. sample rate, operating frequency, propagation speed, carrier wavelength, etc.

A brief introduction of the different system objects used in the radar system with their respective input and output will be described in the following section.

The antennas used in the PAS toolbox are created with mainly two different properties, the frequency range, and the antenna pattern response. Examples of pattern response can be isotropic, cosine, or custom. If more than one antenna element is used they have to be arranged in some kind of geometry. They can be aligned in a ULA, a URA, or a UCA. For both ULA and URA is the number of elements specified together with element spacing. The ULA is just one row while the URA is defined with \( M \times N \) identical sensor elements separated uniformly in vertical, and horizontal direction. Where \( M \) is rows and \( N \) is columns. The UCA does also have the number of elements as an input but instead of the length of the array you specify the radius of the array. All three of these have the option to use a tapering matrix in order to suppress the side lobes.

The modulation of a moving object is done by creating a platform with two different options motion models, "velocity", or "acceleration". For both cases are the velocity or acceleration constant and in a translational motion. An object could be a vehicle or aircraft, or a sonar or a radar transmitter and receiver. Other properties of the platform can be initial position, initial velocity, velocity, and acceleration which all are self explanatory. All positions and velocities are always defined in the global coordinate system.

\(^1\)https://se.mathworks.com/products/phased-array.html
Furthermore, in equation (2.4) one can see how the reflected power depends on RCS ($\sigma$), so each target have to be associated with properties correlated to RCS. This is made with the system object \texttt{RadarTarget} that models how an incoming signal is reflected from a radar target. The properties that will be used depend on whether polarization is enable or not. For a non-polarized signal the reflected signal, $Y$, is

$$Y = \sqrt{\frac{4\pi\sigma}{\lambda^2}} X,$$

where $X$ is the incoming signal. And for polarized waves the incoming signal $X$ is replaced with a vector signal

$$\begin{bmatrix} E^{(\text{scat})}_H \\ E^{(\text{scat})}_V \end{bmatrix} = \sqrt{\frac{4\pi}{\lambda^2}} \begin{bmatrix} E^{(\text{inc})}_H \\ E^{(\text{inc})}_V \end{bmatrix}.$$  

(2.29)

More properties needs to be set after the enable polarization option. If non-polarization is selected \texttt{MeanRCSSource} and \texttt{MeanRCS} will be used, and for polarization \texttt{ScatteringMatrixSource}, \texttt{ScatteringMatrix}, and \texttt{Mode} is used.

When all other physical components (radar and targets) are created it is possible to proceed with the construction of the signal. However, the received signal is obtained with an array (UCA, ULA, or URA) it is necessary to use a beamformer pointing to the steering direction to obtain the combined signal. The signal will propagate in an environment that causes certain characteristics of the electromagnetic wave. As mention in Section 2.1 the wave will travel in a given medium that determines, for example, its speed. During propagation in free space the \texttt{FreeSpace} system object is used, which is defined as

$$L_{\text{fsp}} = \left(\frac{4\pi R}{\lambda}\right)^2.$$  

(2.30)

One sees in equation (2.30) how the loss only depends on range, but, if it is needed, several other types of losses are available. For example, if we recall to equation (2.10)

$$P_{\text{rec}} = \frac{P_{\text{t}} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L},$$

(2.31)

where $L$ is the total loss. The total loss will factor will be the sum of the individual losses that are due to rain, due to fog and cloud, and due to atmospheric gases

$$L_{\text{tot}} = L_{\text{fsp}} + L_{\text{rain}} + L_{\text{fog}} + L_{\text{gas}}.$$  

(2.32)
In order to simulate a received signal based on the given system, the PAS toolbox uses a for loop that for a scanning radar includes the following steps:

1. Updates the position of sensor and targets by calculate the new position and velocity after movement during a sample time.
2. Calculate the relative angle between the target and the sensor as seen by the sensor.
3. Calculates the steering vector for current scan angle, it can both be an angle or a scanning grid.
4. Forms a beam used for the selected angle, then generate and transmits the pulse with given properties.
5. Reflection of the pulses when interacting with targets, either polarized or non-polarized.
6. Beamform the target returns received at the sensor by first collect the signal that has been reflected from the targets in the given angle. Then store the values from when the transmitter is off (due to monostatic radar), and lastly beamformer is used for given scan grid.
7. Repeat from step 1 until the entire surveillance region is scanned.

The received pulses are stored in a datacube (range bin \times pulse index) described in Section 2.5 and can now be analysed with the signal processing tool. The predefined functions are matched filter, MTI filter, TVG where the theory is described in Section 2.7.

In some cases, it is more beneficial to work in different units. Matlab have predefined commands for conversion between power in W and dB, and radial speed to Doppler shift. Namely, `pow2db()/db2pow()` and `dop2speed()/speed2dop()`, which is preformed in following way

\[
\begin{align*}
\text{Pow}_{\text{db}} &= 10 \log(\text{Pow}_{\text{W}}), \\
\text{Pow}_{\text{W}} &= 10^{(\text{Pow}_{\text{W}}/10)},
\end{align*}
\]

and the conversion for speed

\[
\begin{align*}
V_{s,r} &= \Delta f \lambda, \\
\Delta f &= \frac{V_{s,r}}{\lambda},
\end{align*}
\]

where \(\Delta f\) is the Doppler shift, \(V_{s,r}\) is the radial velocity of the source relative to the receiver, and \(\lambda\) is the carrier frequency.

### 2.9.2 Implementation of a filter

The creation and implementation of system objects are done in similar way (see Figure 2.15), but for each object that is used for this thesis is presented here. The implementation of a matched filter in the PAS toolbox is performed with following code

```plaintext

```
Matchingcoeff = [];  
Matchedfilter = phased.MatchedFilter('Coefficients', Matchingcoeff) 
Filtered_data = Matchedfilter(Received_pulses);  

Matchingdelay = size(Matchingcoeff, 1) - 1;  
sz_FIltered_data = size(Filtered_data);  
Filtered_data = [Filtered_data(matchingdelay+1:end) zeros(1,matchingdelay)];  
Filtered_data = reshape(Filtered_data,sz_FIltered_data); 

Figure 2.16: Code for implement a matched filter.

The matching coefficients are a vector corresponding to the length of the pulse and can be obtained directly with the PAS toolbox (with getMatchedFilter() command), or set manually. A filter is create with desired properties and then perform the matched filter on the simulated data. Lastly a delay is added to compensate for displacement cased by matching.

A MTI filter is implemented with following code

MTI_coeff = [];  
MTI_filter = fiter(MTI_coeff,1,Filtered_data,[]);  

Figure 2.17: Code for implement a MTI filter.

where "Filtered_data" is the data after matched filter. The length of MTI_coeff, or the number of MTI coefficients depends on how many pulse cancellers one want to use.

The time varying gain is implemented

TVG = phased.TimeVaryingGain('RangeLoss',2*fspl(range_gates,lambda),... 
'ReferenceLoss',2*fspl(max(range_gates),lambda));  

TVG_pulses = TVG(Filtered_data);  

Figure 2.18: Code for implement the TVG filter.

2.9.3 Detection

When the signal processing is done one moves on to the detection of targets that are present in the surveillance area. The threshold in PAS is calculated with NP decision rule to achieve a specified probability of false alarm. The total threshold will be a product of noise power in the system, the calculated with NP decision rules, and the increased gain due the matched filter. To located at which indices the detections occur the PAS toolbox uses the Matlab function findpeaks. A peak is defined as when the value at one point is greater than the two adjacent. The command returns a vector of all peaks, but it is only the peaks exceeding the total threshold that are used when estimate the position of a target. Since the data is sampled in a certain rate the estimation will be limited to one half of the sample rate in range and scan grid. Then it is possible to obtain the Doppler spectrum, and hence also the Doppler shift. The Doppler shift is obtained with the Matlab function periodogram. Periodogram is used together with findpeaks in order to obtain the indices related to the radial velocities.
2.10 Point target vs plane target

Definition of a point target is rather clear and does not need further explanation. By using multiple points it is possible to arrange them to create a geometric shape that looks like an aircraft. Using more points to simulate an aircraft, the simulation would be more like the realistic than a point target. The point chosen for the simple plane mode is the parts that are easiest to detect, namely the front, the back, the apex of the wings, and the engines. In total will the plane consist of six points and look as follows

![Plane Target Diagram]

From Figure 2.19 can one see how the plane model is built. The front and back is orientated in a straight line which defines the plane length, the plane width is set as the length from wing apex to wing apex if the wing angle is $90^\circ$. The wings are attached 40% of the plane length from the front and the engines are located on the wing 15% of the plane width away from the plane body.

2.11 Bearing estimation

The extractor used to estimate the bearing angle extract all data at the range bins where the signal exceeds a given threshold. In order to compensate for discrete values (resolution cells), a quadratic interpolation to find a better estimation is used [3]. Therefore, if the maximum value appears to be the first or last in the datacube no comparison between adjacent cells can be made. In the other cases, the three angles with the highest power are used to interpolate a more accurate value.
In order to calculate $k'$ in Figure 2.20, the following formula is used:

$$k' = k_0 + \frac{1}{2} \frac{|X(k_0 + 1) - X(k_0 - 1)|}{|X(k_0 - 1)| - 2|X(k_0)| + |X(k_0 + 1)|}.$$  

(2.37)

We see in equation (2.37) that the interpolation only uses one range bin when estimating the bearing angle (where a local maximum occurs). The reflected signal from a real target will be spread over a larger area than where the maximum occurs.
If a target is located in the way shown in Figure 2.21 the received signal strength will be stronger in the range bin behind than in front. By taking more cells into account a better estimation of the bearing angle might be found. The new method will include all nine cells that are illustrated in Figure 2.21. In order to evaluate if the new method is more accurate than the current we use standard deviation

$$\gamma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}, \quad (2.38)$$

where

$$\mu = \frac{1}{N} \sum_{i=1}^{N} x_i, \quad (2.39)$$
Chapter 3

Results

In this section the results are presented in chronological order starting with the results from the PAS toolbox, plane target, and bearing angle estimation. The results from PAS goes from waveform, antenna pattern, simulated received signal, and signal processing. These results are compared with results from the current model in order to evaluate the PAS toolbox. The new plane model is presented, and then used in the current model together with the point model to see the effects of using more range bins described in Section 2.11.

3.1 Phased array system toolbox

Plotting the real part of the simulated waveform generated by the waveform generator that is used for the radar system has the appearance shown in Figure 3.1.

![Linear FM pulse waveform: real part, pulse 1](image)

Figure 3.1: The simulated LFM waveform used for transmitting.

When the radar system continuously transmits the waveform it results in a pulse train. The first five pulses are presented in Figure 3.2.
It can be seen in Figure 3.2 that the amplitude is one for a short duration, and then zero until the next pulse is transmitted. An enlargement of one pulse in Figure 3.2 shows that the pulse have a rectangular envelope where the waveform have the same shape as the simulated LFM waveform.

The waveform is completed and it is possible to analyse the antenna and URA radiation pattern. Figure 3.4 shows the radiation pattern of an isotropic antenna transmitting in only in one direction.
Hence, the radiation pattern is a half sphere as expected. Combining multiple antenna elements aligned in a URA changes the radiation pattern in Figure 3.4 to the one presented in Figure 3.5.
It is a clear difference between the radiation patterns in Figure 3.4 and 3.5, where one can see how the main beam is transmitted in a given direction. Furthermore, not only a URA is used to narrow the main beam, but also the tapering process described in Section 2.3. There are also several side lobes, both in bearing and elevation angle. Below is a series of images presented to see how the radiation pattern changes in various directions after the tapering matrix been applied. First, in Figure 3.6, is the bearing angle presented.
It can be seen how the side lobes almost completely disappear. Before, the side lobes lie within 0 to 20 dBi in power, while after they are below -10 dBi. The spread of the radiation pattern also decreased, and is between -30 and 30°. Secondly, the result of elevation angle, can be seen in Figure 3.7.
Before tapering matrix has been applied the power of the side lobes lie within 10 to 20 dBi, and after they are reduced to 0 to 10 dBi. As mention in the description for Figure 3.6 and 3.7, only one specific cut angle (0°) is shown. The complete 3D pattern is shown in Figure 3.8, followed by two side views in Figure 3.9 and 3.10.
One can see the same tendencies over the complete 3D space as for the cut at zero degrees (Figure 3.6 and 3.7). The side lobes are suppressed compared to the main lobe and are at about the same level. There are, however, different levels of the U and V directions, but it depends on the appearance of the tapering matrix. It is easier to estimate the difference in strength for the side lobes and to see that they are about the same size when viewed from the side. The side...
views are presented in Figure 3.9 and 3.10.

Figure 3.9: The directivity in UV-space.

It is shown in Figure 3.9 how the side lobes are reduced compared to the main lobe. By studying the lobes next to the main lobe the estimated change after the tapering matrix was applied was 15 dBi to 55 dBi. The other side is presented in Figure 3.10, and the estimated...
change after the applied tapering matrix was from 15 dBi to 30 dBi.

![3D directivity pattern in u-v space](image)

(a) Without tapering.

![3D directivity pattern in u-v space](image)

(b) With tapering.

Figure 3.10: The directivity in UV-space.

When both the waveform and the antenna pattern are analysed it is possible to move on further with the simulation of environment and targets. Using these settings for the radar system to simulate a received signal from two targets located at a distance of 7569 m and 12807
m from the antenna. They are placed at 7.6° and 38.7° in bearing angle respectively. The speed vectors of the two targets were \([-100; 0; 0]\) and \([10; 80; 0]\) m/s (one of them approaches and the other goes away). This is presented in Figure 3.11.

![Simulated received data with PAS toolbox](image1)

**Figure 3.11:** Simulated received data with two targets.

One can see in Figure 3.11 that there are two areas where the signal strength is stronger than the surrounding. These yellow dots are the return from the targets, while the blue is from noise. It is shown in Figure 3.12 how the simulated data has changed after pulse compression.

![Pulse compression on simulated data](image2)

**Figure 3.12:** The simulated data after pulse compression.

One can see on the scale bar that the signal strength increased with 10 dB. It is also possible to see a narrow line in the centre of the yellow areas. In order to easier see the result of pulse compression, a cross section was done for each target. The cross sections were taken at pulse indices where local maxima occur. In Figure 3.13 is the cross section shown at pulse index 166.

![Cross section of pulse compression](image3)
3.1. PHASED ARRAY SYSTEM TOOLBOX

CHAPTER 3. RESULTS

Figure 3.13: Comparison of the data before and after pulse compression at pulse index 166.

One can see in Figure 3.13 how the signal strength increase for both the target and the noise. An enlargement around the peak is used to easier estimate the magnitude of the change. This is presented in Figure 3.14.

Figure 3.14: Enlargement of the peak.

First of all, one can see that the noise level increased from about -85 to 80 dB. Secondly, the signal strength from the target increased from approximately -35 to -25 dB, and that the width of the peak is roughly doubled. By using the range bin where the signal was strongest (range bin 159) one could estimate the distance between the radar and target to approximately 7625 m. The same investigation was done for the second target, which is presented in Figure 3.15.
In the same manner, as the first target, one can see in that the signal strength increased after pulse compression. Figure 3.16 shows that the noise level is increased with approximately 5 dB, the signal from the target with about 10 dB, and the peak became wider. The maximum value was estimated at range bin 268, which corresponding to a distance of approximately 12850 m from the radar.

Next up are the results of using pulse integration without TVG in order to see how the multiple pulses are used to reduce the noise level. In Figure 3.17 is the two first pulses presented together with an integration over all pulses for the first target.
One can see that the noise level is in the same size for pulse 1 and 2, while it is smaller after integration. It can also be seen that the signal strength has increased from about -80 to -73 dB. The peak increased also in strength, for 1 and 2 it seems to about -40 dB. An enlargement of the peak when the signal is integrated over all pulses are shown in Figure 3.18, where one can see that it lies at about -35 dB.

Using the same procedure on the second target is presented in Figure 3.19, where the same tendencies are shown as for the first target. The noise level increased from -80 to -73 dB and the peak increased from about -40 to -35 dB when integrating over all pulses. An enlargement of the integrated peak is shown in Figure 3.20.
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Figure 3.19: The two first pulses after pulse compression, and the integration over all pulses at pulse index 30.

Figure 3.20: Enlargement of the peak (integrated over all pulses).

Instead of only using pulse integration one could combine it with TVG, to compensate for range losses. The results of using TVG is done in the same order as before. In Figure 3.21 is the first two pulses presented together with the integration over all pulses.
In Figure 3.21, several things can be seen. First of all, the signal strength of the integrated pulse has increased compared to the two the individual pulses. Secondly, the noise level has decreased when integrating. Thirdly, one can see how the signal goes to zero for smaller range bins. Figure 3.22 shows that the noise becomes about 1 to 2 dB and the peak is at about -40 dB.

The same tendencies are shown for the second target in Figure 3.23 and 3.24.
Figure 3.23: The two first pulses after pulse compression, and the integration over all pulses at pulse index 30.

Figure 3.24: Enlargement of the peak (integrated over all pulses).

The data has been processed with the toolbox to increase the signal to noise ratio. Next up is to calculate the threshold in order to identify if a peak is a target or a false alarm. Using the PAS toolbox to calculate the total threshold resulted in a threshold of -133 dB. To get a better idea of what has happened, a polar plot is made over the surveillance area. In Figure 3.25 is the surveillance region plotted when TVG has been applied and with calculated threshold.
Figure 3.25: A 3D illustration of the surveillance region with the calculated threshold.

One can see that there are two large peaks exceeding the threshold level. To simplify one can see the surveillance region from the side. In Figure 3.26 it is shown that the peaks exceeding the threshold level are wide, the threshold level is low (approximately at the middle of the peaks), and that the TVG increased the strength of the signal at longer range to compensate for losses due to range.

Figure 3.26: Side view of Figure 3.25.

The final detection with the PAS toolbox results in an interval for both range and angle which can be seen in Figure 3.26 (wide peaks that exceeds the threshold). For the closest target, 27 points were detected in a range interval from 6619 to 8538 m and an angle interval from 5.625 to 10.125°. The furthest target, there were 35 detections in a range interval 11848 to 13776 m and 34.875 to 43.875°.

Of course, the comparison would be made after all signal processing, but for obvious reasons
it is better to do for the maximum value. The simulated values are compared with the estimated from Figure 3.13 and 3.15 is presented in Table 3.1.

Table 3.1: Comparison between simulated and estimated value from maximum value.

<table>
<thead>
<tr>
<th>Target</th>
<th>Simulated range [m]</th>
<th>Estimated range [m]</th>
<th>Difference [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>7569</td>
<td>7625</td>
<td>56</td>
</tr>
<tr>
<td>Second</td>
<td>12807</td>
<td>12850</td>
<td>43</td>
</tr>
</tbody>
</table>

One can see that using that by using the estimated maximum value, it is more consistent with the simulated. Both targets were estimated to the same radial velocity, namely 46.5 m/s.

The difference in time required for signal processing using the current model and implementation with PAS system is present in the table below.

Table 3.2: Time required for different moments in the signal process.

<table>
<thead>
<tr>
<th>Process</th>
<th>Time current model [s]</th>
<th>Time PAS [s]</th>
<th>Difference [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse compression GCG</td>
<td>0.1240</td>
<td>0.7277</td>
<td>0.6037</td>
</tr>
<tr>
<td>Pulse compression Air</td>
<td>0.2413</td>
<td>1.2293</td>
<td>0.9880</td>
</tr>
</tbody>
</table>

The TVG system object was not successfully implemented in the current system and thus could not compare the time required.

Using the current model to simulate the received signal with a target located 3880.3 m from the radar in order to compared with PAS simulation in Figure 3.11.

In order to compare the PAS toolbox with the current model, a received signal was simulated with the same properties. A target was simulated 3880.3 m from the radar system. The data simulated with the current system that should be compared with the PAS simulation in Figure 3.11, is presented in Figure 3.27.

Figure 3.27: The simulated data with one target present with current model.
The simulated image shows similar behaviour to the one with PAS. One can see an area where the signal is stronger than the surrounding, where the plane is, and the blue area with noise. This data was also treated with pulse compression. The result of it is presented in Figure 3.28.

There is one major difference between Figure 3.12 and 3.28. One can see that the yellow area became much more narrow for the current model than for the PAS toolbox. By taking a cross section at the pulse index where the maximum signal strength occurs, the effect of pulse compression can be shown. It can be seen in Figure 3.29 how the signal increased in strength and became narrower after pulse compression.

After pulse compression, the position of the target is estimated at a distance of 3884.1 m. The difference between the simulated distance and the estimated were roughly 4 m. Using a large number of simulation (1000), the average difference between simulated and estimated distance
CHAPTER 3. RESULTS

3.2 Plane geometry

More points used results in a geometric shape that is more representative for a plane than just a single point. In Figure 3.30 is the new plane model consisting of six points shown together with the effect of different wing angles compared to the plane body.

Figure 3.30: The constructed plane with four different wing angles and fixed rotation angle.

Figure 3.30 shows that the plane, in this case, was 25 m long and that the wing angle could be alternating. It is also possible to see that the shape of the plane in Figure 3.30b is similar to the desired form in Figure 2.19. It is illustrated in Figure 3.31 how the appearance of the plane changes when it is rotated using a fixed wing angle of 110°.
3.2. PLANE GEOMETRY

CHAPTER 3. RESULTS

(a) Plane rotation 0°, and wing angle 110°.

(b) Plane rotation 90°, and wing angle 110°.

(c) Plane rotation 110°, and wing angle 110°.

(d) Plane rotation 180°, and wing angle 110°.

Figure 3.31: The constructed plane with fixed wing angle and four different rotation angles. The markers used is the same as in Figure 3.30.

Figure 3.31 shows how the plane could be rotated and approximately size of the plane. From 3.31b can the width be estimated to about 23 m, and from 3.31d, the length 25 m.

Using these points to simulate two plane targets at random location and thus get a graphical interpretation of the situation is shown in Figure 3.32. The possible locations were limited to be between 500 and 1000 meter from the radar in range. They had to be in front of the radar, i.e. in an interval of 180 degrees, but there was no restriction in plane orientation. For the sake of visibility in the figure the planes are enlarged, or at least large for the context, to one hundred meters. The planes had a fixed wing angle of 110°.
Figure 3.32: Two simulated targets with random rotation and position.

In Figure 3.32 is the radar placed at the centre and one can see how the planes are at different location with different rotations. One plane is approximately 1000 m from the radar and travels north west, while the other is at about 600 m away travelling south west.

A simulation at a fixed position range equals to 4219 m, bearing equals to 15.4, and elevation equals to 0.1 for 10 targets (both point and plane target) is presented in Figure 3.33. The dimension of the plane was 25 m in both length and width, the wing angle was $110^\circ$ and no rotation of the plane.
3.3 Bearing estimation

We used the new method of estimating the bearing angle on both the point target and the new plane model. The position was the same for both targets. Detection of a target occurs in one of the resolution cells. When using two different models for a target, the detection could be in either the same range bin, or different. We present the notations used for showing the differences between using point target and plane target with one range bin, together with using the adjacent range bins in Figure 3.34. All images in this section use the same notations.

Figure 3.33: The estimation of 10 targets simulated at one fixed location, both for point and plane target.

The red circle is the position where the target was placed, i.e. the simulated location. The blue crosses are the estimated position of the point target, while the black crosses represent the plane targets. One can see that the point targets are distributed over approximately 5 m in range, 0.25° in bearing angle, and no distribution in elevation. For the plane targets, the distribution was approximately 30 m in range, 0.23° in bearing angle, and 0.075° in elevation.
Figure 3.34: The notation used in order to see the effect of taking more range bins into account for estimate bearing angle.

This means that, if the colour of the solid and dashed lines are the same, the detection occurs at the same range bin. Figure 3.35 shows both cases when the detection occurs in same range bin and when they do not.

(a) Maximum signal power at same range bin for plane and point target.
3.3. BEARING ESTIMATION

(b) Maximum signal power at different range bin for plane and point target.

Figure 3.35: Two different cases for bearing estimation, one where the maximum signal occurs at same range bin and one where it do not.

Figure 3.35a shows that the detection occurs in the same range bin, the strength of the signal is stronger for the plane and that the signal is stronger in the range bin in front than behind. In 3.35b can one see that the detection occurs at different range bins. The point target is at 122, while the plane target is at 123. For further analysis it is easier to see the images in a different view.

Starting with the case where the detection occurs at the same range bin (Figure 3.35a). One can see in Figure 3.36 how the estimated angle is slightly to the left of the maximum value, as one could expect. Recall the quadratic interpolation described in Section 2.11 where the three largest points are used for estimation. Using the new method including the adjacent range bins results in a shift of the bearing angle to a higher value than before.
(a) Bearing angle estimation with current and new method.

(b) Enlargement of the peak in 3.36a.

Figure 3.36: The estimation in bearing angle with both current and new method, when maximum occurs at same range bin.
For the other case (Figure 3.35b) when the detection occurs in different range bins, the situation is similar to when they are in the same bins. In Figure 3.37 can one see that for the point target the highest and second highest values are almost the same. Hence, the estimated angle is between the two points. Another aspect is that the signal is stronger in the range bin in front when using the plane model, while it is stronger in the one behind for the point target.

(a) Bearing angle estimation with current and new method.
Figure 3.37: The estimation in bearing angle with both current and new method, when maximum occurs at different range bin.

Evaluation of the new method compared to the old was done with standard deviation and the results presented in Table 3.3. There were 1000 trials in total.

Table 3.3: Standard deviation of for plane and point target.

<table>
<thead>
<tr>
<th>Model</th>
<th>Point model</th>
<th>Plane model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current model</td>
<td>0.4136</td>
<td>1.2439</td>
</tr>
<tr>
<td>New estimation model</td>
<td>0.8251</td>
<td>0.5569</td>
</tr>
</tbody>
</table>

The standard deviation decreases for the plane target while it increases for the point target.
Chapter 4

Discussion

The discussion is presented in the same chronological order as the results with comments, thoughts, and other analysis of the results obtained.

4.1 Phase array system

The simulated waveform in Figure 3.1 clearly shows that the frequency changes with time. In the beginning of the wave the frequency is lower than at the end and thus we have up-chirp. Overall, the shape of waveform in Figure 3.1 and the pulse train in Figures 3.2 and 3.3 looks like what we were expecting for an LFM waveform. The simulated pulse train agrees well with what one could expect comparing it to Figure 2.5.

Figure 3.5 shows how the use of multiple antenna elements aligned in a URA creates a directed beam compared to one isotropic antenna element in Figure 3.4. The radiation pattern shows big side lobes that contributes to the signal. Continuing with the radiation pattern and study the effects of the tapering matrix one can see how the side lobes reduce. It is a clear difference in radiation pattern, both for the bearing angle and elevation angle. The side lobes have been reduced and are at the same level as we expected using a Taylor window for suppressing the side lobes.

The data simulated with the PAS toolbox in Figure 3.11 is similar to the one simulated with the current model presented in Figure 3.27. One difference is that the PAS toolbox uses the unit dB, while the current model has a normalization factor (normalized to noise floor). This is, of course, nothing that should affect the results but something to keep in mind when comparing. Otherwise, the received signal when simulated seems to be distributed over an equal amount of range bins This is what we expected since the received signal is related to the pulse length and the same waveform has been used. So far so good, but, when one study how the received signal gets affected by the pulse compression a large difference could be seen. The yellow area becomes narrower in range bin (compare Figure 3.27 and 3.28). It is even more clear in Figure 3.29 where one can see that the signal not only becomes narrower, the SNR also increased.

This is not the case for the PAS toolbox. Figure 3.14 and 3.16 shows that the SNR increased, but also that the width of the peak is almost twice as large as before. It is of course not a wanted behaviour, a wide peak result in an uncertainty in the estimation process. Anyhow, if one looks closely at the peaks in Figure 3.14 and 3.16 it is a smaller peak in the centre. These peaks are located at the left edge of the blue line, just like the current model. Then, it should
results in that, if we used these smaller peaks to estimate the range, we get a more accurate estimation. The overall difference in signal strength after pulse compression seems to be much larger for the current modal than for the PAS toolbox (compare Figure 3.29 with 3.14 and 3.16). But, as mention before, they are presented in different units so we have to look at the tendencies.

The integration process, both with and without the STC/TVG, reduced the background noise. In Figure 3.17 and 3.19, 3.21 and 3.23 can it be seen. One can see how the noise outside the peak is much smaller in the lower part of the image compared to the others. This is what we expected since the clutter from the ground is assumed to be statistical noise. The strange thing about these images is that all peaks seem to be the same (for every pulse on each target). We know that the received signal consists of contributions from both targets and noise. Therefore, the peaks should at least differ from each other with the size of the noise. One can interpret this as the PAS toolbox does not add noise when receiving the signal from a target, which seems strange because it is not a realistic situation. An explanation for not seeing this would be that the noise on the peaks is small enough to not be visible compared to the peaks. But, the noise for each pulse has an amplitude of approximately 10 dB, which should be more than enough to see a difference.

The 3D image in Figure 3.25 shows a good overview of the surveillance region, but one can see that something went wrong when calculating the threshold level. The peaks that exceed the threshold level is large and hence a bad estimation will be done. It is even clearer in Figure 3.26 where one can see that almost half of the peak exceeds the threshold level.

The time required for calculation with PAS compared to the current model was higher. For pulse compression, the time duration increased by almost a factor 10. Most likely this is caused by the creation of a system object and associate with coefficients. For the ADPL system, two pulse compressions are made which will have a big influence on the time duration for a large number of simulations.

Unfortunately, something went wrong somewhere in the simulation, or PAS did not work as intended, which leads to that the final outputs cannot be compared in a fair way. First of all, the range estimation did not provide a range correlated to a single target, but an entire interval. As mention before the peak become wider after pulse compression and its full-width corresponding to about 2000 m which compared to the range cell (about 50 m) is way to large. However, if only the maximum value at the sharpest peak is used (from where the range was estimated) one can see that it is about 5 dB higher than the adjacent values, which narrows down the range interval to 150 m. This is still three times larger than typical resolution cells, but way better than the 2000 m from the entire peak width. Secondly, one can see in Figure 3.26 that the way PAS calculate the threshold level way too low. The low threshold level results in not only the top of the tip exceeding but also a large part of it. One thing that should improve the result would be if pulse compression narrows the peak in the same way as the current model. Also, if the peak is narrower it does not matter as much if the threshold level should be too low since the adjacent values are at approximately the same range. One can see in the figure that the peaks have approximately the same width, which results in roughly the same interval regardless of threshold level as long as it exceeds $-90$ dB. In Figure 3.26 one can see that the peaks are narrower in the middle as mention before. By changing the calculated threshold from $-113$ dB to approximately $-85$ the answer would be more accurate than the calculated.

From one point of view is it hard to see how PAS can be used in a good way with this
4.2 Bearing estimation

Whether the new estimation method is more accurate for bearing angle determination compared to the current model is difficult to say. The standard deviation did not provide an unambiguous
CHAPTER 4. DISCUSSION  4.3. PLANE TARGET

answer. From Table 3.3 can one see how the standard deviation decreased for the plane target, while it increased for the point target. It might be caused by the fact that the plane is larger in space than the point target and therefore obtain a better estimation of the contribution of the two other range bin. The targets were located at the same position but if they are treated differently due the contribution of the two other range bin this could lead to that the point is more affected at short distances.

If the new method of estimating the bearing angle is better for the plane target than the current is even more difficult to investigate. First of all, we used more points, so the simulated plane has no exact location as the point target. We simulate these six points around a centre, which is used for reference when calculating the standard deviation. Then, in most cases, there is no symmetry between the points for the plane and the radar. So the problem with evaluating if the new method is better than the current for the plane target is that we have no point in the centre that could be used. Even with a point in the centre, there will be a contribution from the other points. Depending on which part of the plane that is closest to the radar or its rotation different bearing angle could be estimated. At least we can say that using the new method of estimating the bearing angle resulted in an answer closer to the simulated centre point.

Another effect of using the plane target instead of a point target can be seen in Figure 3.35. In (a) is the maximal signal strength in the same range bin, i.e. both black lines are at the same range. In (b) on the other hand can we see how the detection for the plane target occurs at one range bin behind (the black dashed line is at the same range as the blue solid) the point target. That this may happen is not really strange because the plane model has a larger spread in space than the point target. The plane size in these simulations was 25 meters in both length and width, which correspond to about a half resolution cell. With this in mind, it is easy to understand that if the plane is near the edges of the resolution cell, the detection may occur in a different cell than the point target. One can further assume that if the size of the plane increased, the probability of finding the plane in different cells increases. The detection should be restricted to one of the nearby cells, so the possible outcomes are: the detection is in the same cell, the detection is in any of the adjacent but at the same range bin, the detection is at the same bearing angle but different range bin, or the detection is both at a different range bin and bearing angle.

4.3 Plane target

The estimation in bearing with the quadratic interpolation described by equation (2.37) indicate that \( k' \) will be adjusted to a lower or higher angle depending on the signal strength of the two adjacent cells. This can be seen in Figure 3.37b. The left point is stronger than the right for both plane and point target (this is easier seen for plane (dashed lines) than for point target). However, for the plane target the signal stronger in the range bin in front (the red dashed line is larger than the blue). While for the point target the signal is stronger in the range bin behind. The effect of this is shown with the purple triangles. Compared to the green triangles that are from the current model where only one range bin used one can see a small shift in bearing angle estimation. One of them change to a higher value and the other to a lower.

The main reason for why they are affected differently is probably caused by the relationship between plane and the radar. If we first look in Figure 3.30a we see how the plane consist of six points instead of one. This will of course change where detection occurs from the centre of plane (point target) to one of the other point or multiple points.

For example, if the plane is in front of the radar, the front will be detected first and it will
be in the same angle as the point target. The only difference in this case is that the front is positioned a short distance closer to the radar than the point target. It can also be some contribution from the engines and wing apex that are symmetric. The same applies if the plane is rotated so that the rear part is straight in front of the radar or the wing apex (in the case of 90° wing angle). In all other situations the contribution of signal is from one, or multiple points and no symmetry. From Figure 3.30 and 3.31 is it possible to see which points that are of interest depending on the angle of incident waves. For most cases will it be a wing angle which is seen in Figure 3.30.

As mentioned the plane target consists of six points which are distributed in space over a volume. When using the current model for a plane target it is shown in Figure 3.33 how the estimated position in range and bearing angle are more spread than the point target. In the lower left part can it be seen how the point target is spread approximately 5 meters in range, while for the plane target is 30. However, this must be consider as a good result since the plane had the size of 25 m, and with the additional 5 m from the point target we end up with 30 m. So in that sense the estimation should be equally good for the plane target as for the point target. In the lower right part can we see that the same behaviour exist for bearing angle. The blue crosses are very close the red circle compared to the black ones. Actually, this is not strange at all because the plan is structured in such way that no points exists at that particular position. Therefore it is difficult to estimate how good this is compared to the point target. We can at least see that seven of the ten detections are at lower angle than the simulated. This is expected since the incident waves interact with one side of the plane and reflects from front, engine, and wing apex.

From Figure 3.33 can one also see that the estimation in elevation was lower than the simulated value for all points but one. The exact reason for this is hard to say, but it might be caused by the interpretation of the scanning beam. The low elevation angle will most likely result in detection in one of the lowest beams and it is possible that the outlying point is estimated from an other beam that have some overlap.
Chapter 5

Final remarks

5.1 Conclusion

During this thesis, we have investigated the possibilities to implement the PAS toolbox within the current model to increase its overall performance. The main task was to reconstruct the ADPL channel with functions from the toolbox, but the results were not better than the results with the current model. First of all, the simulation time increased with one order of magnitude for both ground and air channel. Then, the matched filter did not narrow the peak in the way one expects from the theory which resulted in problems with detection. Using the PAS toolbox for other purposes than in the signal processing were more successful. The design of the waveform and antenna were easy to implement, and the simulated received data is comparable to the one simulated in the current model.

A new plane model was successfully created with properties length, width, wing angles, and rotation. Using this plane in the current model one could state that the detection was in the size of the plane. One could also see that the plane target gets lower standard deviation when estimating the bearing angle while using more range bins. A tendency that not appear when using a point target.

5.2 Future work

In this thesis work all the points of the plane were treated equally. A more realistic situation would be to adjust the contribution from the different parts depending on how much they reflect and remove contributions from points that are not exposed to the electromagnetic pulse. That means that if the left side of the plane is faced to the radar, there will be no contributions from the right side. It would also be of interest to see if a classification system for identifying different parts, or even rotation. Since the sign of the Doppler shift tells if the plane approaches or departs from the radar which may be a good starting point for determining the rotation.

Other things one can further investigate are the dependencies of other variables. For this work, the elevation level was held constant for all simulations while a more realistic scenario would be if even this variable was random.

The waveform, antenna, and radiation pattern was successfully created in the PAS toolbox which opens up possibilities to continue work with these objects. By simulating new waveforms and URA panels one can develop new radar models in a price efficient way.
Bibliography


